THE

Ground of Arts,

TEACHING

The Perfect Worke

AND

Practice of Arithmetike.

Tables of

&c. &c. &c.

By M. Robert Recorde.

Imprinted at London in Pauls church yarde, at the Signe of the *Brasen Serpent, by R. Wolfe.*

M.D.LXXIII.

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To the moſt mighty PRINCE EDWARDE the ſixt, by the grace of God Kinge of Englande, Fraunce, and Irelande. &c.

The excellencye of mans nature, beynge ſuche, as it is by gods diuine fauoure (moſt mighty prince) not onely created in highneſſe of degree far aboue al other temporall thinges, but by perfection of reaſon and ſearche of witte muche approching toward the image of God, as not onely the holy ſcriptures do teſtify, but alſo thoſe naturall Philoſophers, whiche exactly did conſider the nature of man, & namelye the farre reache and infinite compaſſe of the workes of the minde, were inforced to confeſſe, that man ſcarſely was able to know himſelfe. And if he woulde duelye ponder the nature of himſelfe, he ſhoulde finde it ſo ſtrange,

that it might seme vnto hym a verye miracle. And thereof sprang that saying: Magnum miraculum est homo, maximum miraculum sapiens homo. For vndoubtedly as man is one of the greatest miracles that euer God wrought, so a wyse man is plainely the greatest.

And therefore was it, that some dyd accounte ẏ head of a man to be ẏ greatest miracle in the world, bicause not onely of ẏ strange workemanship that is in it, but muche more of the efficacy of reason, wit, memory, imagination, and suche other powers and woorkes of the minde, whiche can more easily conceaue any thing in a maner, than vnderstand it selfe. And amongst all the creatures of God it fyndeth none more difficulte to be perceyued then these same powers of it selfe, whereby it wthe conceaue and iudge, as it may be well coniectured by the diuersitye of opinions that the wisest Philosophers dyd vtter, touching the spirit of man and the substaunce of it: whereof at this present I intend to make no rehearsall, but who so listeth to reade thereof, maye finde it largely set forth not onely in Aristotle hys bookes De anima, but also in Galen hys booke called Historia Philosophica, and agayne in Plutarche his worke, De Philosophorum placitis,

citis, whose wordes are also repeated of Eusebius in the 15 booke, τῆς εὐαγγελικῆς προπαρασκευῆς, vnto whom I remit them that haue desire to vnderstande the intricate difficultie of knowing our owne selues, as touching our beste parte, and that parte whereby we deserue to beare the name of menne.

This matter seemed so obscure and difficulte, in knowledge, that Galen, who for his excellent wisdome and iudgement in naturall workes, is called of manye men à Miracle in nature, yet in searching the nature and substaunce of the spirite of man, hee not onelye confesseth himselfe ignorannte, but counteth plain temerity to attempt to finde it: so far aboue ỹ hope of mans knowledge is that parte wherby mã wth know & iudge of things. And although ỹ ignorant sort (which hate al things ỹ they know not) do litle esteme ỹ profoũdnes of mãs spirit, & of reason, the chiefe pwer & facultye of it, yet as there is à kinde of feare & obedience of all vnreasonable beastes vnto man by the worlinge pwer of God, so is there in those small reasoned persons à certaine kinde of reuerence towarde wisedome and reason, whiche they doe shewe oftentimes, & by power of perswasion are inforced to obeye reason,

wyll they nill they.

And hereby came it to passe, that the rudenesse of the firste age of man was brought vnto somme more ciuille trade, as it ys well declared by Cicero in the beginninge of his first boke, de inuentione Rhetorica, where hee saythe thus.

Nam fuit quoddam tempus quum in agris homines passim bestiarum more vagabantur, & sibi victu ferino vitam propagabant, nec ratione animi quicquam, sed plæraque viribus corporis administrabãt. Non dum diuinæ religionis, non humani officij ratio colebatur. Nemo legitimas viderat nuptias, non certos quisquam inspexerat liberos, non ius æquabile quid vtilitatis haberet, acceperat: ita propter errorem atque insciti̇am cœca ac temeraria dominatrix animi cupiditas, ad se explẽdam viribus corporis abutebatur, pernicioſissimis satellitibus. Quo tempore quidam, magnus videlicet vir & sapiens, cognouit quæ materia esset, & quanta ad maximas res opportunitas in animis inesset hominum, si quis eam posset elicere, & præcipiendo meliorem reddere. Qui dispersos homines in agris, & in tectis syluestribus abditos, ratione quadam compulit in vnum locum, & congregauit: & eos in vnamquanque rem inducens vtilem atque honestam primo propter insolentiam reclamantes. Deinde propter rationem atque orationem studiosius audientes, ex feris & immanibus mites reddidit, & mansuetos.

This long repetition of Tullies wordes wil seme tedious to them whiche loueth but little, and

and careth muche lesse for the knowledge of reason, but vnto your Maiesty (I dare saye) it is a delectable remembraunce, and vnto me it semed so pleasaunt, that I could scarse stay my pen from wrytinge all that mine eyes did so greedely read.

This sentence of Cicero am I lothe to translate into English, partely for that vnto youre Maiestye it needeth no translation, but especially knowing how farr ỹ grace of Tullies eloquence doth excell any Englishe mans tongue, and muche more exceedeth the baseneſſe of my barbarous style, yet for the fruite of the sentence, I hadde rather vnto my mere English country men vtter the rudenes of my translation, then to defraude them the benefite of so good a lesson, trusting, they will so learne to loue reason, that they will also gladlye and greedelye imbrace all good sciences that maye helpe to the iuste furniture of the same, when they consider that informed reason was the onelye instrumente, or at leaſte the chiefe meane to bringe men vnto ciuile regiment, from barbarous manners and beastlye conditions.

For the time was (sayth Tullie) that men wandred abroade in the fieldes vp and downe

like beastes, & vsed no better order in feedinge then they, so that by reasons rule they wrought nothing, but most of their doing dyd they atchieue by force of strengthe. At this time there was no iuste regard of religion toward God, nor of duety toward man. No man had sene right vse of mariage, nother did anye manne knowe their owne children from other, nor no man had felte the commodity of iust Lawes: so that through errour and ignoraunce, wilfull lust, lyke a blynde and heady ruler, abused bodyly strengthe as a moste mortall minister for the satisfyinge of his desire. At that time was there one, whiche not onely in power, but also in wyledome was greate, And he considered how that in the myndes of men was bothe apt instrumentes, and great occasion to the due accomplishement of moste weightye affayres, if a man coulde applye them to vse, and by teaching of rules, frame them to better trade. This manne with persuasion of reason, gathered into one place the people that were wanderinge about the feildes, and laye lurkinge in wylde cotages and woodes: And brynginge them into one common societye, dyd trade them to all suche thinges, as eyther weare profitable or honest, although not without

out repining at the first, by reson that they had not been so accustomed before. Yet at length through reason & perswasion of wordes they obeied him more diligently, and so of à wilde and cruell people, hee made them curteous and gentle.

Thus hathe Tullye set foorthe the efficacie of reason and perswasion, howe it was able to conuerte wylde people to à mylдnes, and to chaunge their furious cruelnes into gentle curtesye: were it not now à great reproche in thys oure time (when knowledge raygneth so large) that men shoulde shewe them selfe lesse obsequious to reason? Unlesse it maye bee thoughte, that nowe euerye manne hauing sufficient knowledge of him selfe, needeth not to hearken to the perswasion of others.

In dede he that thinketh himselfe wise, will not esteme the reason of any other, be he neuer so wise, so that of such one it may wel be said: Hee that thinketh himselfe wyser then hee is, may iustly be counted à double foole: wherfore such men are not to be permitted in open audience to talke, but must be put to silence, and made to giue eare to reason, which reason consisteth not in à multitude of woordes, heaped

rashelye togither, and applyed for one purpose, but reason is the expressinge of a iuste matter w̄ witty persuasions, furnished w̄ learned knowledge: suche knowledge had Moyses, beynge experte in all learninge of the Egyptians, as the Scriptures declare, and therefore was able to perswade the stubborne people of the Iewes, although not without greate payne. Suche knowledge and suche reasōs did Druys shewe, whiche was the firste lawemaker of all the west parte of Europe. Lyke reason & wisedome did Samolxis vse amongest the Gothes: Lycurgus vnto the Lacedemonians, Zaleucus to the Locrians, Solon to the Athenienses, and Dumwallo Molmitius 2000 yeres paste, amongeste the olde Britaines of this Realme. And thereby came it to passe, that theire lawes continued longe, till more perfect reason altered manye of them, and willfull power oppressed moste of them.

Druys was sōne to Kinge Sarron & succeded him in his kingdome.

At the beginninge when these wise men perceiued how harde it was to bringe the rude people to vnderstande reason, they iudged the beste meanes to attayne this honeste purpose to expende of learninge in euery kynde, for by learning, as Ouide sayeth: *Pectora molliscunt, asperitásque fugit*: Stoute stomackes we ware mylde,

mylde, and sharpe fiersenes is exiled. Therfore as Berosus wthe testify, Sarron, that was ỳ thirde Kinge ouer all this weste parte of Europe, for to bringe the people from beastly rage to manly reason, did erecte Schooles of liberal artes, which toke so good successe, ỳ his name continued in that sorte famous aboue 2000 yeares after: for Diodorus Siculus, whiche was in the time of Julius Cesar, maketh mẽtion of the learned men of the Celtes, and nameth them Sarronides, that is to say, Sarron his Scholers and folowers.

Sarron died before Christes byrthe, 1913 yeares, after he had raigned 48 yeares.

Amonge these artes ỳ thẽ were taught, some did informe the tongue, and make men able bothe to vtter aptely their minde, and also to perswade, as Grammar, Logyke, and Rhetorike, although not so curiously as in this time: some other did appertaine to the iuste order of partition of Landes, the trew vsinge of weyghtes, measures, and reckeninges in all sortes of bargaynes, and for order of buyldinge and sundry other vses, those were Arithmetike and Geometry. Agayne to incourage men to the honoure of God, they taught Astronomye, whereby the wonderfull workes of God were so manifestly set forthe, that no mans tongue nor penne can in lyke sorte expresse his infinite

power,

power, his vnspeakable wysedome, & his exceedinge goodnesse towarde man, wereby he doth bountifully prouide for mā all necessaries, not onely to lyue, but also to lyue pleasantly. And so was theire confidence in Gods prouidence strongly stayed, knowinge his goodnes to bee such, that hee woulde helpe man as hee coulde, and his power to bee so greate, that hee coulde do what he would: and thirdely his wisedome to be so pure, that he woulde doe nothinge, but that that was best. Beside these scyences they taught also Musike, which moste commonlye they didde applye partelye to Religious Seruices, to drawe men to delighte therein, and partely to Songes made of the manners of men, in prayse of Uertue, and discommendation of Uice, whereby it came to passe, that no man woulde displease them, nor doe anye thinge euill that might come to theyr hearing: for theyr songes did make euill men more abhorred in that tyme, then anye excommunication dothe in this tyme. The posterity of these Musitians continewe yet dothe in Wales and Irelande, called Bardes vnto this daye, by the aunciente name of Bardus, theyr fyrst founder.

This Bardus Druid

And as these sciences did increase, so dyd vertue

vertue increase thereby. Againe as these sciences did decay, so vertue loste her estimation, and consequently was little in vse: wherof to make a full declaration were a thing mete for a Prince to heare, but it would require a peculiare treatise. wherefore at thys presente I count it sufficient lightly to haue touched this matter in generall woordes, and to saye no more of the particularities thereof, but onelye touching one of those sciences, that is Arithmetike, by whiche not onely iuste partition of landes was made, but also touching buyinge & selling, all Assises, weightes, and measures were diuised, and all recknings and accountes driuen, yea by proportion of it were the true orders of Justice limitted, as Aristotle in his Ethikes dothe declare, and the degrees of estates in the common wealthe established. Althoughe that proportion bee called Geometricall, and not Arithmeticall, yet dooth that proportion appertayne to the arte of Arithmeticke: and in Arithmetike is taughte the Progression of suche proportions, and all thinges thereto belonging. Wherfore I may well saye, that seing Arithmetike is so manye waies needefull vnto the firste plantyng of a common wealth, it muste needes be as muche

dius the fifth king of the Celtes reigned 69 yeares and dyed 1832 yeares before Christ.

required

required to the preseruation of it also, for by the same meanes is any common wealthe continued, by whiche it was erected and established. And if I shall in smale matters in appearance, but in deede very weighty, put one exaumple or two, what shall wee saye for the Statutes of this Realme, whiche bee the onely staye of good order in manner nowe? As touching the measuring of grounde by lengthe and brodthe, ther is a good and an auncient Statute made by arte of Arithmetike, and nowe it shalbe to little vse, if by the same art it be not practised and tryed: for the assise of breade and drynke, the two most common and most necessary thinges for sustentation of man, there was a goodly ordinaunce in the lawe made, which by ignoraunce hath so growen out of knowledge and vse, that fewe men do vnderstande it, and therfore the Statute bokes wonderfully corrupted, and the commons cruelly oppressed: notwithstandinge some men haue written, that it is to doubtfull a matter to execute those Assises by those Statutes, by reason they depende of the standerde of the coyne, whiche is much chaunged from the state of that tyme whan those Statutes were made. Thus shall euerye man reade (that listeth) in the Abridgement of Statutes,

tutes, in the tytle of Weightes and measures, in the seuenth numbre of the Englysshe booke, where he shoulde haue translated a good ordinaunce, whiche is set foorthe in the Frenche booke: but no meruayle if the Abridgemente dothe omitte it, seyng the greate boke of Statutes doth omitte the same Statute, as it hath done diuers other very good Lawes. And this is the fruite of ignoraunce, to reiecte and condemne all that it vnderstandeth not, although they vse some clokes for it: but suche clokes as beynge allowed, might serue to repell all good Lawes, whiche God forbyd.

Againe there is an auncient ordre for Assise of fyrewood, and coales, whiche was renued not many yeares paste, and nowe how Auarice and Ignorance doth canuas that statute, it is to pitifull to talke of, and more myserable to feele.

Furthermore for the statute of coynage, and the standerde thereof, yf the people vnderstoode rightly the statute, they should not nor would not (as they often do) gather an excuse for theire folly thereby: but as I saide, these statutes by wisdome and good knowledge of Arithmeticke were made, and by the same muste they be continued. And let ignoraũce no more

meddle

meddle with the vse of them, then it did with the makinge of them. Oh in howe miserable case is that Realme, where the ministers and interpreters of the Lawes are destitute of all good sciences, which be ỹ keyes of the Lawes. Howe can they eyther make good Lawes or mayntayne them, that lacke that trew knowledge whereby to iudge them. And happy may that Realme be accompted, where the Prince himselfe is studious of learninge, and desireth to vnderstande equitie in all Lawes. Therfore moste happy are wee the louinge subiectes of youre maiestie, whiche may see in youre highnes not only suche towardnesse, but also suche knowledge of diuers artes, as seldome hathe ben seene in any Prince of suche yeares, whereby wee are inforced to conceaue this hope: Certaynly, that hee whiche in those yeares seketh knowledge when knowledge is leaste esteemed, and at suche an age can discerne theim to bee enemyes bothe to his royall person and to his Realmes, whiche labourde to withdrawe him from knowledge to exercise pastime, and from reasonable studye to idle or noysom pleasures, he must nedes when he commeth to more mature yeares bee a moste prudente Prince, a moste iuste Gouernour, and a righte Iudge,

not

not onely of his subiectes commonly, but also of the ministers of hys lawes, yea and of the lawes them selfe. And to be able to conceaue the true equity and exact vnderstanding of all his lawes and statutes, to the comforte of his good subiectes, and the confusion and reproche of them whiche laboure to obscure or peruerte the equitye of the same lawes and statutes. How some of those statutes maye be applyed to vse as well in this our time, as in anye other time, I haue peculiarely declared in this booke, and some other I haue omitted for iust considerations till I may offer them firste vnto your Maiestie, to waye them, as to your highnes shall seeme good: for manye thinges in them are not to be published without your highnes knowledge and approbation, namely bicause in them is declared all the rates of aloies for all standerds from one vnce vpward, with other misteries of mynte matters, and also most part of ỹ varieties of coynes ỹ haue ben currant in this your Maiesties realme, by the space almoste of vi. hundred yeares laste paste, and many of them that were currant in the time that the Romaines ruled here.

All whiche with the aunciente discription of England and Ireland, and my simple cen-

ſure of the ſame, I haue almoſte completed to be exhibited to your highnes. In the meane ſeaſon moſte humblye beſeeching youre Maieſtye, to accept this ſimple treatiſe, not worthie to be preſented to ſo highe a Prince, but that my lowely requeſt to your Maieſtye is, that this amongeſt other of my bookes maye paſſe vnder the protection of youre highneſſe, whom I beſeche God moſt earneſtlye, and daily, accordinge to my duety, to aduaunce in all honour and Princely regality, and to increaſe in all knowledge, iuſtice, & godly pollicy.

AMEN.

Your Maieſties moſt obedient ſubiecte and ſeruaunt, Robert Recorde.

TO THE LOVINGE READERS.

The Preface of M. Robert Recorde.

ORE OFTENtimes haue I lamẽted wyth my self the infortunate condition of England, seing so manye great Clerkes to aryse in sundrye other partes of the world, and so fewe to appeare in this our nation: where as for pregnancy of naturall wit (I thinke) few nations do excell English men. But I cannot impute the cause to any other thing, than to the contempt or misregard of learning. For as Englishe men are inferioure to no men in mother witte, so they passe all men in vaine pleasures to whiche they maye attaine wythoute greate paine or laboure: and are as slacke to anye neuer so greate commoditye, if there hange of it anye painefull study or trauelsome labour.

Howbeit, yet all men are not of that sorte, though

though the moste parte be, the more pitye it is: but of them that are so glad not onely wyth painefull study and studious payne to attaine learninge, but also with as great studye and paine to communicate their learning to other, and make all England (if it mought be) partakers of the same, the moste parte are suche, that vnneth they can suppoꝛte their owne necessarye charges, so that they are not able to beare any charges in doing of that good, that els they desire to do.

But a greater cause of lamentation is this: that when learned menne haue taken paynes to doe thinges for the ayde of the vnlearned, learle they shall be allowed for their well doing, but deryded and scorned, and so vtterly discouraged to take in hand anye like enterpriee againe. So that if any be found (as there are some) that doth fauour learning and learned wittes, and can bee contente to further knowledge, yea onely with their woorde, suche persons, though they be rare, yet shall they encourage learned menne to enterpriee some thinges, at the leaste, that Englande may reioyce of. And I haue good hope that England will (after shee hathe taken some litle taste of learning) not onely bringe forthe more fauou-

rers of it, but also suche learned men, that she shall be able to compare with any Realme in the worlde. But in the meane season, where so few regarders of learning are, how greatly they are to be esteemed that doe fauoure and further it, my pen will not suffice at full to declare.

Therefore gentle reader, where as I doe vppon moste iuste occasion iudge, yea and knowe assuredlye, that there be some men in this realme whiche bothe loue and also muche desyre to further good learninge, & yet am not well able to write their condigne prayse for the same, I thinke it better with silence to ouerpasse it, then other to saye to little of it, or to prouoke againste them the malice of suche other, whiche doe nothinge them selfe that is prayse worthye, and therefore can not abide to heare the praise of any other mannes good deede. And considringe theire great fauoure vnto learning, though I my selfe be not worthye to be reckned in the numbre of great learned men, yet am I bolde to put my selfe in preasse with suche abilitye as God hath lente mee, thoughe not with so great cunninge as manye men, yet with as greate affection as

anye

anye man, to healpe my countrye men, and will not ceaſe daylye (as muche as my ſmall abilitie will ſuffer mee) to endite ſome ſuche thinge, that ſhall bee to the inſtruction, though not of learned menne, yet at the leaſte of the vulgare ſorte, whoſe argument alwaies ſhall be ſuche, that it ſhall delite all learned wittes, though they do not learne anye great thinges out of it.

But to ſpeake of this preſent booke of Arithmetike, I dare not nor will not ſet it forthe with anie woordes, but remit it to the iudgement of all gentle readers, and namely ſuche as loue good learning, beſeeching them ſo to eſteme it, as it doth ſeme worthye. And ſo, eyther to accept the thing for it ſelfe, other at the leaſte to allowe my good endeuour. But I perceaue I neede not vſe anye perſwaſions vnto theim, whoſe gentle nature and fauourable mynde is readye to receaue thankefully, and interprete to the beſte all ſuche enterpriſes attempted for ſo good an ende, though the thinge dw not alwayes ſatiſfie mens expectation. Thys conſidered, did bolden mee to publiſhe abroade this little booke of the Arte of noumbringe, whiche if you ſhall receaue fauourablye, you ſhall encourage mee to

gratify you hereafter with som greater thing.

And as I iudge some men of so louinge a minde to their natiue countrie, ỳ they would muche reioyce to see it to prosper in good learning and wittye Artes, so I hope well of all the reste of English men, that they will be not vnmyndefull of hys due prayse, by whose meanes they are healped and furthered in anye thynge. Neyther ought they to esteeme this thinge of so little valewe, as manye men of lyttle discretion oftentymes dooe: for who so setteth small pryce by the wittye deuyse and knowledge of nombering, hee little considereth it to be the chiefe poynte (in manner) whereby men differ from all brute beastes: for as in all other thynges (almoste) beastes are partakers wyth vs; so in numberinge wee differ cleane from them, and in manner peculiarelye, sithe that in many thinges they excell vs againe.

The Foxe in crafty witte exceedeth moste men,
A Dogge in smelling hath no man his peere,
To foresight of weather if you looke then,
Many beastes excell man, this is cleere.
The wittinesse of Elephantes doth letters attayne,
But what cunning doth there in the Bee remayne?

The

The Emmet foreſeing the hardnes of winter,
Prouideth vitailes in tyme of ſummer.
The Nightingale, the Linet, the thruſhe, the larke,
In Muſicall harmony paſſe many à Clerke.
The Hedghogge of Aſtronomy ſeemeth to know,
And ſtoppeth his caue wher the winde doth blow.
The Spider in weauing ſuche arte doeth ſhowe,
No man can him mende, nor follow I trowe.
When a houſe will fall, the Myſe right quicke,
Flee thence before, can man do the like?

Manye thinges els of the wittinelle of beaſtes and bydes might I heare ſaye, ſaue that an other time of them I intende to write wherein they excell in manner all men, as it is dayly ſeene: but in numbre was there neuer beaſte founde ſo cunninge, ỳ coulde know or diſcerne one thinge from many: as by dayly experience you maye well conſider, when a Bytche hathe mannye Whelpes, or a Henne mannye Chyckens: and lykewayes of other what ſo euer they bee, take from them all their younge, ſauynge onelye one, and you ſhall perceaue playnelye, that they myſſe none, though they will reſiſte you in taking them away, and will ſeeke them againe if they maye

knowe where they be, but els they will neuer misse them truely: but take awaye that one that is left, and then will they crie and complaine: and restore to them that one, then are they pleased againe: so that of numbre this may I iustelye say. It is the onelie thing (almost) that seperateth man from beastes. Hee therefore that shall contempne numbre, he declareth himselfe as brutishe as à beaste, and vnworthie to be counted in the felowshippe of men. But I truste there is no man so foule ouerseene, though manie right smallye do it regarde.

Therfore will I now stay to write againſt ſuche, and returne againe to this my booke, whiche I haue written in the forme of à Dialogue, bicause I iudge that to bee the easiest waye of instruction, when the Scholer maye aske euery doubt orderly, and ỹ Maysster may aunswere to his question plainely.

Why the Authour wrote in Dialogue wyse.

Howbeit, I thinke not the contrary, but as it is easier to blame an other mans worke then to make the like, so there will bee some that will finde faulte, bicause I wright in à Dialogue: but as I coniecture, those shall be suche, as do not, cannot, either will not perceaue the reason of right teaching, and therfore

fore are vnmeete to be answered vnto, for such men with no reason will be satisfied.

And if any man obiecte that other bookes haue bene written of Arithmetike already so sufficiently, ỳ I needed not now to put pen to the booke, except I will condemne other mens writinges: to them I aunswere. That as I condemne no mans diligence, so I know that no one man can satisfie euery man: and therfore like as many doth esteeme greatlie other bookes, so I doubt not but some will like this my booke aboue anie other Englishe Arithmetike hitherto written, and namelie suche as shall lacke instructers, for whose sake I haue so plainely set forthe the examples, as no booke (that I haue seene) hath done hitherto: whiche thing shall be great ease to the rude readers.

Therefore gentle reader, though this booke can be but small ayde to the learned sorte, yet vnto the simple ignorannt (which nedeth most helpe) it maye be à good furtheraunce and meane vnto knowledge.

And though vnto the Kinge his Maiestye priuately I do it dedicate, yet I dout not (such is his clemencie) but that hee can be content, yea and muche desirous, that all his louinge

subiectes

ſubiectes ſhall take the vſe of it, and employe the ſame to their moſte profite: whiche thinge if I perceaue that they thankefully we, and receyue with as good will as it was writen, then will I ſhortely with no leſſe kinenelle ſet forthe ſuche introductions into Geometrye and Coſmography, as I haue at times promiſed, and as hitherto in Engliſhe hath not been enterpriſed, wherewith I dare ſaye, all honeſte heartes will be pleaſed, and all ſtudious wits greatly delighted.

I will ſaye no more, but lette euerye man iudge as hee ſhall ſee cauſe. And thus for this tyme I will ſtaye my penne, committinge you all to that trewe fountayne of perfecte number, whiche wrought the whole worlde by number and meaſure: he is Trinity in Unitye, and Unity in Trinity: to whome be all praiſe, honor, and glorye.

AMEN.

The contentes of the firſte *Dialogue.*

The declaration of the profite of Arithmetike.
Numeration with an eaſie and large Table.
Addition, Subtraction, Multiplication, Diuiſion: } with diuers examples, and ſom new fourmes of worſkinge.
Reduction with diuers declaratiõs of coynes, weightes, and meaſures of ſundry formes, nowe newly added to theſe other rules.
Progreſſion bothe Arithmeticall and Geometricall, with certayne queſtions touchinge the ſame.
The Golden rule, and the Backar rule, with diuers queſtions thereto belonginge.
The duble rule of proportion.
The rule of Felowſhip, bothe with Time, and without Time.
Unto all theſe is added theire proffe.

The ſeconde Dialogue.

The firſte v. kyndes of Arithmetike, wrought by Counters.
The common kindes of caſting accomptes after Marchantes faſhion and Auditors alſo.
Numbring by the hand, newely added.

The

The contentes of the second parte touchinge Fractions.

What á Fraction is.
Numeration in Fractions.
The order of worke in Fractions.
Multiplication.
Diuision.
Reduction of
- Diuers Fractions into one.
- Denomination in 3 varieties.
- Fractions of Fractions.
- Improper Fractions.
- Fractions to the smallest denomination.
- Fractions into other partes of thinges.

Agayne of Multiplication.
Duplation.
Agayne of Diuision.
Mediation.
Addition.
Subtraction.
The Golden rule with diuers questions.
The proofe of the Golden rule.
The Backer rule.
A question of Lone.
The Statute of Assise of Bread and Ale, recogniled and applyed to this time, with newe Tables thereto annexed.
The Statute of measuringe of Grounde, with á Table

à Table and questions.
Questions of Society, with the reason of the rule, and proofes of their workes.
To finde three numbers in any proportion.
The rule of Alligation with diuers questions, and the proofe of their workes, and manye variations of suche solutions.
The rule of Falsehode or false position, with diuers questions, and their proofe.

Before the Introduction of Arithmetike, these Figures must bee learned.

		1	2	3	4	5	6	7	8	9
Figures of Number.		j	ij	iij	iiij	v	vj	vij	viij	ix
10	x.									
20	xx.									
30	xxx.									
40	xl.									
50	l.									
60	lx.									
70	lxx.									
80	lxxx.									
90	xc.									
100	c.									

C. C. two hundred. &c.
D. fiue hundred. &c.
D. C. six hundred. &c.
M. à thousande. &c.

M.D.LXXIII.

c, à cee, the xvj.
q, à kewe, the viij. } parte of à penny.
q̃, à farthing, the iiij.
ob, an halfepenny.
j ō, à penny.
j s, à shillinge.
j li, à pounde.

Thou (ō God) hast ordred all thinges in Measure, Number & Weight. Sapientiæ. xi.

A dialogue betweene THE MAYSTER AND the Scholer, teachinge the arte and vse of Arithmetike with the pen.

THE SCHOLER SPEAKETH.

YR, SVCHE YS youre authoritye in mine estimation, ẏ I am content to cõsent to your saying, and to receaue it as truthe, though I see none other reason ẏ dothe leade me there vnto: whereas els in mine owne cõceit it appeareth but vaine, to bestow any time priuately in learning of that thinge, that euery childe may and dothe learne at all tymes and houres, when he dothe any thinge him selfe alone, and muche more when hee talketh or reasoneth wͭ other.

MAYSTER. Lo this is the fashion and chaunce of all them that seeke to defende theire

blinde ignorance: that when they thinke they haue made stronge reason for themselfe, than haue they proued the quyte contrarye. For yf numbringe be so common (as you graunte it to be) that no man can doe any thinge alone, and muche lesse talke or bargayne with other, but hee shall still haue to doe with Number: this proueth not Number to bee contemptible and vile, but rather right excellent and of high reputation, sith it is the grounde of al mens affayres, so that without it no tale cã be tolde, no communication without it can bee longe continued, no bargaining without it can duely bee ended, nor no busynesse that man hathe, iustely completed. These commodities (if there were none other) are sufficient to approue the worthinesse of Number. But there are other innumerable farre passing all these, whiche declare Number to exceede all praise. Wherfore, in all greate workes are Clerkes so much desired? Wherfore are Auditours so richly feede? What causeth Geometrians so highlye to bee enhaunced? Why are Astronomers so greatly aduaunced? Bycause that by Number suche thinges they doe finde, which elles shoulde farre excell mans mynde.

Scholer. Verelye Syr if it be so, that these men

men, by Numbring their cunning do attaine, at whose great woorkes moste men doe wonder, then I see wel I was much deceyued, and Numbring is a more cunninge thinge then I tooke it to be.

Mayster. If Numbre were so vyle a thinge as you did esteeme it, then neede it not to bee vsed so muche in mens communication. Exclude Numbre and aunswere me to this question. How many yeares olde are you?

Scholer. Mum.

Mayster. How many daies in a weeke? howe many weekes in à yeare? What landes hathe youre father? How many men dothe he keepe? Howe longe is it sythe you came from him to mee?

Scholer. Mum.

Mayster. So that if Numbre wante, you aunswere all by Mummes: How many myle to London?

Scholer. A poke full of Plummes.

Mayster. Why, thus may you see, what rule Numbre beareth, and that if Numbre bee lackinge, it maketh men dombe, so that to moste questions, they must aunswere Mum.

Scholer. This is the cause Syr, that I iudged it so vyle, bycause it is so common in tal-

king euery while: For plenty is not deinty, as the common sayeng is.

Mayster. No, nor Store is no sore: perceaue you this? The more common that a thinge is, being nedefully required, ỳ better is the thing, and the more to be desired. But in Numbring as some of it is light and playne, so the moste part is difficult, and not easy to atteine. The easyer parte serueth all men in commune, and ỳ other part requireth some learning. Wherefore as without Numbring a man can do almoste nothing, so with the help of it, you may attayne to all thinge.

Scholer. Yea syr? Why? then it were beste to learne the arte of Numbringe first of all other learning, and then à man neede learne no more, if all other come with it.

Mayster. Nay not so: but if it be firste learned, then shall a manne be able (I meane) to learne, perceaue, and attayne to other sciences, whiche without it, he shoulde neuer gette.

Scholer. I perceiue by your former wordes, ỳ Astronomie and Geometrie depende muche of the helpe of Numbringe, but that other sciences, as Musicke, Philicke, Lawe, & Grammar and suche like, haue any helpe of Arithmetike, I perceiue not.

Mayster.

Mayſt. I may perceaue youre great clerklines by the ordringe of youre ſciences: but I will let that paſſe nowe, bycauſe it toucheth not ỹ matter that I entende, and I will ſhew you how Arithmetike doth profit in all theſe, ſomewhat groſſely, accordinge to youre ſmall vnderſtanding, omittinge other reaſons more ſubſtantiall.

Firſte (as you recken them) Muſicke hathe not onely great healpe of Arithmetike, but is made & hath his perfectnes of it: for all Muſicke ſtandeth by Numbre and Proportion. Muſicke.

And in Phyſicke, beſide the calculation of Criticall dayes, with other thinges whiche I omitte, howe can anye man iudge the Pulſe rightlye, that is ignorant of the proportion of Numbres? Phiſicke.

And as for the Lawe, it is playne, that the man that is ignorant of Arithmetike, is neither meete to be a Judge, neyther an Aduocate, nor yet a Proctour. For howe can he well vnderſtande an other mans cauſe appertayninge to diſtribution of goods, or other dettes, or of ſummes of monye, if hee bee ignorant of Arithmetike? This oftentimes cauſeth righte to bee hindred, when the Judge either deliteth not to heare of a matter that hee perceaueth The lawe

not, either can not iudge it for lacke of vnderstandinge: This commeth by the ignorannce of Arithmetike.

Grammar

Now as for Grammar, me thinketh you should not doubt in what it needeth Numbre, sithe you haue learned ỳ Nounes of all sortes, Pronounes, Uerbes, and Participles, are distinct diuerlly by Numbres: besides the varietye of Nounes of Numbre, & Aduerbes. And if you take away Numbre from Grammer, then is all the quantity of Syllables lost. And many other wayes withe Nũbre helpe Grammer. Whereby were all kindes of meters found and made? Was it not by Numbre?

Philosophie.

But howe needefull Arithmetike is to all partes of Philosophye, they may sone see, that readeth either Aristotle, Plato, or anye other Philosophers writinge. For all their examples almoste, and their probatiõs depende of Arithmetik. It is the sayinge of Aristotle, that hee that is ignorante of Arithmetike, is meete for no sceyence. And Plato his Mayster wrote a like sentence ouer his Scholehouse dore. Let none enter in hither (quoth hee) that is ignorante of Geometrie. Seeynge hee woulde haue all his Scholers experte in Geometrie, muche rather hee woulde the same in Arithmetike, without

without whiche Geometry can not stande.

And how needefull Arithmetike is to Diuinity, it appeareth, seying so many Doctors gather so greate mysteries out of Number, and so muche doe write of it. And yt I shoulde go about to write all the commodities of Arithmetike in ciuille actes, as in gouernaunce of common weales in tyme of peace, and in due prouision and order of armyes in tyme of warre: For numbring of the hoste, summinge of their wages, prouisions of victayles, bewing of artillerye, with other armoure: Beside the cunningest poynt of all, for casting of grounde for encampinge of men, with suche other like. And howe many wayes also Arithmetike is conducible for all pryuate weales, of Lordes and all possessioners, of marchauntes, and all other occuppyers, and generally, for all estates of men, besides Auditours, Treasorers, receyuers, stewardes, bayliffes, and suche like, whose offices without Arithmetike is nothing. If I shoulde (I say) particularlye repeate all suche commodities of this noble science of Arithmetike, it were ynough to make a very greate booke. Diuinity.

Scholer. No, no syr, you shall not neede: For I doubt not, but this that you haue sayde,

were ynough to perſwade any man to thinke this arte to be right excellent and good, and ſo neceſſary for man, that (as I thinke nowe) ſo much as a man lacketh of it, ſo muche he lacketh of his ſenſe and witte.

Mayſter. What? are you ſo farre chaunged ſince, by hearing the fewe commodities in generall? By likelihode you wold be far chaũged, if you knew all ỹ cõmodities particular.

Scholer. I beſeche you ſir, reſerue thoſe commodities that reſt yet behinde, vnto theyre place more conuenient. And if yee will bee ſo good as to vtter at this tyme this excellent treaſure, ſo that I may be ſomewhat enriched thereby, and if euer I ſhall be able, I will requite your payne.

Mayſter. I am very glad of your requeſt, and I will doe it ſpeedely, ſith that to learne it you be ſo ready.

The duty of a ſcholer.

Scholer. And I to youre authoritye my witte do ſubdue, whatſoeuer you ſay, I take it for trewe.

Mayſter. That is to much, and meete for no man, to bee beleued in all thinges, without ſhewinge of reaſon. Though I might of my Scholer ſome credence require, yet except I ſhew reaſon, I do it not feare. But nowe ſith

Perſeue-

you

you are so earnestlye set this arte to attayne, best it is to omitte no tyme, leaste some other passion, coole this greate heate, and then you leaue of before you see the ende.

Scholer. Though manye there bee so vnconstant of minde, that flitter and turne with euery winde, whiche often beginne, and neuer come to the ende, I am none of their sorte, as I trust you partly know. For by my good will what I once beginne, tyll I haue it fully ended, I woulde neuer blyn.

Mayster. So haue I founde you hytherto in deede, and I truste you will increase rather then go backe. For better it were neuer to assaye, then to shrinke and flee in the middle way. But I truste you will not so do, therefore tell me briefly, What call you the science that you desyre so greately?

Scholer. Why syr? you know.

Mayster. That maketh no matter, I woulde heare whether you knowe, and therefore I aske you. For greate rebuke it were, to haue studied a science, and yet cā not tell how it is named.

Scholer. Some call it Arismetrik, and som Augrime. Arithmetike.

Mayster. And what dothe those names

betoken?

Scholer. That if it please you, of you woulde I learne.

Mayster. Bothe names are corruptlye written, Arsmetrike for Arithmetike, as the Greekes call it, and Augrim for Algorisme, as the Arabians sounde it, whiche bothe betoken the science of Numbringe. For Arithmos in Greeke, is called Number: and of it commeth Arithmetike, the arte of Numbringe. So that Arithmetike is à science or arte teachinge the manner & vse of Numbringe. This arte may be wrought diuersly, with Penne or wͤ Counters. But I will first shew you the workinge with the penne, and then the other in order.

Αριθμητική.

Αριθμός

Scholer. This I will remember. But how many thinges are to bee learned, to attayne this arte fully?

Mayster. There are reckened commonlye vij. partes or workes of it.

Numeration, Additiō, Subtraction, Multiplication, Diuision, Progression, and Extraction of rootes: to these some men adde Duplation, Triplacion, and Mediation. But as for these last three, they are contained vnder the other seuen. For Duplation and Triplation, are contained vnder Multiplication, as it shall appeare

appeare in their place. And Mediation is contayned vnder Diuision, as I will declare in his place also.

Scholer. Yet than there remayne the firste seuen kindes of numbringe.

Mayster. So there wthe: Howe be it, If I shall speake exactly of partes of Numbring, I must make but fiue of them: For Progression is a compounde operation of Addition, Multiplication and Diuision. And so is the extraction of Rootes. But it is no harme to name them as kindes seuerall, seynge they appeare to haue some seuerall worckinge. For it forceth not so much to contende for the Numbre of them, as for the due knowledge and practising of them.

Scholer. Than you wil that I shall name them as seuen kindes distinct. But nowe I desire you to instruct me in ỳ vse of eche of them.

Mayster. So will I, but it muste be one in order: for you may not learne the laste as sone as the firste: but you must learne them in that order, as I did rehearse them, yf you will learne them speedily and well.

Scholer. Euen as you please. Than to begyn, Numeration is the firste in order: what shall I we with it?

Mayster.

Mayster. Firste you muste knowe what the thinge is, and than after learne the vse of the same.

NVMERATION.

NVMERATION, is that Arithmeticall skyll, whereby we may duely valew, expresse and reade anye Number or summe propounded: or els in apte figures and places, set wwne any Number knowen or named.

Scholer. Why? than me thinketh you put a difference betwene the valew and ỹ figures?

Mayster. Yea so doe I: For the valewe is one thing, and the figures are an other thing: and that commeth partely by the diuersitye of figures, but cheifly of the places wherein they be set.

Scholer. Than I muste knowe here three things: the Valewe, the Figure, and ỹ Place.

Mayster. Euen so: but yet adde Order to them as the fourth. And fyrst marke, that there are but ten Figures, that are vsed in Arithmetike: and of those ten, one doeth signify nothinge, whiche is made like an o, and is called priuatelye

priuately à Cypher, though all the other some- A Cipher
time be likewise named. The other nyne are
called Signifyinge figures, & be thus figured. Figures.

1 2 3 4 5 6 7 8 9

And this is their valewe.

j. ij. iij. iiij. v. vj. vij. viij. ix.

But here muste you marke, that euery Fi- ☞
gure hathe two valewes: One alwayes cer-
taine, that it signifieth properly, whiche it hath
of his Forme: and ỳ other vncertayne, whiche
he taketh of his Place.

A Place is called the seate or rowme that à A place.
Figure standeth in. And loke how many Fi-
gures are written in one summe, so many pla-
ces hathe that whole numbre. And the fyrste
place muste be called that, that is nexte to the
right hande, and so reckening by order towarde
the lefte hande, so that that place is laste, that
is next to the lefte hande. As for exaumple: If
there stode before you six men in a rowe, side
by side, and you shoulde tell them as they stãde
in order, beginninge with the man that were
next to youre right hande: than hee that were
nexte him should bee called the seconde, and so
forthe to the farthest from youre right hande,

whiche

whiche is the firt and the laste.

Scholer. Sir I perceaue you well: so might I recken letters or any other thinge. As if I shoulde write. viij. letters after this order, a, b, c, d, e, f, g, h, nowe must I saye: h, is the firste, g the ij, f the iij, e the iiij, d the v, c the vj, b the seuenth, and a the eight.

Mayster. That is well done. And after the same sorte vse hereafter, that what I declare by one example, the you expresse by an other, and so I shall perceaue whether you vnderstande it or no. And so passe ouer nothinge, till you perceaue it well, and be experte therein.

Scholer. Syr, I praye you howe many of these places be there in all?

Mayster. There is no certaine numbre of them, but they are somtimes more & somtimes fewer, accordinge to the sum that is expressed. ☞ For so many as the figures are, so many are ẏ places: and the laste place is so called, not bycause it is laste of all other, but it is ẏ laste of that present summe, and it may bee the middle place in an other summe.

Scholer. Me seemeth I perceaue this very well, as touchinge the order of reckeninge of the places: But as for the numbre of them, you saye there is no certainty. Nowe there resteth

to

to declare the valew of the figures by diuersity of places, which you called, The valewe vncertaine.

Mayster. But firste let me heare whether you know perfectly the certayne valewe. Valewe vncertaine.

Scholer. Yes syr as you wrote them, so I marked them.

Mayster. Howe write you then fiue?

Scholer. By this figure. 5.

Mayster. And howe sixe?

Scholer. Thus. 6.

Mayster. Write these three numbers eche by it selfe as I speake them. vij. iiij. iij.

Scholer. 7. 4. 3.

Mayster. How write you these four other, ij, j, ix, viij?

Scholer. Thus (I trowe) 2, 1, 6, 8.

Mayster. Nay, there you misse: Looke on mine exaumple agayne.

Scholer. Syr troth it is, I was to blame, I toke 6 for 9, but I will be warer hereafter.

Mayster. Nowe then take heede, these certayne valewes euery figure representeth, when it is alone written without other figures ioyned to him. And also when it is in the firste place, though manye other doe followe: as for exaumple: This figure 9 is ix. standinge nowe alone.

alone.

Scholer. How? is he alone and ſtandeth in the middle of ſo many letters?

Mayſter. The letters are none of his fellowes. For if you were in Fraunce in ỹ middle of a M. Frenche men, if there weare none Engliſhe man with you, you woulde recken youre ſelfe to be alone.

Scholer. So it is. Then 9 without more figures of Arithmetike, betokeneth ix, whatſoeuer other letters be about it.

Mayſter. Euen ſo, and ſo doth it, if it be in the fyrſte place ioyned with other, howe many ſoeuer do followe, as in this example, 3679. you ſee 9 in the firſte place, and dothe betoken nyne, as if he were alone.

Scholer. I perceaue that. And dothe not 7 that ſtandeth in the ſeconde place, betoken vij? and 6 in the thirde place, betoken vj? And ſo in the fourth place, betoken three?

Mayſter. Theire places bee as you haue ſaide, but their valewes are not ſo. For as in ỹ fyrſte place euery figure betokeneth his owne ☞ valewe certayne onely, ſo in the ſeconde place euery figure betokeneth his owne valewe certaine x. times: as in the example, 7 in the ſeconde place is vij. times x. that is, lxx. And in the

the thirde place, euery Figure betokeneth his owne valewe à hundreth tymes, so that 6 in ỳ place betokeneth vj. C. And in the iiij. place, euerye Figure betokeneth his owne valewe à M. times, as in the foreslaide number 3. in the fourth place, stãdeth for iij. M. And in the fifth place, euerye figure standeth for his owne valew. x. M. times. And in the vj. place, à C. M. tymes. And in the vij. place, à M. M. tymes. And in the viij. place, x. M. M. times: so that euery place exceedeth the former, ten tymes.

Sc. As thus: if I make this Numbre at all aduentures, 91359684, here are viij. places. In the firste place is 4, ỳ betokeneth but foure: in the seconde place is 8, and betokeneth x. tymes eyght, that is, lxxx. In the thirde place is 6, and betokeneth, vj. C. In the fourth place 9. is ix. M. And 5 in the v. place, is x. M. times 5, that is, l. M. So 3 in ỳ firt place, is à C. M. times 3. that is CCC. M. Then one in the vij. place, à M. M. And 9 in ỳ viij. place. x. M. M. tymes 9, that is, xc. M. M. But nowe I canne not easily nor quickly reade it in order. A general rule.

Mayster. That shall you practise by thys meanes. Firste put à pricke ouer the fourth Figure, and so ouer the vij. And (if you haue so many) ouer the x, xiij, xvj, and so forthe, still

leauing two figures betweene eche two pricks. And those two roomes betweene the prickes, are called Ternaries.

Ternarie. Trinity.

Then begin at the laste pricke, and see how many figures are betweene him and the ende, whiche can not passe three, reckeninge himselfe for one: then pronounce them as if they were written alone from the reste, and adde at the ende of their valewe so many times thousand, as your number hath prickes.

After that come to the nexte three figures, & sounde them as if they were à parte from the reste, and adde to theire valew so many times thousandes, as there are prickes betweene them and the first place of your whole number. And so doe by euery other, iij. figures folowinge, if you haue mo. As in example, 91359684. this was your number.

Put a pricke ouer 9 in the fourth place, and ouer one in the bij. place, and then no more, (for youre places come not to tenne) as thus:

9̇1359̇684.

Now go to the laste pricke ouer 1, and take it and the figure 9 that foloweth it, and valew them alone.

Scholer. 91 that xcj.

Mayster. So is it: but than adde for the numbre

numbre of your prickes twise M.

Scholer. That is xcj. thousand thousande.

Mayster. So is it. Then take the three other figures from one to the nexte pricke, and valewe them.

Scholer. 359. that is CCC.lix.

Ma. Nowe adde for the one pricke, that is betweene them and the first place, M.

Scholer. CCC.lix. thousande.

Mayster. Then come to the other three figures that remayne.

Scholer. 684. that is, vj C.lxxxiiij.

Ma. Nowe haue you valewed all. And at the ende of this laste numbre you shall adde no thinge, bycause there remayneth no pricke nor numbre after it: yet proue in an other numbre, as thus. 230864089105340.

Sc. 230864089105340. I haue pricked them as you taughte mee: but I am in doubte, whether I haue wne well or no, because of the Cyphers: For I remembre, you tolde mee that they we signifye nothinge, and therefore I wubte whether I shoulde recken them for a figure in setting of the prickes: and againe I knowe not wherfore they serue.

Mayst. That will I tel you nowe. In deede

The vse of Cyphers.

they are of no valewe themselfe, but they serue to make vp numbre of places, and so maketh the figure folowinge them, to bee in à further place, and therefore to signify the more valew: as in this example, 90, the Cypher is of no valewe, but yet hee occupieth the firste place, and causeth 9 to bee in the seconde place, and so to signifye x. times 9, that is xc. so that two Cyphers, thrusteth the figure folowing them, into the thirde place, and so forthe.

Sc. Than I perceaue in the example aboue, I haue pricked well ynough: for though that Cyphar that is pricked signifye nothinge, yet must he haue the pricke, bycause he came in the xiij. place. Than will I proue to numbre that summe. Fyrst there is 230 M. M. M. M. and then foloweth 864 M. M. M. And what shal I nowe we? There is à Cyphar in the thirde place, and no figure after him, but they that I haue reckened.

Ma. Hee did serue for them that you haue allready reckened, to make them in à place further then they should be if he were away: and therefore nowe you shall let him go. And so we alwayes when he occuppeth ỳ place next before any pricke, which is ỳ laste of ỳ Ternary, and à cyphre in ỳ laste place wth nothinge.

Schol.

Sc. Then shall I say but 89 thousande M.

Ma. So: but go forth.

Sc. 105 thousand. Now are all my pricks spente, and yet remaine 340, so that I muste valewe them CCC.xl.onely.

Ma. Now can you recken after this sorte: And remembre that euerye suche rowme so parted, is called à Ternary or Trinity. Trinitie.

Some doe parte suche great numbres with letters, after this manner.

c b a c b a c b a c b a f b a

230864089105340. In whiche exaumple yee may see, ỳ a supplieth the roume of youre pricke. And some we parte the numbres with lynes, after this forme.

230|864|089|105|340. where you see as manye lines as you made prickes, & to one intente, saue that the lines withe more playnelye parte euerye three figures, accordinge as they shoulde be valewed vnder one denomination.

Sc. Yea syr, but if you shoulde shewe me à numbre so parted, I shoulde take it for manye numbres, and not for one.

Ma. So might you we, not knowinge my meaninge. But what if I did set forthe ỳ numbre without lynes, and youre selfe (for the ease of reckeninge) did so parte it with lines, wolde

you forget wherefore ye did it, and then take them for many numbres?

Sc. No I trowe not, but yet I wubte.

Ma. Then vse that that you like beste, for all the three wayes are to one intent, saue (as I sayd) ỹ the lines we more playnlie distincte the Denominations.

Denominations.

Sc. What call you Denominations?

Ma. It is the laste valewe or name added to anye summe. As when I saye: CC xxij. poundes: poundes is the Denomination. And likewayes in sayinge: 25 men, men is the denomination, and so of other. But in this place (that I spake of before) the laste numbre of euery Ternary, is the denomination of it. As of the firste ternary, the denomination is vnites: and of the seconde ternary, the denomination is thoulandes: and of the thirde ternary, thoulande thoulandes, or millions: of the iiij. thoulande thoulande thoulands, or thoulande Millions: and so forth.

Sc. And what shall I call the valew of the iij. figures that may be pronounced before the denominators? as in saying: 203000000. that is CC iij. millions. I perceiue by your wordes that millions is the Denomination: but what shall I call the CC iij. ioyned before the Millions?

Mayst.

Mayster. That is called the Numerator or valewer, & the whole summe that resulteth of them bothe, is called the Summe, valewe or number. Numerator. Valewer. Summe.

Schol. Now, is there any thinge else to be learned in Numeration? or else haue I learned it fully?

Ma. I might here shewe you who where ỹ first Inuentors of this arte, and the reasons of all these thinges that I haue taught you, but that will I reserue till yee haue learned ouer all the practise of this arte, leaste I should trouble your witte with ouer many thinges at the firste.

But yet this must you marke, that there are three kindes of numbre: one called digits: another articles: and the thirde mixte numbers. Three kindes of number.

A Digit is any number vnder 10, as this: Digites.

1, 2, 3, 4, 5, 6, 7, 8, 9.

And 10 with all other that may bee diuided into x partes iuste, and nothing remaine, are called Articles: suche Articles.

are 10, 20, 30, 40, 50, &c. 100, 200, &c. 1000. &c.

And that number is called mixt, that containeth articles, or at the leaste one article and a digitte: as 12, 16, 19, 21, 38, 107, 1009. and so forth. And for the more ease of vnderstandinge Mixte.

and remembrãce, marke this: The diget numbre is neuer written with more thẽ one figure, but the article and the myxte number are euer written with more then one figure. And thus they differ, that the article hath euermore thys cyphre ○, in his firste place: and the myxt number hath euer there some diget.

Scholer. By theſe laſte wordes, I perceaue it muche better than I did before, and now (I thinke) I will neuer miſſe to know thoſe three à ſunder.

Mayſter. If you remember nowe all that I haue ſayde, you haue learned ſufficientlye this firſte kinde of Arithmetike, called Numeration. Howbeit, I will yet exhorte you now, to remember bothe this that I haue ſayde, and all that I ſhall ſaye, and to exercise your ſelfe in the practiſe of it: For rules whout practiſe, are but à lyght knowledge: and practiſe it is, that maketh men perfecte & prompte in all thinges.

Vſe maketh maiſtery.

And as you haue learned to gather and expreſſe the valewe of à ſumme propounded & ſet downe before you: ſo muſte you practiſe to mark, note, or write downe, with apt Figures, and in due places, any number, onely named or recited to you, or of your ſelfe imagined. as for à proofe: Howe note you, or write downe

this

this ſumme, Fiue thouſande, two hundreth, fifty and ſeauen.

Scholer. This troubleth mee nowe, weether I ſhoulde beginne at the fyrſte figure or at the laſte. For reaſon (me thynketh) ſhoulde cauſe me to beginne at the firſte: and yet if I write it as you ſpake it, I muſt beginne at the laſte.

Mayſter. When you knowe youre places perfectly, you may beginne where you liſt. But the more eaſe for youre hande is to begin with the laſte, that is to ſay, as I did ſpeake them. Yet for the more ſurety, a whyle you maye begynne with the firſte, repetinge my wordes backewarde, thus: Seuen, fyfty, two hundred, fyue thouſande: or elſe ſoundinge them all by their dygit or valower: as thus: Seuen, fyue, two, fyue: for that waye is eaſieſt. But than muſte you looke well, whether there bee any Cypher in youre ſumme, that hee may be ſette in his place. As if youre laſte valewer of youre ſumme (as you ſpeake it) bee about 9, than is there a Cypher in the fyrſte place. And if it bee a hundred or aboue, than is there twoo Cyphers, one in the firſte place, and an other in the ſeconde, and ſo forthe.

But bicause this thinge is suche that canne not be sette forthe without manie wordes, I thinke best here now at the ende of Numeration, to adde à table easie, and ready for the first exercise of it. *Lo, this is that table.*

The left side or hande.

The names of the digits, valewes certaine, or valowers.

The denominatours of the place or valewe vncertain	Unities.	Tennes.	Hundredes.	Thousandes.	r. Thousandes.	Millions.	C. of thousandes.	r. of millions.	C. of millions.	H. of millions.	r. H. of millions.
Nine.	9	9	9	9	9	9	9	9	9	9	9
Eyght.	8	8	8	8	8	8	8	8	8	8	8
Seuen.	7	7	7	7	7	7	7	7	7	7	7
Sixe.	6	6	6	6	6	6	6	6	6	6	6
Fiue.	5	5	5	5	5	5	5	5	5	5	5
Foure	4	4	4	4	4	4	4	4	4	4	4
Three.	3	3	3	3	3	3	3	3	3	3	3
Two.	2	2	2	2	2	2	2	2	2	2	2
One.	1	1	1	1	1	1	1	1	1	1	1
Ciphar.	0	0	0	0	0	0	0	0	0	0	0
The order of the places.	Firste.	Seconde.	Thirde.	Fourthe.	Fifte.	Sixte.	Seuenth.	Eyghteth.	Ninth.	Tenth.	Eleuenth.

The right side or hande.

This

This table (as you maye see) hathe aleuen places, and in eche of them are sette all the digites, whose certayne valew is witten on the right hand of the table, and the valewe vncertaine on the lefte hande. So that by this table you may learne both howe to expresse any nũbre that you liste, (if that it exceede not aleuen places) that is to say, lxxxx. thousande millions, and so may you by the helpe of it, valewe all summes proposed vnder the saide numbre.

For example: Take the summe that I proposed before, whiche was fiue thousande, two hundred, fifty and seuen. And if you will expresse it, take the firste numbre (as I speake it) whiche is fiue M. whose valower or certayne valew is .v. and his vncertaine valewe or denomination is M. First you shall seeke at the right hand the valower .v. Than seeke alonge vnder the title of denomination towarde the lefte hande, till ye finde thousandes; and vnder it right at the foote of the Table, is the numbre of the place, that is the fourth, wherein you must write your digitte or valower, fiue.

Afterwarde come to the second part of ỹ nũbre, two hundred, whose valower is .ii. and his denomination C. Seeke two at ỹ right hande

of the table, and go alonge vnder the denominations, towarde the left hande, till you come vnder C: than looke to the foote of the table, & there shall you see the numbre of the place, y is to saye, iij, wherein you must set youre digette 2.

Than woe so by youre other two numbres that remaine, and you shall finde fiue in the seconde place for youre fiftye, and 7 in the firste place for youre seuen. And thus may you woe with other numbres.

Scholer. Mayster I thanke you hartelye. I perceaue you seeke to instructe mee moste plainly & briefly, and not to hyde your knowledge with subtile wordes as many do. For this rule is so plaine, that I can desire it no playner. And though it seeme somwhat longe, yet I perceaue it to be a sure way.

Mayster. So is it, & though it bee longe, yet it is neyther to longe, neither to playne for yong learners that lacke practise: for this table is in steede of a teacher, to them that lacke one. But now I trust I haue sayd ynough of Numeration; whiche after you haue well practised, then may you learne forthe.

Why nũbers are placed

Scholer. Yet I praye you in one thinge to tell mee your iudgemente. Why we men reck-

ken

kni the ordre of the places backewarde, from the right hande to the left? backewarde.

Mayster. In that thinge all men wagree, that the Chaldays, whiche firste inuented this arte, did set these figures as they set all theire letters: for they wryte backewarde as you terme it, and so we they reade. And that maye appeare in all Hebrewe, Chaldaye and Arabyke bookes, for they bee not onely writtē from the righte hande to the lefte, and so muste bee read, but also the right ende of the booke is the beginninge of it: where as the Greekes, Latines, and all nations of Europe, wwrite and reade from the lefte hande towarde the righte: And all theyr bookes begin at the lefte side.

Scholer. That reason wthe satisfye mee.

Mayster. Jt nother satisfieth mee, nother liketh mee well, bycause J see that the Chaldays & Hebrewes woe not so vse theire owne numbres, as at an other tyme J will declare. But this playne reason may best satisfye you presently: That seeynge in pronouncinge of numbres wee keepe ỹ order of oure owne readinge, from the lefte hande to the ryght: And agayne, wee we euer name the greater numbres before the smaller: it was reason, that the lesser places contayninge the lesser numbres, shoulde

should be set on the right hand, and the greater places cõtaining the greater numbers, to proceede towarde the left hand.

Scholer. This reason to me is so playne, that it seemeth now against reason to make à doubt of that order. So that nowe for Numeration, I am clearely satisfied: so that onely practise shall make me fully ready and experte in it. And in the meane season, I desyre to learne the other kindes of Arithmetike.

Mayster. That is well saide: but what should you next learne, can you tell?

Scholer. I remember, you said that Addition was next.

Mayster. Euen so: and what that is, must you first knowe.

ADDITION.

Addition is the gathering together and bringinge of two numbers or more, into one totall summe: as if I haue 160 bookes in the Latine tongue, & 136 in the Greeke tongue, and would know how many they bee in all, I must write those two numbers, one ouer an other, writinge the greatest number highest

highest, so that the first figure of the one, be vnder the firste figure of the other. And the seconde vnder the seconde, and so forth in order.

When you haue so done, draw vnder them a right line, then will they stande thus.

Now begin at the firste places, toward the right hand alwaies, and put together the. ij. first figures of those two numbers, and looke what cōmeth of them, write vnder them, right vnder the line. As in saying, 6 and 0, is 6. Write 6 vnder 6: as thus.

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160
136
---
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```
160
136
---
  6
```

And then go to the seconde figures, and do likewise: as in sayinge, 3 and 6 is 9: write 9 vnder 6 & 3, as here you see.

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160
136
---
 96
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And likewise doe you with the figures that be in the thirde place, sayinge: 1 and 1 bee 2: write 2 vnder them, and then will your whole sum appeare thus.

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160
136
---
296
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So that now you see, that 160, and 136 doe make in all, 296.

Scholer. What? this is very easy to doe, me thinketh I can do it euen sithe.

There came through Cheapeside two droues of cattell: in the firste was 848 sheepe, and

and in the seconde was 186 other beastes.

	848
	186

Those two summes I muste write as you taught mee, thus.

Than if I put the two firste figures together, sayinge: 6 and 8 they make 14. That must I write vnder 6 and 8, thus.

	848
	186
	14

Mayster. Not so, and here are you twyse deceaued. Fyrst, in goyng about to adde together two summes of sundry thinges, whiche you ought not to doe, except you seeke onelye the number of them, & care not for the thinges. For the summe that shoulde result of that addition, shoulde be a summe nother of sheepe nor other beastes, but a confused summe of bothe. Howbeit sometimes yee shall haue summes of diuers denominations to bee added, of whiche I will tell you anone: but firste I wyll shewe you, where you weare deceyued in an other poynte, and that was in writinge 14, (whiche came of 6 and 8) vnder 6 and 8, whiche is impossible. For howe can two figures of two places bee written vnder one figure and one place?

Scholer. Truthe it is: but yet I did so vnderstande you.

Mayster.

Mayster. I saie in deede that you shoulde write that vnder them, that did resulte of them bothe together: which saying is alwaies trew, if that summe do not exceede a Digit. But if it be a mixt number, then muste you write the digit of it vnder your figures, as I haue saide before: but and if it be an Article, then write a vnder them, and in bothe sortes you shall keepe the article in your minde. And therfore when you haue added your second figures, which occupy the place of tennes, you shall put that 1 thereto, whiche you kepte in your minde: for though it were ten in deede, yet in that place it is but as one, bicause that euery 1 of that place is ten, for it is the place of tennes. And in like manner: if you haue in the seconde place so greate a number, that it amounteth aboue 9, than write the digites, and reserue the article in your minde, euer adding it to the next place followinge: and so of all other places, howe manye so euer you haue. And if you haue a mixt number, when you haue added your last figures, then write the digite vnder the laste figures, and the article in the nexte place beyonde them: so shall your number resulting of addition, haue one place more than the numbers whiche you shoulde adde together.

Scholer. Now do I perceaue you, and the reason of this is, (as I vnderstande) bicause that no one place can containe aboue 9, which is the greatest figure that is, & then all tennes or articles muste be put to the next place folowing: for euery place (as I may see) exceedeth the other place next before him, by .x.

Nowe (if it please you) I will returne to my example of cattell. But I remember you sayde, I myght not adde summes of sundry thinges togither, and that myght I see by reason.

Mayster. Truthe it is, if you seeke the due summe of any thinges, but if you onely seeke a bare summe, & haue no respect to the thinge, than were it better to name the summe onelye without any thinge, as in saying 8 4 8. without naming sheepe or any thinge els. And like wales 1 8 6, naming nothing.

Now let mee see: how can you adde those two summes.

Scholer. I must first set them so, that the two first figures stande one ouer an other, and the other eche one ouer his fellowe of the same place: than shall I drawe a lyne vnder them bothe. And so likewise of other figures, setting alwaies the greatest number highest,

thus

thus,

Than must I adde 6 to 8, whiche maketh 14, that is mixte number: therefore must I take the digit whiche is 4, & write it vnder 6 & 8, keping ỹ article 1 in my mind, thus.

Next that we I come to the seconde figures, adding them together, saying: 8 and 4, make 12, to whiche I put the one reserued in my mind, and that maketh 13, of whiche number I write the digit 3 vnder 8 and 4, and keepe the article in my minde, thus.

Than come I to the third figures saying: 1 and 8, make 9, and 1 in my minde maketh 10. Sir shall I write the ciphar vnder 1 and 8? Ma. Yea.

Scholer. Than of 10 I wryte the ciphar vnder 1 and 8, & keepe the article in my minde.

Mayster. What needeth that, seinge there followeth no more figures?

Scholer. Sir, I had forgotten, but I will remember better hereafter. Then seinge I am come to the laste figures, I muste write the ciphar vnder them, and the article in a farther place after the ciphar, thus.

Mayster. So now ye see, that of 848, and 186 added together, there amounteth 1034.

Scholer. Nowe I thinke I am perfect in Addition.

Ma. That will I proue by this example.

There are two armies of soldiours: in the one are 106800, and in the other 9400: How many are there in both armies, say you?

Scholer. Firste I set them one ouer an other, beginninge with the firste numbres, at ỹ right hand, thus. But the nether numbre will not matche the ouer numbre.

106800
9400

Mayster. That forceth not.

Scholer. Than we I adde 0 to 0, and there amounteth 0, that muste I write vnder the firste place, thus.

106800
9400
0

Mayster. Well sayde.

Scholer. Then likewayes in the seconde place I adde 0 to 0, and there ariseth 0, whiche I write vnder the seconde place thus.

106800
9400
00

Than I come to the thirde place sayinge: 4 and 8 make 12, of whiche I write the digitte 2, and keepe ỹ article 1 in my minde, thus.

106800
9400
200

Then

Then adde I 9 to 6, whiche maketh 15, to that I adde the article 1 ẏ was in my minde, and it is 16. I write 6 vnder 6 and 9, and keepe one in my minde, thus.

106800
9400
16200

Mayster. Why doe you not write bothe figures, seeinge you are come to the last couple of numbres?

Scholer. Nay, reason sheweth mee that I must adde that article that is in my minde, vnto the nexte figure of the ouer summe, thoughe there bee no more in the nether summe.

Ma. That is well considered: then doe so.

Scholer. Then say I, 0 in the ouer summe, and 1 in my minde, maketh 1, that I write vnder 0: Then foloweth there yet one more in the ouer summe whiche hathe none to bee added to it, for there is none in the nether summe, nor yet in my minde, therefore I thinke I muste write that euen as it is.

Mayster. Yea.

Scholer. Then doth my whole summe appeare thus.

106800
9400
116200

Mayster. If you marke this, you haue learned perfectlye the commō addition of all summes

whiche are of one denomination: so that ye obserue this also, that in Addition you muste haue two numbres at the leaste, or else how can you say that you we adde? And euer let the greatest numbre bee written highest, for that is the best way, though it be not necessary.

And forget not this, that if you haue many numbres to adde togither, you shall haue oftentimes an Article of a greater valewe then 10: sometimes 20, sometimes 30, sometimes more, yea paraventure, 100. Therefore as you did wt the article 10, so we with them, reserving them in youre minde, and adding to ỹ numbre next folowinge so many as theire valewer or valewe certaine is: that is to say, 2 for 20, 3 for 30, and so forth of other. But if the article be 100, then must you not adde the article to the nexte figures folowing, but to the third figures from them, as I will shew you anone by exaumple. And if it chaunce the numbre to bee such that it doe comprehende two sundry articles (that is one of tennes, and an other of hundredes) then muste you reserue them bothe in youre minde, and adde the article of tennes to the figures that followe nexte, and the article of hundredes, to the figure of the third place from thence.

Nowe

Nowe take this example for æl. I would ad these xiij. sūmes in one, whiche I sette after this manner. Then we I begin and gather the summe of the firste figures, whiche commeth to 107. For firste I take 9 there x. times, and that is 90: then 9 and 8 is 17, that is in all 107. of whiche summe I write the 7 vnder the firste figures, and than haue I an article of an hundred in my minde, whiche either I muste keepe in my minde tyll I come to the thirde figures, whiche are in the rowmes of hundredes, or els I maye for feare of forgettinge, write this one (beinge of the thirde place in youre accome) vnder ỹ thirde rowe of figures, makinge two lines, as you see here one. And then muste I write the digites vnder the lowest lyne: and this is the suresste way, when the summe is so greate, that the addition of one rowe passeth 100.

4	8 8 9
4	5 9 9
2	2 9 9
3	6 9 9
2	3 9 9
4	0 9 0
1	0 9 9
3	1 9 8
	2 9 9
	6 9 9
	8 9 9
	4 9 9
	3 8 9

When I haue so done, I muste than come to the seconde rowe of figures, and adde them together, whiche dothe make 115, of whiche

summe I write the vygitte 5 vnder the same seconde rowe, and then I haue a mixte numbre remaining of two figures, of whiche the 1 (that standeth for 10) muste bee added to the second or next place after them that I did laste adde. And the other (that standeth for 100) must be added to ỹ thirde place from thence.

☞ Scholer. That is to say, the fourth place from the first lyne or rowe of figures.

Mayster. Euen so. And thē will the summe appeare thus.

4889	
4599	
2299	
3699	
2399	
4990	
1099	
3198	
299	
699	
899	
499	
389	
11	
1	
57	

☞ Then adde the thirde rowe of figures, with the two vnities betweene the lyne, and the summe amounteth to 50: of which I write the Cypher vnder the same thirde rowe, and the 5 vnder the nexte figures towarde the lefte hande. And with my pen I giue a dashe to the two vnities betweene the lines, whose valewe I haue already added vnder the lowest lyne.

Then I adde the figures of the fourth rowe, with the 1 and 5 that are vnder them betweene the

the two lynes, and they make 29: then dashe I the 1, and the 5, w̄ my pen, as I did before the two vnits: and so write vnder the lowest line the 9 (that is the digit) vnder the fourth place: and the 2, that is the article, beyonde it, towarde the left hande. So those summes do make 29057.

Scholer. This seemeth somwhat harde, by the reason of so many numbres together. Howbeit I thinke if I do often proue euen with this same example, I shall be able to w̄ so, shortly, with any other summe.

Mayster. So shall you. For it is often practise that maketh a man quycke and ripe in all thinges: But bicause of suche greate summes there may chaunce to bee some errour, I will teache you how you shall proue whether you haue done well or no.

Scholer. That were a greate helpe & ease.

Mayster. Begin firste with ỹ highest number, and then to all the other orderly, and adde them together, not hauinge regarde to their places, but as though they were all vnities:

	4889
	4599
	2299
	3699
	2399
	4090
	1099
	3198
	299
	699
	899
	499
	389
	11
	11
	29057

and ſtill as youre number encreaſeth aboue 9, caſt awaye 9. Then go forthe, euer caſtinge away 9, as often as it amounteth thereto: and ſo do till you haue gone ouer all the numbers that you entended firſte to adde, and whatſoeuer remaineth after ſuche addition and caſting away of 9, write it in ſome voide place by the ende of a line for the better remembraunce: and then put together the figures that reſult of the addition, ſtill caſting away 9 alſo. And then that that remaineth, write at the other ende of that line: and if thoſe two figures be like, then haue you well done by likelyhode: but if they be vnlike, then haue you miſſed. As for example in this preſent ſumme: The firſte figure of the ouer lyne is 9, let him go: then 8 and 8 is 16, take away 9, and there remaineth 7, adde to it 4 that followeth, and that maketh 11, from whiche if you take 9, there reſteth 2: then come to the next rowe, whoſe firſt and ſecond numbers are 9, therfore ouerpaſſe them bothe, and take ẏ 5 to the 2 whiche did remaine in the firſt rowe, that maketh 7, put thereto the 4 following, that maketh 11, thence take 9, and there remaineth 2: next that, go to the thirde lyne, whoſe two firſte numbers you maye let paſſe, becauſe they are nynes: then take the

two

two whiche & the other two remained in the second row, make 6: the go to the fourth row, whose two first numbers let goe, and take the 6 to the 6 that remained, and that maketh 12, take away 9, and there resteth 3, whiche with the 3 that is next, maketh 6. And so go throughe all the other numbers, and you shall finde that there remaineth 5, after you haue caste awaye 9 as often as you finde it: therefore write 5 at one ende of a line in a voyde place, thus. 5—— Then gather all the figures of the totall summe whiche is vnder the lowest line, and cast away 9 as often as you finde it, as thus: seuen and 5 make 12, take away 9, and there resteth 3, to that if you adde the 2 that is last (for you may let go the 9) then doeth it make 5, whiche you must write at the other ende of the line that you made in the voyde place, and it will bee thus. 5——5

And then you see the those two figures be lyke, wherby you maye knowe that you haue wrie well, and so may you proue in any other.

Scholer. If it please you, I will proue in an other summe.

Mayster. With a good will.

Scholer. Then will I take one of youre former examples, whiche was this.

Firste

First in the highest line, 8 and 6 make 14, than 9 taken awaye, there re- 1 0 6 8 0 0
maine 5, to whiche I adde the 9 4 0 9
1 that followeth, and that ma- 1 1 6 2 0 0
keth 6. Then come I to the
second line, where I finde firste 4, whiche with 6, maketh 10, from that I take 9, and there resteth 1, the next figure is 9, and therefore I let him alone, so finde I one remaining, whiche I set at the ende of a line thus, 1——
Than I come to the totall summe, and there I finde that all the figures put together, make 10, from whiche I take 9, and there resteth 1 also, whiche I put at the other ende of the line thus, 1——1.
And because they be like, I know that I haue well added.

Mayster. So you knowe nowe both howe to adde two summes or more together: and also howe to proue whether you haue done well or no: whiche thing also you maye doe beste by Subtraction. But because you can not yet skil of it, I will let that passe till anone, and will teache you now how to adde summes of diuers denominations: whiche thinge can neuer bee, but whē the one denomination is suche that it containeth the other certaine times. And yet you

Addition of nūbers of diuers denominations.

you shall adde them to the other, not after this sorte as you did them that were of one denomination, but after suche a sort as I will now shew you, that is to say.

If you haue a summe of diuers denominations, then loke that ye set euery denomination by himselfe, with some note or figure of his denomination, as they be wont to bee written. Then write your other summes so vnder that first, that euery one bee set vnder the other of the same denominations, as for example: if your denominations be pounds, shillings, and pens, write pundes vnder pundes, shillinges vnder shillinges, and pens vnder pens, and not shillinges vnder pens, nor pens vnder pundes.

Scholer. Nowe that you haue spoken it, me thinketh it needeth not to warne mee of it, for it were against reason so to confound sumes: but yet if you had not spoken of it, peraduenture I should haue bene deceiued in it.

Mayster. If you doe saye it is so plaine, I will speake no more of it, but with an example make the matter to appeare euidently.

Firste, one man oweth mee 22 li. 6 s. 8 d.
An other oweth mee 5 li. 16 s. 6 d.
And an other oweth mee 4 li, 3 s. I woulde

knowe

know what this is altoge-
ther. Therfore muste I firste
set downe my greatest sūme
and thē the other, euery one
vnder his denomination a-
greeinge to the greatest sūme, as here you see.

lb	s	d
22	6	8
5	16	6
4	3	

Then muste I begin at the smallest nūbres, (which muste alwaies be set next ỳ right hand) and adde them together, and if the summe of them will make one of the next denominatiō, then muste I keepe it in my minde till I come to that place, or els for more easines, write it vnder that place betweene the double line, and vnder that firste place muste I note the residue, if there remaine any of the same denomination, but if there remayne none, then neede I to write vnder it nothinge. And this is all that you muste marke in this Addition: for all other thinges are like to the other maner of addition before mencioned. Therfore the chiefeste pointe of this addition is, to knowe the valewes of common coynes and rated summes. As howe many shillings be in a pound: how many pens in a shilling, of which & of other like thinges, I will instructe you hereafter, in teachinge of Reduction: But nowe I maye not disturbe youre witte from the thinge that wee are about

aboute.

Therefore let vs returne to that former example, whiche I proposed of three letters, whiche summes when I had sette orderly, they stode thus with a double lyne vnder them.

lb	ß	d
22	6	8
5	16	6
4	3	

Then to adde them vnto one sume, I muste begin at the right hand, where the smallest denomination is, and adde them together firste, saying: 6 and 8 make 14. Nowe seeing these 14 are pennyes, and that 12 pens make one shilling, whiche is the next valewer, I take away 12 from 14, & there resteth 2, whiche I write vnder the pennies, and for the other 12, whiche make 1 shilling, I write 1 vnder the tytle of shillings, thus.

lb	ß	d
22	6	8
5	16	6
4	3	
	1	
		2

Than doe I adde all the shyllinges together, & finde them 25. to whiche I adde ỳ 1, betweene ỳ two lines, ỳ maketh 26, but because that 20 shillings do make 1 pounde, I take away 20

lb	ß	d
22	6	8
5	16	6
4	3	
1	1	
	6	2

from

from 26, and for that 20 I write 1 vnder the pounds betweene the two lines, and the other 6 that remaineth, I write vnder the shillinges, as appeareth in the example before.

Then come I to the pounds, addinge them all together, and finde them to bee 31: thereto I adde the 1 betweene the two lynes, & that maketh 32. whiche summe I write downe whole, bicause there resteth no greater denomination, and then my whole summe appeareth thus.

li	ß	d
22	6	8
5	16	6
4	3	
1	1	
32	6	2

So is my totall summe 32 li. 6 ß. 2 d. And this may you proue in an other like summe.

Scholer. Then will I caste the whole charge of one monethes commons at Oxforde with batteling also.

Mayster. Go to, let mee see howe you can doe?

Scholer. One weekes commons was 11 s. ob. q. & my batlinge that weeke was 2 d. q. q. The seconde weekes commons was 12 d, and my batling 3 d. The thirde weekes commons 10 d. ob. and my batling 2 d. q. c. The fourth weekes commons 11 d, q, and my batlinge

ling 1 ō. ob. c. These summes wolde I ad into one whole sume, and therfore I will set thē one ouer an other, thus.

ōt.	ob.	q̄.	q.	c.
1 1	1	1		
2		1	1	
1 2				
3				
1 0	1			
2			1	1
1 1		1		
1	1			1

But I had forgotten, I shoulde haue set ẏ greatest sum highest.

Mayster. So is yt commonly beste, howe bee it, here it forceth not: and in suche summes as this is, that go by order of weekes, dayes, or yeares, it is better to keepe that order, then to alter them, and to set ẏ greatest numbre highest, for that serueth for suche summes as go not by order.

ōt.	ob.	q̄.	q.	c.
1 1	1	1		
2		1	1	
1 2				
3				
1 0	1			
2			1	1
1 1		1		
1	1			1
			1	

Scholer. Then yt I haue set theim well ynough, I will begin to adde them thus.

First of the smallest valewers at the right hand, whiche are called cees, I finde 2, & seeing ẏ 2 cees, do make one

q, I will write nothing vnder the cees, but will write 1 q for 2 cees, vnder the kewes betweene ye lines, as the example on ye other side sheweth.

Than come I to the nexte valowers, where I finde 2 q, and to thē I adde the q that is betweene the lynes, and so are they 3 q: but bycause 2 q, maketh one q̄, I wryte one q̄ vnder the farthings betweene the lines, and the q that remaineth, must I write benethe the nethermost line vnder the kewes, thus.

d.	ob.	q̄.	q.	s.
1 1	1	1		
2		1	1	
1 2				
5				
1 0	1			
2			1	1
1 1		1		
1	1			1
		1	1	
			1	

Then come I to the farthinges, where I finde 3, and the other q̄ that is betweene the lines, maketh 4 farthinges. And bycause 4 q̄ make iuste 1 peny, I shall write nothinge vnder the farthinges, but must write 1 vnder the pens, betweene the lines.

Next that must I adde the halfepence together, of whiche there are 3. but seeing that 2 ob. make 1 d, I must write 1 vnder the pens, betweene the lines: but howe shall I doe it, for there

there is 1 already?

Mayster. Haue you forgotten how I did in addition of the greate summe before? you must set it vnder the other, so shall they bothe stande for 2. For if you shoulde set it before or behinde the other, they should make 11.

Scholer. I remembre it nowe, and I perceaue the reason. Then I will write 1 ob. vnder ẏ halfepence, and for the other two halfepence, whiche make 1 d̄, I write 1 vnder the pence: Then come I to ẏ pence, and finde that there are of them 52. than put I to them the 2 betweene the lines, and that maketh 54, which amounteth to 4 s, 6 d̄: the 6 d̄, I must write vnder ẏ pens, and the 4 s. I must sette (I suppose) farther toward ẏ lefte hãde by them selfe.

s.	d.	ob.	q̄.	q.	c.
	1 1	1	1		
	2		1	1	
	1 2				
	3				
	1 0	1			
	2			1	1
	1 1		1		
	1	1			1
	1		1	1	
	1				
4	6	ob.		q.	

Ma. Euen so.

Sco. Than appeareth all my addition, thus. And ẏ summe is 4 s. 6 d̄. ob. q.

Ma. Now haue

you wne this well. But tell mee, why did you wzite kewe, cee, thus, q,c, and not rather thus q^c, as the fashion is?

Schol. Because I thoughte it was the best way foz due gathering of euery denomination by him selfe.

Mayst. So was it in deede. Well nowe, can you tell howe to pzoue this addition, and sudze other like of diuers denominations, and to trie whether you haue wne well oz no?

Scholer. I would I coulde.

Ma. That shall you we by this meanes: Firste as you did beginne to adde, so recken agayne euery denomination by it selfe, & when you finde so many small that we make any other denomination, let them goe, and keepe in minde onely the residue ẏ will make no greater denomination, and loke whether there bee any sudze like valewe vnder the nether line, & if there bee, you haue well done, and so goe forth from one denomination to an other, vnto the ende.

But heere must you note, that in gathering of the summes, yee must recken those figures that are written betweene the lines, with them that are written aboue them: as for an example. I will examine that summe that I did

did last adde, whiche stode thus.

lb	ß	d
22	6	8
5	16	6
4	3	
1	1	
32	6	2

Fyrste I finde 6 and 8, whiche maketh 14, from whiche I take 12, bycause it maketh one of the next denomination, and there remaineth 2, and vnder that place I see a like figure, therfore I knowe that well to bee wne. Then come I to the ß, where I finde 1, 3, 16, and 6, that maketh 26, I cast away 20, for they make an other denomination, that is to say pundes: and the 6 whiche remayneth, is like to the 6 that is wrytten vnder them benethe the loweste line, therefore that is well wne also. And thence I go to pundes, where I finde 1, 4, 5, 22, that is 32, to whiche summe agreeth an other like vnder it. Therefore I iudge all well wne.

Scholer. I perceaue reason in this probation. Nowe will I attempte the same in the summe that I did adde, whiche whan I had ended addinge, stode as you maye see in the page folowinge.

Firste amongest the cees I finde but two, whiche make one q euen, therefore there muste nothinge bee vnder the line for theim: And

amongst the kewes I finde 3, of whiche two make 1 q̃, therfore I let them go, and the one q, that is lefte, hathe an other like vnder his place, therefore that is well done.

ß.	d.	ob.	q̃.	q.	c.
	1 1	1	1		
	2		1	1	
	1 2				
	3				
	1 0	1			
	2			1	1
	1 1		1		
	1	1			1
	1		1	1	
	1				
4	6	ob.		q.	

Than the farthinges are iuste 4, whiche make 1 d, & therfore I let them go. Amongest the halfe pence there is one odde (for 2 muste I caste away, bycause they made one peny) & vnto it aunswereth à like summe vnder it. The pence are 5 4, from whiche I take away 4 8, that makes 4 ß, and the 6 remaining agree to à like figure set vnder thē. and laste of all remayneth the 4 ß, whiche the abiected pence did make: so I perceaue that I haue well done. Nowe this will I not forget. But will this examination serue in all Addition?

☞ Mayst. It serueth for all Addition of sundry denominations, if the Addition bee made with two lines, (as these were) els it will not serue,

ſerue, bicauſe that thoſe ſummes whiche are here added betweene the lynes, in Addition by one lyne, are vnderſtanded and not witten: but I let ẏ waies paſſe, bicauſe as it is cõmon, ſo is it moze deceiuable thẽ this waies, namely if a mans memozy be either dul oz troubled.

Scholer. Yet it were good to knowe that waies alſo.

Mayſter. If you deſire to knowe it, this it is in fewe wozdes. Do euery thing as you did in this ſozte of Addition, ſaue that where you made here two lynes, you ſhall make there but one: and thoſe ſummes that you did here write betweene the lines, you muſt keepe in your memozy, and vſe them (as you did here) eche one when you come to his place. An other forme of Addition

Scholer. Then they differ not, but in this, ẏ this addition ẅ two lynes leaueth nothinge to memozy, but writeth downe all: and the other way committeth certaine numbers to memozy, as you taught me in the firſte examples of addition of ſmall ſummes of one denomination. But what if a man vſe it (as you ſay men do commonly) how ſhall it be examined?

Mayſter. Seinge you are ſo deſirous of it, I will ſhew bothe an example of the addition, and alſo the manner to examine it.

I propose these three summes to be added, and I gather firste the pens, as I did in the other sorte, and I finde of them 8, 3, 9, that is 20, of which summe I bate away 12, whiche make 1 ſ, & keepe that 1 in my minde, and the reſt, that is 8, I write vnder the pens.

	li	ſ	d
	12	8	9
	6	7	3
	3	6	8

Than do I adde the shillings together, and finde of them 6, 7, 8, that is 21, whereof I bate 20, that make 1 li, which I keepe in minde, and to the other 1 that remaineth, I adde that one that came of the pens, & was in my mind, whiche make 2, and them I wryte vnder the shillinges.

Than do I recken the pounds together, 3, 6, 12, that is 21, and to them I adde the 1 in my mynde that remayneth of the shillinges, whiche make 22, them dwe I wrote vnder the pounds, and than my summe totall appeareth to bee 22 li, 2 ſ, 8 d.

	li	ſ	d
	12	8	6
	6	7	3
	3	6	8
	22	2	8

An other forme of proofe.

Now to examine this sume and all suche like, you shall doe thus. Firſt begin at the left hand with the pounds, and take from them that are aboue the lyne, 9, as often as

as you can: then that that remaineth, shall you double, and ioyne it with the shillinges, & take away 9 from that as often as you can, and whatsoeuer remaineth, ye shall take for it three times so much, and put to the pens: then take from all that sum 9, as often as you can, and what so remaineth after you haue withdrawē 9 as often as you can, write that at the ende of a line, as I taught you in ỳ other addition.

And then come to the summe vnder the line, beginnige with the poundes, and doe euen as you did with the summes aboue the line, tyll you come to your pennies: and if the figure of the summe that remaineth after castinge away 9, (as often as you can) doe agree with the other that remayned before of the other summe, which you did write at the end of the line, then haue you done well, els not. and for an example, I wyll examine that laste summe which was thus set.

	lb	ß	d
First I shall begin at the			
left hand with the pountes,	12	8	9
putting thē together, which	6	7	3
make 21, in whiche summe	3	6	8
I finde 9 twise, (for twyse	22	2	8

9 is 18) that I deduct, and there remayneth 3: that 3 muste I double (as

I saide) bicause it is the remainer of ỹ poũdes, and it will bee 6. Than gather I the summe of the shillinges, whiche is 21, to the whiche I adde the aforesaide 6, and than is it 27, wherin I finde 9 three tymes, and there remayneth nothing. This remayner shoulde I take three tymes, but three tymes nothing, is nothinge: therefore in this place is there nothinge left to bee added to the pennyes. Wherefore I must take the summe of pennies alone, whiche is 20, from thence if I take 9 twise, there remayneth but 2, whiche I put at the ende of a lyne, thus. 2——

Then I come to the poundes of the vnder number or totall summe, and there I finde 22, from whiche I take away 9 twise, and there remayneth 4: that 4 I double, and it is 8, then doe I adde that 8 to the shillinges, and it maketh 10, from whiche I withdrawe 9, and there resteth one: then doe I take that 1 three tymes, and it maketh 3, whiche I adde to the 8 ů, and it maketh 11, from whiche if I bate 9, there resteth 2, whiche is equall to the number noted at the ende of the line: & thereby I perceiue that I haue done well.

Scholer. But I doe not see the reason of this.

Mayster.

Mayster. No? no more doe you of many things els, but hereafter will I shew you the reasons of all Arithmeticall operations. for this I iudge to bee the best trade of teachinge, first by some briefe preceptes to instruct a learner somewhat in the vse of the arte, before hee learne the reasons of the arte, & then may you afterwarde more soner make him to perceaue the reasons: for harde it is for to occupy a yong learned witte with bothe the arte and the reasons of it all at once: howbeit hee shall neuer be cunninge in deede in an arte, that knoweth not the reason of euery thing touching it. But for this worke, because the reason is easy, I will shew it you nowe. You know that if one pund w remayne, it beinge tourned into shillinges, woulde make 20 ß, in whiche numbre there is 9 contained twise, and 2 ß beside. And therefore for one pund you shal take 2 ß, and so for euery one pund 2 ß.

The reason of this profe

The best trade of teaching.

Scholer. I see it well, for if there remained 7 li, after the nynes were cast away, I must take 14 ß, for that 7 li. And so haue I cast away 14 times 9 ß, and yet remayneth of euery punde 2 ß, whiche maketh 14 ß.

Mayster. Likewayes in shillings, whiche containe 12 d: for euery shilling, if you abate 9 pence

9 pens, there resteth 3 pens.

Scholer. It is plaine inough. And so if 5 shillings w remaine, I must take for it 15 d, that is 3 pence for euery shillinge, and yet in that so doinge, I haue caste away fiue times nine pens.

Mayster. Other workes haue as good reason, but I will not stande about yelding reasons nowe.

Scholer. Yet one thing more I praye you shew mee, why did you write youre numbre y remained (after you had withdrawen all the nynes) at the end of à line? for I saw no reason why that line did serue.

Mayster. Did you euer marke à Carpenter when he wrought?

Scholer. Yea many times.

Ma. And haue you not seene him when hee hath taken measure of à boarde, that hee hathe pricked it, and hath with à twitche of his hande drawen à line from the prick that hee made?

Scholer. Yes I haue marked that, & haue seene some make 3 or 4 lines by the pricke, and some also haue I seene make à crosse by it, but that I perceiued was for the easye findinge of their pricke.

Mayster. And euen so is this line for the easie

ealle findinge of your remainer, and therefore
som we make a crosse, thus.

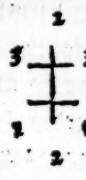
And set the one remainer aboue
the crosse, and ỹ other vnder the
nether part of the crosse, as if I should set my
two remainers thus.

But there is an other sort of
proofe of Addition, to whiche
the crosse serueth more meeter: & that is when ỹ addition is of diuers denominations, and I would examine euery denominatiõ by it self, whiche waies though it be not much vnlike to the first proofe that I brought of suche diuers summes, yet I will declare it, least you should thinke that I woulde hyde it from you. An other sorte of proofe.
You must make so many lines in your crosse, as you haue sundry denominations: as if you haue but two denominations, than you maye make it thus, that the ouer part and the nether part may serue for one denomination, and the two sides for the other. And if you haue three denominations, as pounds, shillinges, and pennies, than must you make three lines thus. The vpright line may serue for the poũds, and the highest thwart line for shillings, and the lowest for pens: as for

example

example I will take a summe thus added.

li	s	d
16	12	5
12	8	1
9	2	7
38	3	1



For the proofe of the whiche, because it containeth three denominations, I must make a crosse of three lines, thus. Than I recken firste at the right hand the pennies: 7, 1, 5, make 13, from which I take 12 for the next denomination, that is to say, a shilling, and there resteth 1, whiche I must write at one ende of the nether thwart line.

After that I gather the summe of the shillings, 2, 8, 12, whiche maketh 22, to them I put one that I toke of the pennies, and that maketh 23: from those I take 20 the quantity of the next greater denomination, that is to say, a pound, and there resteth 3, whiche I write at the ende of the highest thwart line.

Thirdly, I adde together the pounds, 9, 12, 16, whiche make 37, to them I adde the 1 that came of the shillinges, and than there is 38, wherein I finde 4 times 9, and 2 ouer, that 2 I write on the vpright line.

That done, I come to the totall summe,

and

and examyne it, beginninge at the pennyes, where I finde but one, and can not take 9 from him, therefore I set him at the other ende of the nether thwart line: Than come I to the shillinges, where I finde onely 3, whiche because it is lesse then 9, I sette it at the other ende of the line of shillinges, that is, the ouermost thwart line.

Laste of all, of the 38 li. I take out foure times 9, whiche is 36, and there remaineth 2, whiche I write vnder the vpright lyne.

Then I consider euery number, comparing it to the number that is against it, and bicause I finde thē to bee euery one like his matche, I know that I haue well done.

Scholer. This crosse I perceiue doth serue for those three denominations, pounds, shillinges, pennyes, but what if I had ob, q̄, q, and c?

Mayster. You thinke you bee at Oxforde still, you bring forth so faste your q, &c. These lines, as I haue sayde, do serue for three denominations, such as they be: as here they doe serue for poundes, shillinges, and pennies: but if ye haue no poūdes in your summe, then may they serue for shillings, pennies, and halfepennies, yea for q̄, q, and c, if you haue no greater

ter denomination, so that you remember that the vpright line serueth for the greatest denomination, and the highest thwart line, for the next, and the lowest for the least.

And so if you haue foure denominations, you must make your crosse with so many lines. And if that your summe bee of more denominations, make so many lines in your crosse. And thus will I make an ende of Addition.

Examples of Addition.

li.	s.	d.
2625876		
416351	12	
28124	2	
4711	4	
3328184		

li.	s.	d.
340	17	10
28	6	8
13	13	4
382	17	10

The proofes.

An other example.

li.	s.	d.	q.
22		6	2
2	3	4	
	10	2	3
24	14	1	1

SVB-

Scholer.

Hen haue I learned the two firste kyndes of Arithmetike, nowe as I remember, dothe folowe Subtraction, whose name mee thinketh dothe sounde contrary to Addition.

Mayster. So is it in deede: for as Addition encreaseth one grosse summe by bringinge many into one, so contrary waies, Subtraction diminisheth a grosse summe by withdrawinge of other from it, so that Subtraction or Rebating is nothing els, but an arte to withdrawe and abate one sume from an other, that the Remainer may appeare. Subtraction.

Scholer. What call you the Remainer?

Mayster. You may perceaue by the name.

Schol. So me thinketh: but yet it is good to aske the trouth of all sudr thinges, leaste in trusting to myne owne coniecture, I bee deceaued.

Mayster. So is it the surest way. And as

I see cause, I will still declare thinges vnto you so plainly, ỳ you shall not neede to doubt. Howbeit, if I doe ouer passe it sometimes (as the mãner of men is to forget the small knowledge of them to whome they speake) then doe you put me in remembraunce your selfe, and that way is surest.

And as for this worde that you laste asked me, take you this description. The Remainer is a sum left after due Subtraction made, whiche declareth the excesse or difference of the two other numbers: as if I woulde abate or subtract 14 out of 18, there shoulde remayne 4, whiche is called the Remayner, and is the difference betwene those two numbres 14 & 18.

Remainer

Scholer. I perceaue than what Subtraction is: Nowe resteth to knowe the order to worke it.

Mayster. That shall you doe by this meanes. First you must consider, that if you should go about to rebate, you must haue two sundry summes proposed, the first whiche is your grosse summe or summe totall: (and it must bee set highest) and then the rebatemente or summe to be withdrawen, whiche must bee set vnder the first (whether it bee in one parcell or in many) and that in such sort, that the first figures

figures be one iust ouer an other, and so the seconde and thirde, and all other folowinge, as you did in Addition: then shall you drawe vnder them a line, and so are youre summes duely set to begin your workinge.

Then begin you at the righte hande (as you did in Addition) and withdrawe the nether number out of the higher, and if there remaine any thing, write that right vnder them beneth the line: and if there remaine nothing (by reason that the 2 figures were equall) then write vnder them a ciphar of nought. And so do you with all the other figures, euermore abatinge the lower out of the higher, and write vnder them the Remainer still, till you come to the ende. And so will there appeare vnder the line what remayneth of your grosse summe, after you haue deducted the other summe from it, as in this example.

I receiued of your father 48 ß, of whiche I haue layde out for you 36 ß: nowe woulde I know what dothe remayne? and therefore I set my numbres thus in order: First I wryte the greatest summe, and vnder him the lesser, so that the figures at the right side bee euen one vnder another, and so the other, thus.

ß
48
36

Than we ℐ rebate 6 out of 8, and there resteth two, whiche ℐ write vnder them right beneath the line, thus.

```
ß
48
36
--
 2
```

Than ℐ go to the second figures, and we rebate 3, out of 4, where there remaineth 1, whiche ℐ write vnder them right, and thē ỹ whole summe and operation appeareth thus.

```
ß
48
36
--
12
```

Whereby it appeareth, that if ℐ withdrawe 36, out of 48, there remaineth 12.

Scholer. Now will ℐ proue in a greater summe: And ℐ will Subtract 2367924 out of 3468946. Those sumes ℐ set in order thus.

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3468946
2367924
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Then we ℐ begin at the right side, and reducte 4 out of 6, and there resteth 2, whiche ℐ write vnder them. Then goe ℐ to the second figures, and withdraw 2 out of 4, and there remaine two, whiche ℐ set vnder them also: then ℐ take 9 out of 9, and there resteth 0, whiche ℐ write vnder them: for you say, that if the figures be equall so that nothing remayne, ℐ must write this ciphar 0 vnder them.

Mayster. ℐt was well remembred, nowe goe

go forth.

Scholer. Then come I to the fourth place and draw 7 out of 8, and there remayneth 1, whiche I write vnder them also. Then in the fift place I take 6 from 6, and there resteth nought, for it I write vnder them a ciphar, 0: Than in the sixt place 3 rebated from 4, there remaineth 1, whiche I write vnder them: and likewise in the vij. & last place, 2 taken from 3, there is lefte 1: whiche I write vnder them, to haue I done my whole worckinge, & my summes appeare thus.

3468946	
2367924	
1101022	

Whereby I see, that if I rebate 2367924, out of 3468946, there remayneth 1101022.

Mayster. This is well wne. And that you maye bee sure to perceaue fully the arte of Subtraction, let me see howe can you subtract 52984732 out of 8250003456.

Scholer. Firste I set wwne the greatest summe, and after that I write vnder it the lesser number, beginning at the right side: and then my figures will stand thus.

8250003456	
52984732	

Than take I 2 from 6, and the reste is 4,

whiche I write vnder them: then we I withdrawe 5 from 5, and there remaine 0, whiche I write vnder them. Then take I 7 out of 4, but that I cannot, what shall I now we?

Note. Mayst. Marke well what I shall tell you now, howe you shall we in this case and in all other like. Yf anye figure of the nether summe be greater then the figure of the summe that is ouer him, so that it can not be taken out of the figure ouer him, then muste you put 10 to the ouer figure, and then consider how mucke it is, and out of that whole summe withdrawe the nether figure, and write the reste vnder them. Can you remember this?

Scholer. Yes, that I truste I shall. Now then in mine exaumple where I shoulde haue taken 7 out of 4 and coulde not, I put 10 to that 4, whiche maketh 14, from it I take away 7, and there resteth 7 also, whiche I write vnder them.

Note. Mayster. So haue you done well, but now muste you marke another thing also: that when soeuer you doe so put 10 to any figure of the ouer number, you muste adde one still to the figure or place that followeth nexte in the nether line, as in this example there followeth 4, to whiche you muste put 1, and make him 5,

and

and then go on as I haue taught you.

Scholer. Then shall I saye: 4 & 1 (whiche I muste put to him for the 10 that I added to 4 before) make 5, whiche I shoulde take out of 3, but that can not bee, therefore must I put to it also 10, and then it will be 13, from whiche I take 5, and there resteth 8 to bee written vnder them: and bycause of that 10 added to the 3, I muste adde 1 to 8 that followeth in the nether line, and that maketh 9, whiche I shoulde take out of 0, and can not, therefore I put thereto 10, and that maketh 10: from 10 I take 9, and there remaineth 1, whiche I write vnder them.

Then we I adde 1 likewise to the nexte figure benethe, whiche is 9, and that maketh 10: that 10 shoulde I take out of the figure aboue, but I can not, for it is 0, therefore I put 10 to it, and so take I 10 out of 10, and there resteth 0 to be written vnder them. Thē come I to the nexte figure whiche is 2, and to him we I adde 1, whiche maketh 3, that 3 I can not take out of nought, therefore of that nought I make 10, and thence we I take 3, so remayneth there 7 to bee written vnder them. Likewise we I put 1 to 5 that foloweth, and then is it 6, ỹ woulde I take out of 5, and can not, therefore I adde

10 to that 5, and make it 15, from whiche I rebate 6, and there remaineth 9, whiche I write vnder them. Now haue I spent all the nether figures, and what shall I doe more?

Mayster. You shoulde haue added 1 to the nexte figure folowinge (if there had been any) bycause you added 10 to the laste figure before of the ouer lyne: but seeing there is no figure folowinge, you muste adde that 1 to the place folowing, and then deduct that 1 from ỹ number aboue.

Scholer. Then shall I say, bycause I borowed 10 to the ouer 5, I muste put 1 in the next place benethe, that is vnder 2: then muste I subtract that 1 from 2, and there resteth 1, to be written vnder that 2 in ỹ ninth place. Now I haue no more to subtract, for there is neuer any figure remaninge benethe, nother yet any vnitie to bee added, bycause I borowed not 10 to the figure laste before, and yet is there 8 remaininge in the ouer lyne, whiche (I thinke by reason) shoulde bee set at the ende of the figures in the lowest rowe whiche is vnder the line, for bycause there was nothing taken from it.

Mayster. That is well considered, and reason teacheth so in deede.

Scholer. But syr I beseeche you, shall I alwayes

alwayes when any number so remayneth alone (as this 8 did) write him vnder the lyne straight agaynste his owne place?

Ma. Yea, what else? whether they bee one or many: and this well remembred, you haue sufficiently learned Subtraction. Howe be it, bicause of certaine thinges that myght deceiue you, if you did not take good heede to youre worckinge, I will propose to you an other example of many numbers to bee subtracted, as thus.

I receaued of a frende of myne to keepe 2869 crounes, of whiche at one tyme I deliuered him agayne 500, at an other tyme 368, and at an other time 440, and an other time 80, and an other tyme 64: nowe woulde I knowe howe many dothe rest behinde. Therefore first I set downe my grosse summe, and a lyne vnder it: and vndernethe it I set all the parcelles, thus: and vnder them a double line.

Then firste I beginne at the first place, and gather together the summe of all those lynes (saue the ouermoste) in their

2869	
500	}
368	}
440	}
80	}
64	}

firſte figures, and ſo doe I with all the figures of the ſeconde place, & ſo forthe, as I did in Addition, ſaue that I leaue out the higheſte rowe of numbers, (as the line warneth mee) and that ſumme ſo gathered betweene the double lyne, we I ſubtract out of the higheſt rowe of numbers, and the remainer we I ſet vnder the nethermoſte lyne: as for example.

I ſet the ſummes as before: than we I gather the firſt figures together, where I finde but 4 and 8, that make 12, (for three cyphres increaſeth no ſumme in addition, as you learned before) of the 12 therefore doe I write ỹ digit 2 betweene the double lyne, and keepe the article in my minde, till I come to the ſeconde places, where I finde, 6, 8, 4, 6, ỹ make 24, to them I put the article in my minde, and it is 25, of whiche I write 5 vnder the ſeconde place, and keepe the digit 2 in my minde for the thirde place, where I finde 4, 3, 5, that make 12, to the whiche I adde the 2 in my minde, and that maketh 14, thereof I write the 4 vnder the thirde place: and becauſe there remaineth

2	8	6	9
	5	0	0
	3	6	8
	4	4	0
		8	0
		6	4
1	4	5	2
1	4	1	7

remayneth no more figures to bee added, I write the digit 1 in the fourth place, as you see in the example.

Thã come I to subtracting of this summe betweene the lines, for by Addition it is equall to the fiue parcels ouer it. Therefore I procede to subtracte it from the ouermoste summe, saying: 2 from 9, remayne 7, to be written vnder them beneath the lowest line. Than in the seconde place I take 5 from 6, and there resteth 1, to be written vnder them. Than in the thirde place, 4 from 8, resteth 4. Laste of all in the fourthe place, 1 from 2, remayneth 1. And thus I see that after those 5 summes are subtracted from 2869, the Remainer is 1417.

Scholer. This I perceaue: but is there no shorter way and more spedier?

Mayster. Yes, whan you are a while exercised in it: for you may as fast as you can gather the numbres togither, withdrawe them out of the highest summe, if so be it, that all the parcelles which you do gather, do not exceede nine, but and if they exceede nine, than muste you subtracte onely the digitte that is in it, & reserue the article till the nexte place, where you shall adde it with the other figures, and so subtracte the whole out of the figure aboue them

An abridgement of the former manner of Subtraction.

them: but and if in this place the summe of ẏ parcels we exceede 9, than (as I sayde before) subtract the digit onely, and reserue the article to the next place: and so still go forthe, till you haue ended your worckinge.

As for example: In the laste summes proposed, I gather firste in the firste place 4 and 8, that maketh 12, of whiche I reducke the digite 2 out of 9, and write vnder the remayner, whiche is 7, & the article 1 I kepe in my minde. Than in the seconde places I gather the parcelles 6,8,4,6, whiche amount to 24, to that I adde the article 1, whiche I haue in my minde, and than is it 25. than do I take 5 (that is the digitte in this numbre) from 6, that is in the seconde place of the highest summe, and there remayneth but 1 to bee written vnder theim, & now we I keepe the article 2 in my mind stil. Than in the thirde place, 4,3,5, maketh 12. and the article 2 in my minde maketh 14: thē take I 4 (whiche is the digite) from 8 that is ouer them, and there resteth 4, whiche I write vnder them. Then haue I the article 1 yet in my minde, whiche I shoulde adde to ẏ parcels next folowing, but seeing there is no number folowing, I take ẏ digit alone, & deduct him out of ẏ nexte summe aboue, whiche is 2, & than is the

remayner

remainer 1, whiche I write in the fourthe place vnder 2. Loe, now haue you à shorter waye.

Scholer. I lyke bothe wayes well, and I perceiue bothe well, yet as in the one the worcking seemeth somewhat longe, so in the other it leaneth very muche (mee seemeth) to remẽbrance, and therefore may cause errour quickly, excepte à man haue à quicke and an exercised remembraance.

Mayster. What? woulde you then haue suche à waye that shoulde not bee so longe as the one, nor so shorte as the other?

Scholer. Yea, if there were any suche.

Mayster. Than we thus: still as you gather your parcels, when they exceede a digit, & maketh him 10 or more, take the article, and write him betweene 2 lines (as in the firste example) vnder the next place towarde ỹ lefte hande: and then deducte the digite from the figure that is ouer him, and write the remainer. And then when you gather the next parcelles, you shall adde to them the figure that is vnder them, betweene the two lines. And if it exceede 9, we as I sayde before, write the article vnder the next place betweene the lines, and subtract the digit from the figure that is ouer those parcels: and if that all the parcelles together &

the

the numbre betweene the lynes we make but a digite, then deducte it wholly from the fygure aboue: as in this example. I woulde subtracte out of 40308964, these three parcelles,

20003428.	40308964
10002342.	20003428
10101461.	10002342
	10101461

Therfore I set them fyrste in ordre due: and then I gather the parcelles of the firste place, which are 8, 2, 1, that is 11: of whiche I take away the article, and set him vnder the second place betweene the lynes: and the digit 1 that remayneth, I deduct out of 4, and there resteth 3 to bee written vnder the fyrste place benethe the lowest line. Then come I to the seconde place, and gather the parcels of it, 6, 4, 2, & the 1 betweene the lynes, whiche make 13. of whiche I take the article, and set him vnder ẏ thirde place betweene the lines, and the digit 3 I take from 6, and there remayneth 3, whiche I wryte vnder the seconde place benethe the lowest line. Than in the thirde place I finde 4, 3, 4, whiche with the 1 betweene the lines, we make 12, therfore I write the article agayne vnder the fourth place, and the digite 2

I

I take from 9, and there remaineth 7, whiche I write vnder them beneth the lowest lyne.

And than come I to the fourth place, where I gather 1,2,3, and the 1 betweene the lynes, ẏ maketh 7, whiche bycause it is but à digite, I plucke from 8, and the Remayner is 1, and muste bee written vnder them in the fourthe place. After that come I to the fifte place, where are onely three cyphars, whiche make nothinge, than shoulde I take that, that is to say nothinge, from ẏ figure ouer them, whiche is also à cyphre, therefore I muste saye thus: yf I take nought from nought, there remayneth nought: so muste I write à cyphre vnder them. Thā in ẏ firt place I finde but 1, whiche I take out of 3 ouer him, and the Remainer is 2, that must bee written benethe the lowest lyne in the firt place. So goe I to ẏ seuenth, where I finde onely cyphres, and in the grosse summe ouer them à cyphre also, therefore must I write their remayner (whiche is nothinge) with à cyphre also. Than in the eyghte and laste place, I gather 1,1,2, that make 4, whiche if I take out of that 4 that is ouer theim, there will nothing remayne. And that must bee noted with à ciphre benethe the lowest lyne, as I haue often sayde, and so haue I ended my

worke,

worke, and the figures stande thus.

40308964	
20003428	}
10002342	}
10101461	}
1	
00201733	

Scholer. Sir, I remember you taught me that ciphers shoulde not come in the laste place, for bycause they serue onelye to encrease the valewe of other fygures whiche followe them, and serue not for those fygures that goe before them: and nowe in youre example you haue set two ciphers in the two laste places.

Mayster. I commende you for your remembraunce. And truth it is, I shoulde not haue set them here, but onelye bicause that I would make you playnely to perceaue the arte of Subtraction. Therefore seeing that you do nowe perceaue it, whensoeuer you shoulde write downe a cyphar, looke whether anye other figures bee yet behinde. And if not, than let go the cyphar also, for it needeth not to write him in any latter places, where no other figure doothe followe, except it bee (as I did) to teache the vse of Subtraction the playner.

Therefore

Therefore my figures muste stande thus when I haue ended my worke.

40308964	
20003428	
10002342	
10101461	
	1111
	201733

Scholer. So I woulde thinke by ỹ you taught mee before. And now I beleue I could subtract any summes.

Mayster. So may you, if you haue marked what I haue taught you. But bicause this thinge (as all other) must be learned surely by often practise, I will propounde here two examples to you, wherein if you often exercise your selfe, you shalbe ripe and perfect to subtract any other summe lightly, for in them is contained all the obseruances of whole nũber. And because you shall perceaue somewhat bothe howe to doe it, and also whether it bee well done whan you haue proued to doe it, therefore haue I written vnder them, bothe the Remayners.

308964	125614
103145	342
102597	681
101024	201
11	11
2198	124390

H.j. Scholer.

Scholer. Sir, I thanke you. But I thinke I might the better doe it, if you did shewe me the working of it.

Mayster. Yea, but you muste proue youre selfe to doe some thinges that you were neuer taughte, or els you shall not be able to doe any more than you were taughte: And that were rather to learne by rote (as they call it) then by reason. And agayne there is nothinge in this exaumple or any other of whole numbre, but I haue taughte you the rules of them already.

Scholer. Then I truste by practise to attaine the vse of it. And is this all that I shall learne of Subtraction?

Mayster. Yea, saying that (as you haue seen in Addition) there are nũbers of diuers denominations, in whiche the worke is not muche vnlike, yet without some instructions be geuẽ of it, it might seeme to a learner more difficult, then in deede it is. Therefore I will briefly shewe you the vse of it onely, by one exaumple or two.

A certaine man owed to me 14 li, 12 s, 8 d. of whiche hee payde me at one tyme 4 li, 6 s, 8 d: at an other tyme 3 li: and at an other 2 li, 3 s, 4 d, and laste of all, 6 s, 8 d.

Nowe

Nowe woulde I knowe what remaineth vnpayed yet. therefore I sette my summes thus.

li	s	d
14	12	8
4	6	8
2	5	4
	6	8

Scholer. Sir, I pray you why do you write .li. for the common speache vseth rather to say .40 s.

Mayster. Wee must here vse the denomination that is greatest in anye summe, so that wee maye not write accordinge as wee vse to speake, sayinge: 16 d, 18 d: or likewayes, 3 grotes, 8 grotes: 24 s, 40 s, 48 s, and suche other, but wee must write euery denomination that is in any summe by it selfe, namely shillinges and pundes. So muste wee write for these summes now named, 1 s, 4 d: 1 s, 6 d: 2 s, 4 d: 2 s, 8 d: 1 li, 4 s: 1 li: 1 li, 8 s: and so forth of other like.

Scholer. So that wee maye not write in Arithmetike pennies, when y summe amounteth to shillings, nor shillinges when y summe maketh pundes. Nowe if it please you, ende your example.

Mayster. When my summes are so set as I shewed, then muste I begin with the smallest denomination, sayinge 8, 4, 8, are 20.

which summe because it is pens, and 12 pens doe make 1 s, I muste take from that 20 (whiche commeth of the 3 parcels) 12, & for them write 1 betweene the lines vnder ye shillings, than the 8 d, that remaineth, I take out of the highest summe, whiche is 8 also, and then remayneth nought: wherefore vnder the pens I write nothing. Then come I to the shillings, & gather the parcels 6, 3, 6, whiche with the 1 betweene the lines, make 16, that must I take out of the summe that is ouer it. But seinge that summe is but 12, I cannot take 16 out of 12, I must borowe one of the 14 li, and put to the 12, and that maketh 32, for 1 li, is worth 20 s: then take I 16 out of 32, and there resteth 16 to bee written vnder the shillinges. Then come I to the poundes, whose parcels are 2, 3, 4, that is in all 9, and one more muste I adde thereto, because of the 1 that I borowed before vnto the 12 s, and that is there 10, whiche I must take out of 14, so doth there remaine 4 to bee written vnder the poundes; so doth my remainer appeare to bee

lb	s	d
14	12	8
4	6	8
3		
2	3	4
	6	8
	1	
4	16	

4 li,

4 li, 16 f.

Scholer. This doe I perceaue very well, and if there be none other thing to bee learned in Subtraction, then maye I come to Multiplication, for that you reckened to bee in order nexte.

Mayster. We haue one in deede with the arte of Subtraction, as touchinge the worcking. But yet before we go to Multiplicatiō, I wil instruct you how to examine your worke whether it bee well done or no, and that is by casting away 9 as often as you can finde it, as you did in Addition, sauinge that you muste heere examine the higheste number alone, and note the residue of it at a lines end, as you did in Addition. A proofe of Subtraction in numbers of one denomination.

And when you haue done with the highelte numbre, then examine all the other together, casting thence 9 as often as you can: and if the laste remayner bee like the other, then haue you done well.

But if you haue diuers denominations in your summe, yet for them all shall you make but one seuerall line, as you did in Addition, remembringe to begin the examination at the greatelte denomination, and to double the remayner of pounds, and triple the remayner of A proofe in Subtraction amonge diuerse denominations.

shillings, as you did also in Addition.

As for a proofe, I will examine this worke wherein the highest line I finde of poundes 14, from thence I take 9, & there resteth 5, which I doe double, bycause they are pounds, and then are they 10: thereto I adde the 12, & it maketh 22, from which I take 9 twise, and there resteth 4, which bycause they are shillinges, I triple, and then are they 12, thereto I adde the 8, & then are they 20, thence take I twise 9, and yet resteth 2, which I write at the one ende of a line thus. 2

lb	ʃ	d
14	12	8
4	6	8
3		
2	3	4
	6	8
4	16	

Then I examine all the other parcels and the remaynder together, euery e denomination by it selfe. And first of poundes I finde 4, 1, 2, 4, that is 13, from which I take 9, and there resteth 4, that we I double, and it maketh 8, to it we I put the shillinges, 6, 3, 6, 16, that is 31 (for the one betweene the lines must not bee reckened, nor none in that space) and that maketh in all 39. Where hence I take 9 foure times, and there remayneth 3, that we I take three times, and it is 9, wherefore I call it awaye: than we I take the pennies 8, 4, 8, that

that maketh 20, from whiche I take 9 twyse, and there resteth 2, whiche I write at the other ende of the proofe lyne. And bicause I see that those two numbers are equall, I say that I haue well wrought.

And if you will, you may make for euery denomination a lyne, as you learned in Addition: but then must you begin your examination at the smallest denomination, as you did in Addition, for their proofe is altogither like, sauing that in Addition you examined the nethermoste summe alone, and all the other togither: and in Subtraction ye must examine the highest number alone, and all the other together. And if you marke it well, it is euen all one, for that summe that in Addition is lowest, in Subtraction is highest: and that summe is called the Grosse or Totall summe. Grosse or Totall summe.

Therefore if you marke what I sayde in Addition, you maye easilye perceaue what is to bee wone for the proofe of Subtraction. And to the intent that you may perceaue it the better, I will shewe you an other proofe of Subtraction, and that shall bee by Addition, thus. An other proofe of Subtraction.

Draw vnder the lowest nuber, (which is your remaynder) a lyne: than adde that number, and all the other that you did subtracte before, to-

gether, and write that that amounteth, vnder the lowest line: and if the summe that commeth thereof, bee equall to the highest of the subtraction, than was the subtraciō well wrought, or els not. As for example: in the laste summes, whiche stode thus.

li.	s.	d.
14	12	8
4	6	8
3		
2	3	4
	6	8
1	1	
4	16	

First I adde 8,4,8, that maketh 20. whereof I take 12 awaye, bycause they make one shillinge, and write for them 1 vnder the shillinges: and the 8 that is lefte, I write benethe the lowest line, than adde I the shillinges 6. 3. 6. 1. 16. that make 32: from whiche I take 20, and for it I write 1 vnder the pundes, and the 12 that remayneth, I write vnder the shillinges. Thā come I to ỹ pundes, adding them togither, whiche are 4. 3. 2. 1. 4. that maketh 14: than we I write 14 vnder the li, and so haue I ended the Addition. And I see that the lowest lyne of number and the highest bee lyke, wherefore I knowe that I haue well doone. For my fygures appeare as you may playnely see in the page folowinge.

And

And thus nowe haue I taughte you the arte of Subtraction, and the meanes to proue whether it bee well wroughte or not.

lb.	ß.	d.
14	12	8
4	6	8
3		
2	3	4
	6	8
1	1	
4	16	
14	12	8

Nowe and you remember, I omitted in teaching the proofe of Addition one waye, which I saide was by Subtraction.

Scholer. Truth it is, and than was it deferred, because that I had not then learned the feate of Subtraction, whereby I should haue proued it, but nowe I thanke you, I haue well learned the arte of Subtraction, & the prowes of it, bothe by 9, and by addition. And nowe I woulde be glad to knowe, howe I may prooue Addition by Subtraction.

Mayst. Then marke you this. When you haue ended youre Addition, take the numbers all that you did adde, to the highest summe, and deduct or subtract them from the grosse summe that bothe resulte, and if the remayner bee like to the highest number, then haue you wone wel, els not. The proofe of Addition by Subtraction.

As for example. I take one of the summes

that I did adde before, whiche was this that foloweth here.

1 0 6 8 0 6	
9 4 0 0	
1 1 6 2 0 0	

Then we I come to ỹ middle number (bycause here in this example are onelye three numbers) and subtract that from the nether number, beginninge at the righte hande, and firſt I ſay, 0 out of 0, there remayneth 0: that write I vnder an other lyne. The agayne, 0 in the ſeconde place from 0, remayneth 0, vnder it I write 0 alſo. Next that in the thirde place, 4 out of 2 will not bee, therefore I adde to that 2, 10, and make it 12, from that I take 4, and there reſteth 8. Then ſaye I farther: 9 in the fourthe place, and 1 (whiche I muſte adde for the 10, borowed before) make 10, that muſt I take from 6: and bycaule I cannot, I adde to the 6, 10, and then is it 16: from thence I take 10, and there reſteth 6, to bee written vnder them. Agayne in the fifte place where I finde nothinge written, I muſt ſet 1 for the 10 laſte borowed, and that 1 w I take from the 1 vnder hint, and ſo remayneth nought, wherefore I write wonne a cipher 0. Nowe haue I done with the ſubtraction: & yet in the groſſe ſumme remayneth 1, whiche I muſte ſet righte in the ſame place, in the remayner, and ſo the remayner appeareth to bee

like

like vnto the higheste summe of the Addition, as heere appeareth. Wherefore I saye that the Addition was well wroughte. And note, that if

106800
9400
116200
106800

you had subtracted the vppermoste from ye product or totall summe, than the residue thereof would bee equall to that middlemoste number. But if the parcels whiche you added, bee more then two: (as three, foure, fiue, sixe, or more,) than from youre grosse or totall summe subtract first one of the percelles: and note that new residue. Out of that new residue, subtract an other of youre percelles, (whiche you will) and Note that second new residue. And if you haue no mo percelles added, but three, than is that seconde new residue equall and alike to ye thirde parcell, whiche you haue not (as yet) subtracted, if you haue wrought well: bothe in youre firste Addition, and nowe in youre subtracting. And so in this wise, (if you haue four, fiue, or more percelles) may you proceede to make youre selfe sure of youre totall summe, first, by Addition of the sayde percelles, produced and gathered. And thus may you doe in any other summe of one denomination or manye. Therefore nowe will I make an ende of

Subtraction,

Subtraction, and will instruct you in Multiplication.

MVLTIPLICATION.

Multiplicatiō is such an operation, that by two summes produceth the thirde: whiche thirde summe so many times that containe the first, as there are vnities in the second. And it serueth in the stede of many Additions. As for example. When I would know how many are 30 times 48: if I shoulde adde 48, thirty times, it would be a long worke. Therfore was this worke of Multiplication deuised, whiche shal we that at once, that Addition should doe at many times.

Scholer. I perceaue the commodites of it partely, but I shall not see the full profit of it, till I know the whole vse of it. Therfore sir I beseche you, teache me the working of it.

Multiplication of digites.

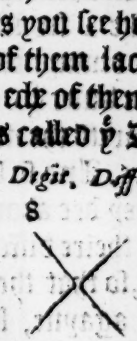
Ma. So I iudge it best, but because that greate summes can not bee multiplyed, but by the multiplication of digites, therfore I thinke best to shew you first the way of multiplying them: As when I saye, 8 times 8, or 8 times

9. &c.

9. &c. And as for the small digits vnder 5, it were but folly to teadx any rule, seing they are so ealy, that euery childe can doe it. But for the multiplication of the greater digits, thus shall you we.

Firste set youre digittes one ouer the other righte, than from the vppermoste wwnewarde, & from ỳ nethermoste vpwarde, drawe straight lines, so that they make à crosse, commonlye called saynt Andrewes crosse, as you see herre. Then loke howe many edx of them lacketh of 10, and wryte that against edx of them, at the ende of the lines, and that is called ỳ Differences: as yf I would know howe manye are 7 times 8, I must wite those digits thus.

The difference.



Then we I loke howe muche 8 dothe differ from 10, and I finde it to bee 2, that 2 we I wite at the right hande of 8, at the ende of the line, thus.

After that I take the differences of 7 likewise from 10, that is 3, and I write that at the right side of 7, as you see in this example.

Then we I drawe à line vnder them, as in Addition, thus.

Laste

Last of all I multiply the two differences, saying: 2 times 3 make 6, that must I euer set vnder the differences, benethe the line: then must I take the one of the differences (which I will, for all is lyke) from the other digit (not from his owne) as the lines of the crosse warne mee, and that ỳ is lefte, must I write vnder ỳ digits. As in this example. If I take 2 from 7, or 3 from 8, there remayneth 5: ỳ 5 must I write vnder ỳ digits: & thẽ there appeareth the multiplication of 7 times 8, to be 56. And so lykewayes of any other digits, yt they bee aboue 5, for if they bee vnder 5, then will theire differences bee greater then themselfe, so that they can not bee taken out of thẽ. And agayne, such little summes euery childe can multiply, as to say: 2 times 3, or 4 times 5, and suche like.

Digit.	*Differen.*
8	2
7	3
5	6

Scholer. Truthe it is. And seing me seemeth that I vnderstande the multiplying of the greater digittes, I will proue by an example how I can woe it. I would knowe how many are 9 times 6.

Mayster. It is all one in valewe to saye 9 times 6, or 6 times 9: but yet the order is best

to

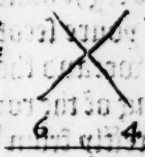
to put the lesse summe first, saying: 6 times 9, and so of all other summes.

Schol. Then would I know, howe many are 6 times 9: therefore I set the digites thus, and make the crosse thus.

Then we I set their differences at the right side: the difference of 9 whiche is 1 against it, and the difference of 6, whiche is 4 againste it also, as in this example.

And vnder them I drawe a line. Then doe I multiply the digites together, saying: one time 4, maketh 4, that 4 doe I write vnder the differences thus.

Then take I one of the differences from the other digite, as one from 6, or else 4 from 9, and eche wayes there resteth 5, whiche I do write vnder the digits thus. And so appeareth the multiplication of 6 tymes 9, to bee 54. Thus I see the feate of this maner of multiplication of digits.



Mayster. Now might you go streight to the multiplication of greater numbers, saue y

bothe for youre ease and suretye in worckinge, I will draw you here a table, whereby shall appeare the multiplication of all digites, and this is it.

1	1	2	3	4	5	6	7	8	9
	2	4	6	8	10	12	14	16	18
		3	9	12	15	18	21	24	27
			4	16	20	24	28	32	36
				5	25	30	35	40	45
					6	36	42	48	54
						7	49	56	63
							8	64	72
								9	81

In whiche figure, whan you woulde knowe the produkte in anye multiplication of digites, seeke youre firste or laste digite in the greate figures, and from it go right forthe towarde the right hande, till you come vnder the numbre of youre seconde digite, whiche is in the highest rowe: and than the numbre that is in the meeting of the rowes of little squares (whiche come directly from bothe youre propounded digits) is the multiplication that amounteth of theym. As yf I woulde knowe by this table the multiplication of 7 times 9, seeke firste 7 in ỹ greate-

ter

ter figures, and than go right forthe towarde the right hande, tyll you come vnder 9 of the highest rowe, in whiche place, where you so come vnder the other digitte (as here for example you come vnder 9) is alwaies contayned the ofcome or product, whiche you seeke: and that place wee terme to bee in the commō angle, in respect of the two numbers so taken on the outsides, as here in that common angle, where the rewes of little squares (directly proceeding from 7 and 9) doe meete, you haue 63: whiche 63, is the summe of the multiplication of 9 by 7.

Scholer. This is very good and ready. And so may I finde the multiplication of anye dygittes. But nowe how shall I dwe in greater summes?

Mayster. When you woulde multiply any summe by an other, you shall marke that it is the meetest order to sette the greatest number highest, whiche is the place of the number that must be multiplied: and likewaies the lesser number vnder it, for that is the place of the Multiplyer or multiplicatour, that is to say, y number by whiche multiplication is made: & is in Englishe alwayes put before this worde, Tymes: in suche speakinge when I saye, 2 or

times 70. And the number that foloweth this worde Tymes, is that whiche muste bee multiplyed.

Therefore when I would multiply one nũber by an other, I muste write the greatest, highest, and the lesser vnder it, as in Addition. And vnder them muste I drawe a lyne. As for example: If I would multiply 264 by 29, I must set them thus.

264
29

Than must I multiply euery figure of the higher rowe, by euery figure of the nether rowe: and that that amounteth, I must sette vnder the lyne, as thus. First I doe multiply 4 by 9, sayinge: 9 times 4 (or 4 times 9, whiche is all one) and that maketh 36, as ỹ table before of digits doth declare. of that 36 I must write the 6 that is the digitte, vnder the 9, and the 3 in the next place towarde the left hande.

264
29
36

Then come I to the ij. figure of the higher rowe, and say: 9 times 6 make 54, of whiche I write ỹ 4 vnder the 3, and the 5 vnder the next place (as the reason willeth mee) thus.

264
29
5436

After that come I to the next figure, whiche is 2, and doe multiply

tiply it by 9, and that maketh 18: whereof I write 8 vnder the thirde place, and the article 1 in the fourthe place, thus.

And so haue I ended the firſt figure of the multiplier. Wherfore I giue it nowe a fine dashe with my pen.

Then beginne I with the nexte figure, and multiply it into all the higher figures, as thus.

First, 2 times 4, make 8, that w I write vnder ỹ seconde place: for euermore the digitte or firſte figure of the multiplication that amounteth of the firſte figure of the higher nũber, muſt bee ſet vnder the multiplier of it, and the other in their order, towarde the left hande.

Scholer. I vnderſtande you thus: that the digit of the ſumme amounting of the multiplication of the firſte figure of the higher rowe, by the firſt fygure of the lower rowe or multiplyer, muſt be ſet vnder the firſt place: and that that amounteth of the ſame firſte figure by the ſeconde multiplyer, muſt bee ſet vnder the ſeconde place: and ſo of ỹ other, if there be more multiplyers.

Mayſter. So meane I indeede: and if

there amount but a digit, then muste it bee set vnder the multiplier.

And now to go foorth: I multiplye by the same 2, the seconde figure of the higher rowe, whiche is 6, sayinge: 2 times 6, make 12: whereof I wryte the digit 2 vnder the third place, and the article 1, I write vnder the fourth place.

```
  2 6 4
    2 9
-------
1 5 3 6
1 8 4
  2 8
```

Then do I multiply the laste figure of the higher summe, by that same 2, sayinge: 2 times 2 is 4: whiche I write vnder the fourth place. And so haue I ended the whole multiplication: wherfore I also giue the 2 à dash with my pen, thus: and so I do euer assone as I haue dispatched any digit by whiche I multiply. And the summes stande thus.

```
  2 6 4
    2 9
-------
1 5 3 6
1 8 4
4 2 8
```

Than muste I drawe à line vnder all those summes that amounte of the multiplication, and must adde all them into one summe, as in this example you may see.

```
  2 6 4
    2 9
-------
1 5 3 6
1 8 4
4 2 8
-------
7 6 5 6
```

Where in the first place I finde but 6, and therefore write I it vnder the line. Then in the seconde

place

place 843, make 15, whereof I write 5, and keepe one in my minde, and so forth, as you learned in Addition. And so appeareth the whole summe to be 7656, whiche amounteth of the multiplication of 264 by 29.

Scholer. If there be no more to be obserued in it, then can I doe it, I suppose, as by this example I shall proue. I would multiplye 1365, by 236, wherefore I set them thus.

1365
236

Than doe I multiplye 5 by 6, saying: 6 times 5, make 30: of whiche I write the ciphre in the first place, and the article 3 in the seconde place.

1365
236
0

Then do I by the same 6, multiplye the seconde figure of the higher summe, whiche is 6, saynge: 6 times 6, make 36: of whiche I write the 6 vnder the seconde place, and the 3 vnder the thirde place.

1365
236
30
6

Then we I multiply the thirde figure whiche is 3 by the same 6, and they maketh 18: of that I set the 8 vnder the thirde place, and the 1 in the fourthe place.

1365
236
30
86

Than come I to the laſt figure of the higher ſumme, and multiplye it by 6, ſayinge: 6 times 1 make 6: that doe I wryte vnder the fourthe place.

1365
236
1330
686

And ſo haue I ended the firſt multiplier, and dathe him ſlightlye w̄ my pen.

Than begin I with the ſeconde multiplyer, & ſaye firſte, 6 times 5, that maketh 15, of whiche I ſette the 5 vnder the ſeconde place, bycauſe that the multiplyer is there, and the 1 I ſet vnder the thirde place.

1365
236
1330
686
15

Than come I to the ſeconde figure, that is 6, and multiply it by 3, whiche maketh 18, of whiche I ſet the 8 vnder the thirde place, and the article 1 in the fourth place.

1365
236
1330
686
115
8

Than come I to the thirde figure, whiche is 3, and multiply it by 3, ſayinge: 3 times 3, make 9, whiche becauſe it is but one digit, I ſet vnder the fourthe place.

1365
236
1330
686
115
98

And then comminge to the laſte

laſte figure 1, I multiply it by 3, and it maketh 3, whiche I ſet in the fift place, and then haue I ended two of the multipliers, and ỹ ſummes ſtand thus. And then I giue 3 his dalye.

1365	1365
236	236
1330	1330
686	686
115	115
398	398
10	2

Then come I to the thirde multiplier, and multiplye it into euery figure of the higher ſumme, and firſte I ſaye: 2 times 5 makes 10, of whiche I ſet the cipher vnder the multiplier in the thirde place, and the article 1 in the fourth place.

And ſo multiplying the ſeconde figure 6 by that ſame 2, there amoũteth 12: wherof I wyte the digitte 2, vnder the fourth place, & the article 1, vnder in the fifte place.

Nowe doe I multiplye by the thirde figure of the higher ſumme, whiche is 3, and that maketh 6: whiche I ſette vnder the fifte place, as appeareth in the page folowinge.

```
 1365
  236
 ----
 1330
 686
 115
398
110
62
```

Than come I to the laste place, and multiplye that 1 by 2, and there amounteth 2, whiche I sette in the sixt place, & then do the ye summes stande thus.

```
 1365
  236
 ----
 1330
 686
 115
398
110
262
 ------
322140
```

And so haue I ended the whole multiplication.

But now (as you taught mee) to knowe what this whole summe is, I muste adde all those parcels together, and then vnder the line will appeare as you see, the grosse or totall summe, that is, 322140.

Ma. That is well done.

Scholer. Than me thinketh I would call it well done, when I knewe whether I had well done or no.

Mayster. It may be tryed by 9, as addition was; but the surest proofe is by Diuision, and therefore I will reserue that till you haue learned the arte of Diuision.

And before wee passe from Multiplication,

I

I will yet shew you other waies of Multiplication, whiche are counted of some men bothe moze ready and moze certaine, of whiche the one differeth nothinge from this that I haue taught you, saue that it doeth vnderstande alwaies the articles, and ioyne them to the next summe, and therefore I will declare it onelye by an example.

If I woulde multiplye 1542, by 365, I must set them as I saide before, and then doe I multiply 2 by 5, and it maketh 10, of whiche I write the article vnder the first place, and keepe the digit 1 in my minde.

Than say I forth 5 times 4 doe make 20, and the 1 in my minde, are 21, thereof I write the 1 vnder the seconde place, and kepe the 2 in my minde.

Then come I to the thirde figure 5, saying 5 times 5, make 25, and the 2 in my minde, make 27, whereof I write the 7 vnder the thirde place, and keepe the article 2 in my minde.

Then comming to the laste figure,

1542	
365	
	10
1542	
365	
	210
1542	
365	
	7210

I saye: 5 times 1 make 5, and 2 in my minde make 7: that doe I write vnder the fourth place.

And then haue I ended my first multiplier, and therfore I dashe it.

Than doe I likewaies with the seconde multiplier, saying: 6 times 2 make 12, thereof I write the digit 2 vnder the seconde place, and keepe the article 1 in my minde.

Then say I forth: 6 times 4 maketh 24, and 1 in my minde make 25, so I set that 5 vnder the thirde place, and keepe the 2 in my minde.

Than multiply I forth, saying: 6 tymes 5, maketh 30, and 2 in minde make 32, whereof I write the 2 vnder the iiij. place, and keepe the 3 in my minde.

Then doe I multiply the laste figure 1 by 6, and it maketh 6, to that I adde ỹ 3 in my minde, and it maketh 9, which I write in the fifte place.

And so haue I ended ij. figures of the multiplier.

Than with the thirde

1542	
365	
7710	
1542	
365	
7710	
2	
1542	
365	
7710	
52	
1542	
365	
7710	
2312	
1542	
365	
7710	
9252	

laste

laſte multiplier, doe I likewayes, and ſaye firſte: 3 times 2, make 6: whiche I wrize in the thirde place vnder the multiplyer. Than by that 3 doe I multiplye likewayes the ſeconde figure 4, and it maketh 12, whereof I wrize the digit 2 vnder the fourth place, and the article 1 I keepe in minde.

```
1542
 365
7710
9252
  26
```

Than come I to the thirde figure 5, ſayinge: 3 times 5 maketh 15, and the 1 in my minde make 16, thereof I wrize the 6 vnder ẏ fifte place, and keepe the article 1 in my minde.

```
1542
 365
7710
9252
 626
```

Than come I to the laſte figure whiche is 1, and multiply it by 3, and it maketh 3, thereto I adde the 1 in my minde, and it maketh 4, whiche I wrize in the 6 place. And than haue I ended the multiplication, and the figures ſtande in order thus.

```
1542
 365
7710
9252
4626
```

Whiche parcels if I adde into one ſumme,

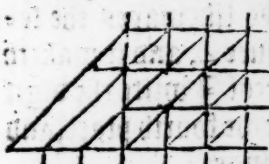
it

it will be 562830, whiche is the grosse or totall summe of all that multiplication.

Scholer Well, this manner of Multiplication I perceaue: but what other sortes haue you?

An other waye of Multiplication.

Mayst. There is one waye that is wrought by a checker table, made thus.



Loke how many places your sume hathe that you woulde multiply, so many squares muste you make in youre table, from the right side to the lefte: and so manye from the higher parte to the lower, as there be places in youre multiplyer. Then set downe youre greatest summe firste on the toppe of the table, euery figure in due order, in a square alone, I meane in those squares that be open & vncrossed. And likewayes in those like squares at the right hand, set downe youre multiplicatour or multiplier, the last figure in the highest place, and so downewarde, that the firste figure maye be in the lowest place.

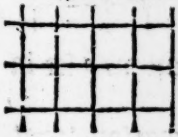
Scholer. Sir if it please you, me thinketh then I vnderstand you best, when you do not stand long in telling the rule before examples: But propose some example, and then in declaringe

ring it, bying in the rules withall.

Mayster. In deede, that way is easiest for a yong learner, therfore will I euen so do. Take this example: nowe I would multiply 2036 by 25.

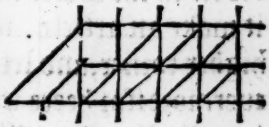
First I consider that my greatest numbre hath foure figures or places, & therfore I make so manye roumes betweene lines, thus.

Than I see that of my multiplyers there are two, wherefore I drawe so manye lines a crosse the other, that there may bee two roumes betweene them, thus.



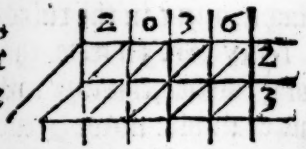
But you muste not forget to let the endes of the lines runne out, as it appeareth in this Patrō, for in those open squares must your two firste numbres, and all ỹ totall summe bee set.

Than drawe a crosse bar through euery close square, so that it maye reache wnne to the lowest ouerthwart lyne, as in this forme. And than is youre checker forme prepared.



Than sette wnne youre firste or greatest

summe



summe on the toppe, and youre multiplier on the right side in the open squares, thus.

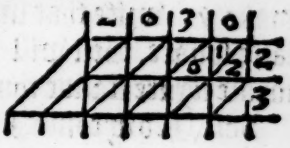
Than begin to multiply the firste figure of the higher summe, by the highest of the multiplier, saying: 2 times 6 make 12, that 12 must you write in the square that is agaynste the 2 and the 6, but in suche manner that the digitte bee set in the nether corner of the square, and the article in the higher corner: as you may see in this example.

And so of euery other multiplication, what euer amounteth you muste write in the common square, whiche is agaynste bothe those figures, by whiche you we multiply. And if that summe we make but one digitte, than muste it bee set in the lower corner of the square, but yf it make an article, than write the article in ỹ higher corner, and let ỹ cyphre go (if you will) euermore: for heere it serueth for nothinge, seinge the lines we distincte the places: but if the summe amounting of suche multiplication we make à mixt numbre, then write the article in the higher corner, and the digitte in the lower

corner,

corner, as I did by that 12.

Then whan you haue multiplyed and ended the first figure, come to the nexte, and multiply it in like manner, as in sayinge: 2 times 3, is 6: that 6, bicause it is but a digit, you shall set in the nether corner of the square, next vnder 3, thus.



Than go forthe, saying: 2 times 0 is 0: set that vnder the barre (if you liste) in the third square.

Than forthe and saye: 2 times 2 make 4, that set in ỹ laste square vnder the barre, so haue you ended the firste multiplier: Washe him.

Come nowe to the seconde multiplier, and say: 3 times 6, make 18, of whiche summe, ỹ article 1 muste be set aboue the barre, in the square that is next to ỹ 3 (as you se here) and the 8 vnder the barre.

Then say: 3 times 3, make 9, set it in the next square beneth the barre. Then 3 times 0 is 0, write it in the nexte square, or let it go, for all is one.

Scholer. I perceaue it well: for here the lines

lines distincte the places, wherefore ciphers we onely serue, and therefore here they neede not to bee.

Mayster. Then say farther: 3 tymes 2, make 6: write that in the laste square, then will the whole figure stande thus.

Sch. Now could I (me seemeth) we like againe. But how shall I we now to gather the summe?

Mayster. Marke firste the order of the places in this figure, and so shall you perceaue the reason of gathering them into a summe.

The sloape barres we part the places, so that the firste place is ỳ lowest corner (in all such figures) of the nethermost square next the right hand: and all the halfe squares betweene that barre and the next, standeth for the seconde place, and so the roome betweene that and the next barre, is the third place: & so forth. Now if you perceaue this, then muste you adde all the figures of one place together, as if you had an Additon of diuers summes.

Scholer. If I vnderstand you right, then must I take here in this example 8 to be in the firste place: 9, 1 and 2, in the seconde: 0, 6, 1, in the third: 6, 0, in the fourth: 4 in the laste: and

the

the firſt place hath no figure.

Mayſter. You ſay well, and the reaſon is becauſe the multiplication ſeruing to ỹ ſquare, made but a digit.

Scholer. Then it is all one, as if they ſtode thus.

Mayſter. Euen ſo it is: and now adde this ſumme, and there will appeare the totall of the multiplication to bee 4 6 8 2 8.

And if you will ſee the agreement of this manner of multiplication, and the other that you learned before, then multiply thoſe two ſummes (that is 2 0 3 6, by 2 3) after the firſte manner without ſquares.

A proofe without ſquares.

Scholer. You meane to ſet them thus in order.

And then multiply 3 into 6 make 18: 3 times 3 make 9, 3 times 0 is 0, then 3 times 2 make 6: which muſt be ſet thus,

Then we ſhall likewaies with ỹ ſeconde multiplier, ſayinge: 2 times 6 make 12, 2 times 3 make 6, 2 times 0 is 0, and 2 times 2 make 4, which when I adde to the other, then will

ẏ whole multiplication stande thus.

2036
23
18
609
12
406
46828

Mayster. So that you may see in euery place the same figures, as they were in ẏ multiplication by squares, though they differ in height an lownesse of places, but being added togither, they make one summe.

And thus nowe ye haue learned three sortes of multiplication, whiche you liketh best, that may you vse.

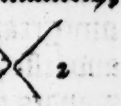
Yet are there other formes, but sith they nothinge differ from these three in effect, but onely in setting of the numbers, I will ouerpasse them till à more meeter place and tyme. And nowe will I instruct you in Diuision, so that you thinke your selfe sufficientlye to perceaue what I haue taught you.

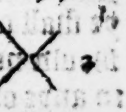
Scholer. Yes sir I thanke you. but I doe not perceaue howe to examine my woorke, to try whether I haue well done or no.

Mayster. That is commonly vsed by the proofe of 9, as you learned before in Addition and Subtraction, saue that it hath this waies diuers from them.

Firste

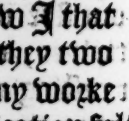
First you must make à crosse after this manner.

Then must you examine youre summe that shoulde bee multiplied, and loke what remayneth after casting away of 9, that set you at the one side of the crosse: then examine the multiplyer, and whatsoeuer remayneth in it, after casting away 9 as often as you can, write that at the other side of the crosse: then must you multiplye those two numbres togither, and loke what amounteth thereof, yf it bee vnder 9, write it at the higher parte of ỹ crosse: but if it bee aboue 9, then take thence 9 as often as you can, and write the reste at the head of the crosse. As in the laste exaumple of multiplication, the numbre to bee multiplied is 2036, wherin is ones 9, and 2 remaineth, whiche I write at one side of à crosse, thus. 

Then do I examine the multiplier, whiche is 23, wherin there is no 9, but 5 in all, that 5 therfore I sette at the other side of the crosse, thus. 

Then doe I multiply 5 by 2, and it maketh 10, from whiche I withdrawe 9, and there resteth 1, that 1 doe I set at the head of the crosse, than doe I examine the grosse

summe amountinge of the multiplication, whiche is 46828, wherein I finde 9 three times and 1 remaining, that 1 I set at the foote of the crosse, and then I see it to agree with the other 1 at the top of the crosse, and so know I that I haue one wel: for if they two did differ, than were my worke vayne, and the multiplication false.



This is the common proffe, but the moste certaine proffe is by Diuision, of whiche I wil anone instructe you.

Scholer. Syr, what is the cheife vse of Multiplication?

Mayster. The vse of it is greater then you can yet vnderstãde: howbeit, the saplayne commodities it hathe, that if you woulde resolue any great and whole nũbre into many small and lesse portions: as if you woulde chaunge pundes into shillinges, pennies, or any other greater or smaller parcels, by multiplication, ye shall do it speedily and easily. Also if you shoulde neede to adde one summe to it selfe, or to anye other oftentimes, you shall do it by Multiplication much more speedily, readily, easilye and surelye, than by often and tedious Additions. Take you these commodities

grossely

grossely shewed for an aunswere at this time, and hereafter I will more abundantlye make you to perceiue the vse of it.

DIVISION.

Scholer.

Ell sir, then in Diuision I praye you to instruct me. But me thinketh by the name of it, that it should be al one with Multiplication: for I call that Diuision, when any thing is parted into diuers and many partes.

Mayster. You take it as it is taken commonlye, howbeit, if you marke well, you shall perceiue that it is quite contrary to Multiplication, and withe not parte one thinge or fewe things into many, but contrary wayes it bringeth manye parcels into fewe, but yet so, that these fewe taken together, are equall in valure to the other many: for by Diuision pennies are tourned into shillinges, and shillinges into pondes: as for example of 120 shillinges, it

maketh 6 pound, so are 1 2 0 tourned into 6, whiche is a smaller number: but then if you consider the denominatours, you shall see that they are sudy, that one of the latter is equall to 2 0 of the first, and so in valewe the summes are one, though in number they do farre differ, and the latter summe is the lesser, and so is it alwayes in diuision, how be it, yet in the working, the summe is parted by an other, & therof doth it take the name.

Scholer. I thinke I shall better understand the reason of the name, when I knowe the vse of the worke, therefore nowe would I gladly learne that.

Diuision what it is

Mayster. Diuision is a distributinge of a greater summe by the vnitees of a lesser. Or Diuision is an Arithmeticall producing of a thyrd number, in respecte of two propounded numbers: whiche third number shall so often contayne an vnit, as the greater of the two propounded numbers doth containe the lesser. So that, euen as Multiplication did seeme to serue in steed of manye Additions, so Diuision may seeme to bee in place of many Subtractions: Because that thyrd number briefly expresseth, how many times the lesser of your two propound numbers may bee Subtracted,

from

from ỳ greater: As in practise will more plainly appeare. Therefore (as you may perceaue) vnto Diuision are required three numbers: the first, whiche should be diuided, and that muste (generally) be the greater: and the seconde, by whiche the other muste be diuided, and that is (generally) the lesser, and is called the Diuisor. And the thirde whiche aunswereth to the question, How many times: and therefore is called the Quotient.

The firste must be firste written, and the seconde so set vnder it, that the laste figure of the lower number bee ryght vnder the laste of the higher, contrary wayes to the worke of the other kindes of Arithmetike: for in them the two firste figures were set euer meete one vnder ỳ other, but in Diuision the laste figures must bee set meete, except it chaunce so, that the laste figure of ỳ Diuisor be greater then the laste of the hygher number, for then you shall set the laste of the Diuisor, vnder the last (saue one) of the higher number, as for example.

A general rule for placinge the figures.

An excepti-on.

If you shoulde diuide 365 (whiche are the summe of the dayes of a yeare) by 28, whiche are the dayes of a common moneth, then shoulde you set them thus.

365	
28	

But if you woulde diuide those 365 dayes, by 52, whiche is the number of weekes in one yeare, then should you set them thus.

365
52

Likewayes if I woulde diuide the same 365 by 4, whiche is the summe of the quarters of a yeare, then muste I set them thus.

365
4

Scholer. Syr, this we I vnderstande, but howe nowe shoulde I we to diuide the one by the other?

Mayster. You must beginne with the laste figure next the left hande, and see howe many times the laste figure of the diuisor may be taken out of the laste figure of the ouer number, and that shall you note within a crooked lyne toward your right hande. As for example.

I woulde diuide 365 by 28, then set I those two summes thus.

365 (
28

And I loke how many times I may finde 2 (whiche is the laste figure of the diuisor) in 3, (whiche is the laste of the number to be diuided) and considering that I can take 2 out of 3 but once, I make a crooked lyne at the right hand of the numbers, e with in it

in it I set 1, and that is called the Quotient number, as I tolde you. Then because that when 2 is taken out of 3, there remaineth 1, I must write that 1 ouer 3, and deface or cancell the 3 and the 2, then wyll they figures stand thus.

Quotient number.

1
3 6 5 (1
2 8

Than muste I go to the nexte figure of the diuisor, and take it likewaies so many tymes out of the figures that bee ouer it, and looke what doth remayne, that I muste write ouer them, and cancell them, as in this example.

Therefore nowe I take once 8 out of 16, and there remayneth 8, whiche I muste set ouer the 6, and cancell or crosse out the 16, and the 8 of the diuisor: And then will the figures stande thus.

1 8
3 6 5 (1
2 8

And so haue I once wrought.

Scholer. So I perceaue that you take the nether figure not onely out of the other that is ryght ouer him, but out of that with the other also that remayneth before, and are written toward the left hande.

Mayster. So must you doe: for you must so take the diuisor out of the ouer number, that there remaine not ouer it so great a summe as

it selfe is, for then were youre worke in vaine.

But yet agayne here must you marke, that when you seeke how manye times the laste figure of the diuisor may bee founde in the number ouer him, that you looke also whether you may as often finde all the figures folowing in those that are aboue them, (considering all the remainders if there bee any) if not, take youre Quotient lesse by one, and then proue againe, & so still, till you finde á meete Quotient: And by that meete quotient must you alwaies multiply your diuisor, and the product sette vnder your diuisor, so that his first figure stande vnder the firste figure of your diuisor, and the seconde vnder the seconde, and so forth: and then subtract that product from the number to be diuided, that standeth directly ouer it, as you haue seene mee doe.

Note.

☞ Whan you haue thus wrought once, than must you begin agayne, and write your diuisour á new, nearer towarde the right hande by one place, as in this example, you shall set 2 vnder 8, and 8 vnder 5, thus.

~~5~~	8		
3	~~6~~	5	(1
~~2~~	~~8~~	8	
	2		

Then (as before) seeke howe many times you may take your diuisor out of the

the number ouer him now.

Scholer. That may I doe here 4 times.

Mayster. Truth it is that you may finde 2, foure times in 8: but than marke whether you can finde the figure following so manye times in the other that is ouer him. Can you finde 8 foure times in 5?

Scholer. No, nother yet once.

Mayster. Therefore take 2 out of 8, once lesse.

Scholer. That is 3 times.

Mayster. Well, then 3 times 2 make 6: if I take 6 out of 8, there remaineth 2: whiche 2 with the 5 folowinge, make 25, in whiche summe I maye finde 8 iij times also, and therefore I take 3 as a trewe quotient, and write it within the crooked line of the quotient, before the one, thus.

Marke howe to consider this kinde of remainer.

8	
365	(13
8	

Then saye I: 3 timus 2 make 6, than 6 out of 8, resteth 2, therefore I cancell the 2 and the 8, and write ouer it the 2 by dothe remaine, thus.

	2
8	
365	(13
8	

Then doe I take 8 as many times out of 25, saying: 3 times 8 make 24, and if I take 24

	x

out

out of 25, there remayneth 1: so then I cancell 25 and 8, and ouer the 5 I set 1, thus. Or you mought (after you had found 3 to bee a fit quotiente) straight way haue multiplyed the whole diuisor 28, by that 3, at once: which geueth 84, whiche beinge set vnder 28, and duely subtracted from 85, of the number diuidend, giueth 1, the remayner of the whole diuisiō: as before you had. worke which way you list: here you see also ye forme.

```
   2
 1 8 1
 3 6 5 (1 3
 2 8 8
   2
```

```
   8 1
 3 6 5 (1 3
 2 8 8
   2 4
   8
```

And now haue I done with diuidinge, for I can finde my diuisour 28 no more in the ouer summe.

Scholer. No, except you woulde parte the 1 that remayneth, into 28 partes.

☞ Mayster. That is well sayde, and so must we doe in suche cases, when there remaineth any thing: but I will let that passe now, & will make you perfecte in diuision of whole numbers, and will hereafter teache you peculiarly of broken numbers, called Fractions.

Now yf you doe perceaue the order of diuision, then doe you diuide this summe, 136280

by

by 452.

Scholer. Firste I sette downe the numbre that should be diuided, than do I set the diuisour vnder it, so ỳ the laste figure of it be right vnder the laste figure of the ouer number. Than will it be thus.

1	3	6	2	8	0	
		4	5	2		

Mayster. Can you take ỳ laste of your diuisour (whiche is 4) out of 1, whiche is the laste of the ouer number?

Scholer. I had forgotten, because the laste of the diuisor cannot bee taken out of the laste of the ouer numbre, in as muche as it is ỳ greater, therfore must I set the diuisor one place more forewarde towarde the righte hande, thus.

1	3	6	2	8	0	
			4	5	2	

And then muste I loke howe often I maye finde the laste figure of the diuisour (that is 4) in 13, whiche thinge I may we 3 times, therefore we I saye: 3 times 4 is 12, whiche I take out of 13, and there remayneth 1. Then we I make at the righte hande of my summes a croked line, and write before it my quottient 3: and I cancell 13 and 4, and ouer the 3 I set the 1 that remaineth, and than the figures stande thus.

1	3	6	2	8	0	(3
			4	5	2	

Than

Than do I multiply the same quotient into euery figure of the diuisor, and withdrawe the summe that amounteth out of ẏ numbres ouer theim, as firste I saye: 3 times 5, make 15, whiche I take from 16, and there resteth 1, I cancel therfore 16 and 5, and wite ouer the 6 that 1 that remaineth, thus.

Than doe I say likewayes, 3 times 2, makes 6, whiche I take out of 12, and there resteth 6, therefore I cancell the 12 and the 2, and ouer the 2 I wite 6 that remaineth, thus.

Then shoulde I set forwarde the diuisor, into the next place towarde ẏ right hande, thus.

Mayster. But you may see, that ouer the 4 is no figure, therfore muste I set the diuisor yet forwarder by an other place.

And marke, whan so euer it chaunceth so, ẏ you should set forwarde the diuisor, and that it can not stande there, bycause there is no nũbre ouer the laste place, or if there bee any, it is

lesser

lesser then the laste figure of the diuisor, than must you remoue the diuisor yet ones agayne: and bicause that his firste place of remouinge serued not to subtracte him so muche as once, therefore shall you write in the quotient à cypher 0. And if you shoulde by chaunce neede to doe so often times, for euery time write à ciphre in the quotient. The reason of this, will I shewe you hereafter.

Scholer. Then must I set my summes thus.

And bicause I remoued the diuisor, so that I ouerskipped one place, I must write à cipher in the quotient: & than must I seeke ànew quotient, as in this example I must say, How many times 4 is there in 6? and sith it can bee but once, therefore doe I write 1 in the quotient, & then say I: 1 times 4, taken out of 6, remaineth 2, I cancell the 6 and the 4, & write 2 ouer them thus.

Then saye I agayne, ones 5 out of 28, remay-

```
 ~~1~~ ~~1~~ 6
~~1~~ ~~3~~ ~~6~~ ~~2~~ 8 0
 ~~4~~ ~~5~~ ~~2~~ 5 2 (30
      4

      2
 ~~1~~ ~~1~~ ~~6~~
~~1~~ ~~3~~ ~~6~~ ~~2~~ 8 0 (30 1
 4 ~~5~~ ~~2~~ 5 2
      4

      2
 ~~1~~ ~~1~~ ~~6~~
~~1~~ ~~3~~ ~~6~~ ~~2~~ 8 0 (30 1
 ~~4~~ ~~5~~ ~~2~~ 5 2
      ~~4~~
```

neth

neth 23, I let the 2 stande as it did, and ouer ỹ 8 I set 3, cancelling the 8 and the 3 vnder it, thus.

2
4 4 6 3

Mayster. You might as well haue saide, ones 5 out of 8, and so remaineth 3, but nowe go forthe.

4 3 6 4 8 0 (301
4 5 4 5 2
4

Scholer. Than ones 2 out of 0, can not be, what shall I now we?

Mayster. Borowe of the nexte number that is behinde (for there is 230) and we as you learned in Subtraction in alike case.

Scholer. Than muste I borowe 1 of the 3 comminge behinde nexte, and make that 0 to bee 10: and then take I 2 out of 10, & there resteth 8. And bicause I borowed one of the 3, I muste cancell the 3, and write 2 ouer it: than with the figure stande thus.

2 2
4 4 6 3 8
4 3 6 4 8 0 (301
4 5 4 5 4

Mayster. Now haue you wone, and yet remayneth 228, and your quotient with shew you, that if you diuide 136280 by 452, you shall finde your diuisour in your greater number 301, that is CCC times, and ones 228 remayninge.

4

And in the other example, where I diuided

365 by 28, the quotient was 13, and 1 remayned: whereby I knowe that in a yeare (which contayneth 365 dayes) there are 13 monethes, reckening 28 dayes (or 4 weekes) iuste to a moneth, and 1 daye more.

Scholer. Why than we wee call a yeare but 12 monethes?

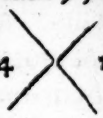
Mayster. Of that at a more conuenient time will I fullye instructe you: but nowe it is not conuenient to entangle your minde with other thinges, than doe directly pertayne to your matter. Therefore if you can remember what you haue hearde, you haue learned a short māner of diuision, which I woulde haue you often to practise, so that you may be perfect in it, and hereafter I will shewe you certaine other proper pointes touching it.

Scholer. Then I pray you, yet tell mee, how I shall examine and trie my worke, whether I haue done well or no, that though no man be by me to tell mee, yet I may perceaue it my selfe.

Mayster. Some men (yea and commonly) doe trie it by the rule of 9, as in all the other kindes, saue that their order is this. First they cast away 9, as often as they can, out of the diuisor, and that that remayneth, they set at Proofe.

one side of à crosse: As in our first example, the diuisor was 28, from whiche you may take 9 three times & 1 remayneth, which they set by à cros, thus.

Than we they likewise examine the quotient (whiche in our example is 13) and from thence they casse away 9 as often as they can, and the remainer they set at the other side of the crosse, and then multiply they together those ij. remainers: and to it that amounteth they adde the remainer of the diuision, if there were any, from that whole summe they withdrawe 9 as often as they can, and the resl they set at the heade of the crosse: as in our example the quotient is 13, from whiche take 9, and there remaineth onelye 4, and therefore must you set 4 at the other side of the crosse, thus.



Then multiply 4 by 1, and it yeldeth but 4, thereto adde the remainer of the diuision (whiche was 1) and it will bee 5, whiche summe doth not amounte to 9, & therefore must be set wholy at the head of the crosse, as you see here.

And this number on the head of the crosse, is the first prwfe, to whiche if you finde

finde an other like in the number that was diuided, then haue you done well.

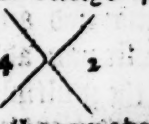
Therfore nowe shall you likewise examine the whole summe that was diuided, and take away 9 as often as you can, and that that remayneth, set at the foote of the crosse: and if it be equall to that in the head of the crosse, than haue you well done, els not.

As in our example the whole summe was 365, whiche maketh 14, from that take 9, and there resteth 5, whiche set at the foote of the cros, thus.

And you shall see that they agree: therefore haue you well done.

Scholer. Nowe will I likewise examine our seconde example, where ỹ diuisor was 452 whiche maketh 11: from thence I take 9, and the 2 that remayneth I sette at the righte side of a crosse, thus.

Than examine I the quotient, whiche was 301, where I finde but onelye 4, that do I set at the other side of the crosse, thus.



Than me I multiply 4 by 2, & it maketh 8, to that me I adde the remainer

of the diuision (whiche was 228, and maketh 12) and they two make 20, wherein I finde twise 9, and 2 remaining, that 2 muste I set at the head of the crosse, thus.

Then we I examine they whole number to bee diuided, whiche was 136280, where I finde twise 9, and 2 remayning, whiche I set at ỹ foote of ỹ crosse, thus.

And because that it wth agree with the figure at the head of the crosse, I knowe ỹ the diuision was well wrought.

Mayster. This is the common proue: how be it, the more certaine working is by the contrary kinde, as to proue Diuision by Multiplication, thus.

A proofe by Multiplication.

Multiply the quotient by the diuisor, and if the summe that amounteth bee equall to the summe that should bee diuided, then haue you well diuided, else not.

Howbeit, this must you marke, that if there remayned anye thinge after the diuision, that muste you adde to the summe that amounteth of the multiplication: as in our first example the quotient was 13, and the diuisor was 28:

Nowe

Nowe multiplye the one by the other, and the ſumme will bee 364: to that if you adde the one that remained after the diuiſion, than will it be 365, whiche was the ſumme that ſhould be diuided, and therefore I know that I haue well wne.

Scholer. Nowe will I proue the ſame in the ſecond example, whoſe diuiſor was 452, and the quotient 301: theſe we I multiply together, and there amounteth 136052, to whiche if I adde the 228 that remayned, then will it bee 136280, whiche was the whole ſumme to bee diuided, and therefore I perceiue that I haue well wne.

Mayſter. This is the ſureſte waye to examine Diuiſion by Multiplication: and contrary wayes ẏ ſureſt proſe of Multiplication, is Diuiſion.

And therefore now will I ſhewe howe you may proue Multiplication by Diuiſion.

When you haue ended multiplication, and woulde knowe whether you haue well wne or not, ſette the groſſe ſumme that amounteth of the Multiplication ouermoſt, and diuide it by ẏ multiplier: and if the quotient bee the ſame number that ſhould bee multiplyed, then haue you well wrought, els not: as in that exam- A proofe of Multiplication by Diuiſion.

ple where we multiplied 264, by 29, the gros ſumme was 7656.

Now if you will know whether that multiplication be trewe, you ſhall diuide that 7656, by the Multiplier, 29: and you ſhall perceiue that the quotient will bee 264, and that is a token that you haue well wrought.

Scholer. By youre pacience I will proue that: and firſte I ſet wwne the groſſe ſumme & the Multiplier, not after the rule of Multiplication, but after the rule of Diuiſion, for nowe that numbre is become the diuiſor, that was before the multiplier, I ſhall ſet theim therefore thus.

7656
29

Then ſhall I ſeeke howe many times 2 in 7, that may bee 3 times, and 1 remayneth: but then may not 9 be found ſo often in 16, therefore muſt I take a leſſer quotient, ỹ is to ſay, 2. then ſay I twiſe 2 maketh 4, whiche I take out of 7, and there remayneth 3. then we I cancell 7 and 2, and ouer 7 I write 3, and in the quotient I ſet 2, ſo ỹ figures ſtande thus.

3
ỹ 656 (2
x 9

Thē ſay I forthe, 2 times 9 make 18, whiche I bate out of 36, and there reſteth 18, then cancell I 3, and

ouer

ouer him set 1, and likewise I cancell 6 and 9, and ouer them I set 8, so that thus stande the figures.

Then we I set forwarde the diuisor by one place, and seeke à newe quotient, that is to say, how many times 2 are in 18, whiche I finde to be 9 times, but then cã I not finde 9 so many times in 5, therfore I take à lesser quotient, as to say, 8: but yet is ỹ to great, for if I take 8 times 2, out of 18, there remaineth but 2, and I can not finde 8 times 9 in 25, therfore yet I take à lesser quotient, that is 7, whiche is also to greate, for if I take 7 times 2 out of 18, there resteth 4, but nowe I can not take 7 times 9 out of 45, therefore yet I seeke à lesser quotient, as to say, 6: then saye I, 6 times 2, make 12, that I take out of 18, and there remaineth 6, so I cancell the 18 & the 2, and write 6 ouer 8, thus.

1

3 8

7 6 5 6 (2

x 9

x 6

3 8

7 6 5 6 (26

x 9 9

x

Then say I forth: 6 times 9, maketh 54, that take I out of 65, and there remaineth 11, and the figures stande thus.

Then must I set forthe the

diuisor againe, and seeke à newe quotient, whiche will bee 4: for though I maye finde 2 in 11 fiue times, and 1 remayne, yet I can not finde 9 so often in 16, therefore I set the figures thus.

And the 4 in the quotient I multiply into the figures of the diuisour, saying: 4 times 2, maketh 8, whiche I take out of 11, and there resteth 3, therefore I cancell the 11 and the 2, and set 3 ouer the first place of 11, thus.

And then doe I say forthe, 4 times 9 maketh 36, whiche I take from 36, and there remaineth nothinge, so that the quotient of this diuision, where 7656 is diuided by 29, is 264, whiche doth declare, that if 264 bee multiplied by 29, the summe will be 7656. And thus I perceaue nowe howe bothe Multiplication is proued by Diuision, and Diuision also by Multiplication.

Mayster. Now haue I ended the fiue most cõmon kindes of Arithmetike: for as touching Mediation, Duplation, Triplation and such other, they are no seuerall kindes of Arithmetike,

like, but are contained vnder the other: for Mediation is contained vnder Diuision, and is nothinge else but diuidinge by 2: and so are Duplation and Triplation contained vnder Multiplication: for Duplation is nothinge else but multiplying by 2, and Triplation is multiplying by 3, of whiche I will only propose examples, for ỹ rules you haue heard already.

If you woulde mediate or diuide into 2, this summe, 4531010, you shall set 2 for the diuisor, & worke as you learned before, as thus. An example of Mediation.

```
4531010 (
   2
```

Then I finde 2 in 4 two times, therefore my quotient must be 2: so I cancell 4 and 2, and remoue the diuisor forwarde, thus.

```
4531010 (2
  x 2
```

Then againe I finde 2 in 5 twise, and 1 remaininge, so I write 2 againe for my secõde number of the quotient, and cancell 5 and 2, and ouer 5 I set 1, thus.

```
  1
4531010 (22
  x x
```

Then remoue I the diuisor forwarde and seeke a newe quotient, whiche is 6: then say I, 6 times 2, make 12, take ỹ out of 13, there resteth 1, so I cancell 2

and 13, and ouer 3 I set 1, thus.

Then remoue I the diuisor forward, and seeke a new quotient whiche is 5, then take I twise 5 out of 11, and there resteth 1, so I cancell the 2 and the laste figure of 1, and let ẏ first stand thus.

Then remoue I the diuisour forwarde, and seeke a new quotient, whiche is 5: then take I 2 fiue times out of 10, and there resteth nothinge.

Then remoue I againe the diuisor forwarde, thus.

But bicause I can not finde the diuisour in the number ouer it, I muste set a cipher in the quotient, and remoue the diuisor to the nexte place, as appareth in the figure before.

Then seke I a new quotient, whiche I finde to

to be 5, for so many times may I haue 2 in 10. Then haue I fully ended this Mediation or diuision by 2, & the quotient is this, 2265505, which is the halfe of 4531010, as you maye try by Duplacion: for double that quotient, or multiply it by 2, and the same number wil amounte. Duplatiõ

I will no longer tarry about these, seynge they are but members of the otherkindes. But here now will I teache you certayne easy formes bothe of Multiplication and of Diuision, and first of Multiplication. Easy formes of multiplication.

If you would therefore multiply any sũme by 10, you shall neede to we no more but adde a cypher before his firste place: as for example: 36 multiplied by 10, make 360.
Likewayes if you will multiply any summe by 100, put two ciphres at his beginninge.
So if you would multiply any summe by a thousande, adde three ciphres to the beginninge of it.

Scholer. This we I well perceaue, and also the reason of it.

Mayster. I will omit all reasons till our next meeting, when I shall tell you the reason of all other partes of Arithmetike also: and as to our matter nowe, loke (as I haue tolde you)

you) that you bothe remembre it, and also often practise it.

But if you would multiply any numbre by 5, marke first whether ỹ nũbre be euen or odde: and if it be euen, take the halfe of it, and write à cyphre at the beginning of it, as for example: I would multiply 2564 by 5, I take ỹ halfe of it, which is 1282 (as you maye knowe by Mediation) & before it I sette à cyphre, thus, 12820, and this is 5 times 2564.

And thus may you we with any other euen summe, that you would multiply by 5.

But if the summe bee odde, as for example 2563, then must you take the lesser halfe of it, or (if you will) take away 1 from the firste figure, (as heere take 1 from 3) and then take the halfe of the reste, and sette before it 5: as of 2563, the lesser halfe is 1281, for heere I take but 1 for the halfe of 3: and if I putte 5 before that lesser halfe, then haue I multiplyed it 5 times, as thus, 12815.

Sc. What meane you by the lesser halfe?

Mayst. There is no iuste halfe of any odde numbre, therfore if wee diuide an odde numbre into two partes as nighe equall as canne bee, yet will the one halfe exceede the other halfe by one, as for example. The two moste neereſt

halfes

halfes of 9, are 5 and 4: and likewise of 15, are 7 and 8, where you see, that the one parte still is greater then the other by one. Nowe it is easie to know which is the greater halfe, and which is the lesser halfe.

Scholer. Then I perceaue you, and can we likewayes (I wubt not) with any summe. For if it bee not very easie to parte into halfe, then will I we it by Mediation easily ynoughe.

Mayster. That is à sure waye. And nowe haue you learned howe to multiply easilye by 5, 10, 100, 1000: and of like manner may you we with any other of that sorte.

But nowe if you will multiply by 20, 30, 40, and so forthe: or by 200, 300, and such like, where there is one ciphre in the firste place, or many orderly in the first places, you shall take awaye those ciphres, and multiply the summe onely by the other figure or figures, (if they bee many) & then at the beginninge of the summe that amounteth, shall you set so many ciphres as you toke awaye.

Example of 2873, which I would multiplie by 300. Firste I cast away the two ciphres from the multiplier, & I multiplye the summe by onely 3 that is lefte, and it amounteth to 8619: before which I put ỹ two ciphres that

I toke away before, and then is it 861900. And that is the summe that amounteth, when 2873 is multiplied by 300.

Scholer. And if there were two or more figures beside the cyphres, I must onely take away the cyphres, and multiply by the other figures, as I learned before: as if I would multiply 93648 by 25000, I shoulde take away the three cyphres, and multiply the same by 25, & then at the beginninge of that totall summe, should I adde the three ciphres agayne.

Mayster. Euen so: but and if it chaunce the numbre that should be multiplyed, or bothe the summes, as well the numbre that shoulde bee multiplyed as the multiplier, to haue cyphres in their firste places, euermore caste away the cyphres, and worke by the reste. But remembre to restore as manye cyphres to the amountinge summe, as you bated before, as in this example: 30200 shalbe multiplied by 206: I shall only take away the two ciphres from the greater numbre, and then multiplye 302 by 206, and afterwarde adde the two cyphres agayne. But if I woulde multiplye the same 30200 by 2060, I shal not onely take awaye the two ciphres from the numbre that shoulde bee multiplyed, but also I maye take

awaye

away the one cipher from the multiplier, and then muste I adde 3 ciphers to the summe that amounteth: but take heede that you take away no cipher that commeth after any signifiynge figure, as in this laste example, you may not take away that in the fourthe place of the higher number, neyther that in the thirde place of the multiplier: howe bee it, yet this you may we: If one cipher or more come in the middeste of your summes, you may multiply by the other figures, and ouerskippe them, but so, that you giue euerye figure his due place, as thus:

I will multiplie 3026 by 2004, therefore I set theim thus.

```
3026
2004
```

And thus I multiplye them: firste 4 times 6, make 24. I set the 4 vnder ẏ firste

```
  24
1208
```

place, and keepe the 2 in my minde, or write it downe for easte remembraunce: than say I againe: 4 times 2 maketh 8, and 4 times 0 maketh 0, then 4 times 3 make 12.

But nowe whan I come to the next cipher, bicause that it multiplyeth nothinge, I let it go, and likewaies the seconde cipher, but then whan I we come to the 2, and multiply it into the 6 of ẏ ouer number, you muste take heede

(according

(accordinge as I taughte you in Multiplication) that the firſt number amountinge of ẏ Multiplication, bee ſet vnder the multiplier right, and the other orderly towarde the lefte hand, accordinge as you may ſee in this example.

Where if you had expreſſed ẏ ciphers after the common rate, than ſhoulde the figures ſtande as foloweth.

But in effecte all is one, ſaue that the firſte way by ouerſkipping of the ciphers, is ẏ ſhorter and eaſier way: for, that, in effecte they bee bothe one, the addition of the parcells will declare, whiche in bothe will appeare thus.

And now will I make an ende of this matter.

Sch. Sir, I thanke you: for I ſe great eaſe in this wayes of Multiplication, and if you canne ſhewe mee ſuche like in Diuiſion, you ſhall greatly further me.

	3026
	x004
	24
	1208
	12
604	

	3026
	x004
	24
	1208
	0000
	0000
	12
604	
6064104	

Mayſter.

Mayster. Yes, I will teache you some easie waies in Diuision also, and first this: If you would diuide any summe by 10, you shall onely with your penne make a square line, betweene the first figure of your summe and the seconde, and than haue you one: for the whole number that followeth the line, standeth for the quotient, and the figure that is before the line, is the remainer: as for example, 3648 diuided by 10, will stande thus. 364|8

Easie formes of Diuision.

Where 364 is the quotient, and betokeneth that so many times are 10 in 3648: and the 8 after the line, is the remainer, whiche can not bee diuided into 10, but by breakinge it into fractions, wherewith I will not meddle yet.

And so like waies if you would diuide any summe by 100, with your penne, you shall cut away the two first figures: and if you would diuide by 1000, you must cut away the 3 first figures: & so of any other diuisor, whose laste figure is 1, and the other be ciphers, loke howe many ciphers the diuisor hath, and so manye figures at the beginninge shall you cut awaye with the squire line, and they stande alwaies for the remainer, bicause they are lesse then the

diuisor, and can not bee diuided by it, and the other figures that be behinde the line, stand for the Quotient.

But now if your diuisor haue any other figure in his laste place than 1, and in all his other places haue ciphers, loke howe many ciphers there bee, cut away so many of the firste figures of the number that should bee diuided, and diuide the reste that followeth the line, by that figure that is in ỹ laste place, as if it were the whole diuisor.

Example of 64284, whiche I woulde diuide by 300, here muste I cut away the two firste figures, (for so many ciphers my diuisour hath) and muste diuide the reste by 3, whiche is the figure in the laste place of the diuisor. Firste therefore I part away the two firste figures, & the sum standeth thus. 642|84

Than doe I diuide 642, by 3, and the quotient is 214, for in 6 I finde twise 3, and in 4 once, and 1 remaininge, whiche with the 2 next before, doth make 12, wherein I finde 3 foure times: and this is a ready way to turne shillinges into poundes: for sithe one pounde doth containe 20 shillings, I muste diuide the whole number of shillings by 20, therefore eallly to doe it, I see that my diuisor hath

one

one cyphre, and therefore I cut away one figure from the beginninge of the whole summe of shillings, and than I we mediate or diuide by 2, the other figures or summe that foloweth.

Scholer. I will put an example.

If you woulde diuide 64287 shillinges by 20, that is to saye: If I woulde turne so many shillinges into pundes, I muste cut awaye the firste figure, that is 7, and diuide the reste, that is 6428 by 2, so shall the Quotient bee 3214, whereby I knowe that 64287 shillings, make 3214 pundes, and 7 shillinges remayninge.

Mayster. Nowe proue by Multiplication whether you haue well wne or no.

Scholer. The quotient is 3214, whiche I we multiply by the diuisoure 2, and it wyll amounte to 6428.

Mayster. Hereby maye you perceaue not onely that you haue well wne, but also howe by diuision you may tourne shillinges easilye into pundes: And contrary wayes, by Multiplication you may tourne pundes into shillinges.

But heere shall you see amongst diuers me, diuers formes of suche diuision, but if you marke what I haue tolde you, you shall per-

ceaue eaſily all their wayes: for ſome men do not cut away ſo many of ẏ firſt figures of the ſum ẏ they would diuide, as there are cyphres in ẏ firſte places of their diuiſor, but they ſet al their cyphres orderly vnder ẏ firſt places of the number ẏ they woulde diuide, and than w̄ the other figure (or figures if they bee many) they diuide the reſte of their ſumme. Example. If they would diuide 725931, by 3400, they ſet their ſummes thus.

An other manner of the abridgmēt.

7	2	5	9	3	1	
3	4			0	0	

And then we they diuide orderlye till they come to the ciphres: for there they ſtaye and ende their worke, as in this example: They ſeeke how often 3 may be founde in 7, whiche is 2 times, and one remayninge, therefore they ſet 2 in the quotient, and cancell 3 and 7, & ouer 7 they ſet that 1 ẏ remayned, thus.

							2
7	2	5	9	3	1	(	
3	4			0	0		

Than we they go forthe ſaying: two times 4 maketh 8, whiche they take out of 12, and there remayneth 4, thus.

							2
7	2	5	9	3	1	(	
3	4			0	0		

Than renew they the diuiſor forwarde, and ſeeke how often 3 maye be founde in 4, whiche

is

is but once, and 1 remayneth, then set they 1 in the quotient, and cancell 3 and 4, & ouer them they set that 1, thus.

1
~~1~~ ~~4~~

Than take they once 4 out of 15, and there resteth 11. Or els more easilie: Take once 4 out of 5, and there resteth 1, so they cancell the 4 and 5, and set 1 ouer them, thus.

~~7~~ ~~2~~ 5 9 3 1 (21
~~3~~ ~~4~~ 4 0 0
~~3~~

1
~~1~~ ~~4~~ 1
~~7~~ ~~2~~ ~~5~~ 9 3 1 (21
~~3~~ ~~4~~ ~~4~~ 0 0
~~3~~

Then set they forthe the diuisor agayne, and seeke howe manye times 3 are in 11, whiche they finde 3 times, and 2 remayninge: so they set 3 in the quotient, and cancell 11 and 3, and ouer them setteth 2, thus.

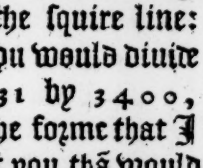
~~1~~ 2
~~1~~ ~~4~~ ~~1~~
~~7~~ ~~2~~ ~~5~~ 9 3 1 (213
~~3~~ ~~4~~ ~~4~~ 4 0 0
~~3~~ 3

Then doe they multiplie 4 by 3, whiche maketh 12, that withdraw they out of 29, & there resteth 17, of whiche ye 7 must be set ouer the 9, and the 1 ouer the 2, thus.

1
~~1~~ ~~2~~
~~1~~ ~~4~~ ~~1~~ 7
~~7~~ ~~2~~ ~~5~~ ~~9~~ 3 1 (213
~~3~~ ~~4~~ ~~4~~ ~~4~~ 0 0
~~3~~ ~~3~~

And now are the two ciphres next ensuing, so that the diuisor can no more be set forward, and therefore is the diuision ended, and the remayner is 1731.

Nowe the quotient, whiche is 213, dothe declare, that if you diuide 725931, by 3400, you shall finde it therein 213 times, and there remayneth 1731, so shall you finde it, if you worke as I taughte you, by cutting awaye the two firste figures, bicause of the two cyphres. But this muste you marke (as you maye perceaue by this laste example) that if there be lefte any other remainer in the sume that was behinde the squire line, that the remayner muste be set to the latter ende of the firste remayner, whiche was cut awaye with the squire line: as if you would diuide 725931 by 3400, after the forme that I taught you, thã would your sumes appeare thus.



So that 17, whiche remayneth after the line, must be set to the 31 (that was cut away with the line) in higher places, as you see heere: where that 17 with the 31, doe make 1731.

And heere would I make an ende of Diuision

sion, sauinge that there commeth to my minde one late inuention of easy Diuision, whiche I will briefly set forthe to you, so that if you finde ease in it, you maye vse it. Bicause that the hardest point in Diuision, is the ready and easie finding of the quotiēt number: and againe, if that be truly knowen, all the rest is but light to bee doone: therefore this waies shall you quickly and truely finde the quotient. An other inuentiō of easie Diuisiō.

First write the nine figures of number: I meane 123456789, not alonge as I haue set them now, but vp and wwne as in this forme. And at the left side of them drawe a longe line, as you see here: Then consider the diuisor, by whiche you intende to worke, and set it on the lefte side of the longe line, right againste 1, and for a distinction draw a line vnder it: then multiply your diuisor orderlye by eche of those figures, beginninge with 2, and so go wwneward till you haue ended all. And loke what doth amount of the multiplication of eche figure into the diuisor, than write it againste the figure whereby you did multiply.

1
2
3
4
5
6
7
8
9

Scholer. By example I may perceaue it better.

better.

Mayster. Take this example 263845 to bee diuided by 64, then muste I set the 9 figures as I saide before, and the diuisour must I set against the 1, thus.

Thã must I multiply ẏ diuisor by edẏ figure orderly: firste by 2, and it maketh 128, whiche I must set against 2 at ẏ left hand.

Than multiply 64 by 3, and it maketh 192, whiche is sette againste 3. Then 4 times 64, make 256, that set I by 4. Thẽ say I, 5 times 64 make 320, ẏ set I Againste 5. Thã 6 times 64, make 384, ẏ sette I againste 6. Then 7 times 64, make 448, whidẏ I set againste 7.

Further I say: 8 times 64, make 512, whiche I set by 8. And laste of all I say: 9 times 64, make 576, whiche I set against 9. And than they will stande thus.

And so is the table ended, by whidẏ you may eaſily finde the Quotient, as you shall see by example nowe.

Doe you set wonne ẏ numbers (as you learned

64	1
	2
	3
	4
	5
	6
	7
	8
	9

64	1
128	2
192	3
256	4
320	5
384	6
448	7
512	8
576	9

ned before) accordinge to ỹ order of Diuision.

Scholer. That is thus. 2 6 3 8 4 5

Mayster. Now loke 6 4

what number standeth ouer the diuisor, reckeninge thereto all theim that bee behinde it towarde the left hande.

Schol. Then are there ouer ỹ diuisor 263.

Mayster. That is iuste: nowe seeke in the table on ỹ left side, whether you can finde 263.

Scholer. It is not there.

Ma. Then take ỹ number ỹ is next to it, beneth it: I meane à lesser number then 263, but of all the lesser numbers that the table hathe, take you that that goeth nighest to 263.

Scholer. That is 256.

Mayster. So is it: and marke this euermore, whan you can not finde iustlye in the table that summe that is ouer your diuisour, then note that that is next beneath it of any summe that is in the table, and loke at the right hande of the line what figure or digit that is against ỹ summe, & take that digit for your quotient, & then worke on, as you learned before: for now haue I tolde you the whole vse of this table.

Howbeit, yet that you may bee sure to vnderstande it, I will see you ende this example of Diuision by it.

Now therefore begin againe.

Scholer. First I set wwne the summes after the common manner, thus.

263845	
64	

Then we I looke ouer the diuisor, and finde there 263. Nowe to knowe how many times 64 may bee taken out of 263, I resorte to the table aforesaide, and seeke for the number 263, but it is not there, therefore as you badde mee, I take à lesser number, the nexte to it that I can finde in the table, and that is 256, which number hathe againste it on the right hande this digitte 4, which I muste take for the firste figure of my quotient.

Then we I (as I learned before) multiply that quotient into euery figure of the diuisor orderly, withdrawinge the summe thereof, amountinge out of the ouer summe: as here I say first: 4 times 6 make 24: so I take that out of 26, saying: 4 out of 6 remaineth 2, whiche I write ouer the 6: then 2 out of 2 remaineth nothinge, then cancell I 2 and 6, and also 6 in the diuisor, and the summe standeth thus.

2	
x63845	(4
64	

Then we I likewise say forthe: 4 times 4 make 16, whiche I take out of

of 23, & there resteth 7 to be set ouer 3, and that 3 with the 2 behinde it & the 4 vnder it, muste bee cancelled, as you see heere.

```
 x7
x63845 (4
 644
```

Then haue I done with the figure of the quotient.

Mayster. Nowe set forwarde youre diuisour, and seeke à new quotiente, as you sought this.

```
 x7
x63845 (4
 644
  6
```

Schol. Than thus standeth the figures, so that ouer the diuisour I set 78, whiche I seeke in the table, and can not finde it: therefore I take the nexte lesser, and that is 64 the diuisor it selfe.

Mayster. So muste you doe when there is none other.

Sc. Than againste it I finde this digit 1, whiche I muste set in the quotient before 4, thus.

```
 x7
x63845 (41
 644
  6
```

Than multiply I 6 by 1, and it is but 6 still.

Mayster. You neede not go about to multiplie Note.

tiplye whan the quotiente is 1, for 1 with nother multiply nor diuide, but in suche case onely subtract the diuisor out of ẏ numbre ẏ is ouer it.

Scholer. Then I take 4 out of 8, & there resteth 4, and 6 out of 7, there remayneth 1, so I cancell those numbres, & write the remayners ouer their places, thus.

Then set I forwarde ẏ diuisour agayne, thus.

Where I see ouer ẏ diuisour 144, whiche I seeke in the table, & finde it not: therefore I take ẏ numbre in ẏ table ẏ is nexte thereto, benethe it, whiche I finde to be 128, againste whiche in the right side I finde 2, whiche I take for my quotient, & that we I multiply firste into 6, and thereof commeth 12, whiche I take out of 14, and then remaineth 2, that 2 I sette ouer 4, and cancell the other figures, 1, 4, and 6, thus.

Then say I forthe: 2

times

times 4 are 8, whiche I take out of 24, & there remaineth 16, of whiche I write the 6 ouer 4, & the 1 ouer 2, and cancell 2, 4, and 4, thus.

Nowe againe I set forwarde ẏ diuisor thus. And seing ouer it 165, I seeke that in ẏ table, but finde it not, therefore I take ẏ next lesser, whiche is 128, against whiche I finde 2: that w I set into the quotient, and by it I multiply firſt 6, & thereof cõmeth 12, whiche I take out of 16 & there resteth 4, then cancell I 1, 6, & 6, and ouer 6 I set 4, thus.

Then we I multiply 4 by 2, and it maketh 8, whiche I take out of 45, and there remaineth 37, as in this example.

And nowe haue I wne.

Mayster. Well, nowe I see that you can worke by this kinde of diuision, as farre forthe as I taught you.

Scholer. Yea sir, I thanke you, and I finde in it muche ease and certainesse.

Mayster. Yet one thinge I doubt whether you perceaue: what if you did finde in the table the number that standeth ouer the diuisor, what would you next we?

Scholer. I thinke I should take the digit against it on the lefte hande for the quotient.

Mayster. So is it: and as often as you seeke in the table and finde your number iuste, the digit against it is your trewe and iust quotient. I call that à trewe quotient also, if it be the right quotient ẏ you should take, thoughe youre diuisor multiplied by the same, we not clearely subtract the number ouer it, but there wthe somewhat remaine, as it chaunced in all the examples that you did worke by. But if it should chaunce (as it wthe often) that your diuisor multiplied by your quotient, we subtract cleane the numbre ouer it, the call I that quotient not onely à trew quotient, but also à iust quotient, bycause it dothe iustlie consume the number ouer the diuisor: and that chaunceth euermore when the number ouer the diuisor is

Marke the diuersitie betweene à trewe quotient and à iust quotient.

iustlie

iustly founde in the table.

Scholer. This I shall remember.

Mayster. But yet one easie pointe more I will tell you in this sorte of Diuision, therefore marke it well.

When you haue founde in the table, other the same summe that is ouer the diuisor, other ẏ next beneath, (for lacke of ẏ other) then looke what digit standeth againste it, take that for your quotient. And bicause it is some paine to multiply the diuisor by the quotient, you shall not neede to doe it, but onely take the number that you founde in the table, and subtract that from the ouer number: for if you doe multiply the diuisor by the quotient, that will be the nũber that shall amounte, therefore is this waye more easier.

Scholer. So is it, and also more certainer for suche as I am, that might quickly erre in multiplyinge, especiallye beinge smally practised therein.

Mayster. Then proue in some briefe example whether you can doe it, and so will we make an ende.

Scholer. I would diuide 38468 by 24, therefore firste I set ẏ table as here foloweth.

38468	
24	

Then

Then set I the two summes of Diuision, thus.

And ouer the diuisor I finde 38, whiche I seek in the table and finde it not, therefore take I the next benethe it, whiche the table hath, and that is 24, the diuisor it selfe: against whiche is set 1, whiche I take for the quotient, whiche I set in his place. And now I neede not to multiply the diuisour by it, but onely to withdrawe the diuisour oute of the 38, that is ouer it, and so remaineth 14, as thus.

Then set I forwarde ỹ diuisour, and finde ouer it 144, as appeareth: then seeke I that number in the table and finde it, against it is 6, therefore I set 6 before 1 for my quotient, and I take that 144 for the iuste multiplication of the diuisor by that quotient, and therefore without any newe multiplication I we subtracte ỹ 144, from the other 144, there resteth nothing, as you may see.

24	1
48	2
72	3
96	4
120	5
144	6
168	7
192	8
216	9

38 | 468 (1
14

38 | 468 (1
144
2

38 | 468 (16
144
144
0

Therefore

Therefore I set forward the diuisor: but seing it will not bee in the next place, (for then ouer 2 would be nothing) I set it forward twise, as you se here.

```
x 4
3 8 4 6 8 (16
x 4 4
x 2
```

And for because that I could not set it in ỹ next place following, therefore I set a cipher in the quotient, as you see.

Then loke I ouer the diuisor, and finde 68, whiche I can not finde in the table, therefore take I the next beneth it, whiche I finde in the table, and that is 48, and againste it standeth 2, whiche I take for the quotient. And then without any multiplyinge of the quotient into the diuisor, I we subtract that 48 from 68, and there resteth 20, as here appeareth.

```
x 4   2 0
3 8 4 6 8 (16 0 2
x 4 4 4 4
x x 2
```

And so haue I ended the whole diuision.

Mayster. In very greate summes to be diuided by greate diuisors, I thinke there is no better way thē this for any man to vse, though he bee neuer so expert. And that especially, yf one great diuisor be often to be occupied about dividing many and diuers greate summes. As

commonly happeneth in Astronomicall workinges, and Geometricall, about the signes, both straight and reuersed: as if it be your fortune and desire to wade to the profoundnes of Geometricall and Astronomicall calculations demonstratiue, you will some confesse. Whereof, an other time shall better serue to speake. Now can you sufficiently skill in these kindes of Arithmetike. And now for the farder vse of these two laste, that is Multiplication and Diuision, I will briefly shew you the feate of Reduction, by the way.

REDVCTION.

Reduction is, by which all sumes of grosse denomination maye bee turned into summes of more subtile denomination: And contrarye wayes, all summes of subtile denomination, may be brought to sumes of grosser denomination.

Scholer. What call you grosse denomination, and subtile denomination?

Grosse denomination.

Mayster. That I call a grosse denomination,

tion, whiche wthe contayne vnder it many o-ther ſubtiler or ſmaller: As a pounde in reſpect to ſhillings, is a groſſe denomination: for it is greater then ſhilling, and containeth manie of them. And ſhillinges in comparíſon to pundes, are a ſubtile denomination, for bicauſe they are leſſer then pounds, and many of them are contained in one of the other: as ſo, likewayes of other thinges, what ſo euer thing is cõpared to other, if it be greater and contayneth manye of them, it is a groſſer denomination: but if it be leſſer, ſo that many of them are in the other, then are they called ſubtile denominations: whereby you may perceaue, that one denominatiõ may bee called a groſſe denomination, and alſo a Subtile, (that is to ſaye, a great and ſmalle) in diuers comparíſons. For ſhillings cõpared to pundes are a Subtile or ſmale denominatiõ: but compared to pennies, they are a groſſe or greate denomination. Subtile denomination.

Scholer. Now I vnderſtande the name, I pray you teache me the vſe.

Mayſter. The vſe is eaſily learned, if you remember what you haue learned before. For if you will reduce anye ſumme of a groſſe denomination, into a ſumme of a ſmaller or ſub- To reduce groſſe de-

nomination to subtile.

tiler denomination, you muste consider howe many of that Subtiler denomination to make one of the Grosser denomination, and by that number or numerator we you multiplye the other summe: as if you would reduce 20 pũdes into shillinges, you muste consider that in à pounde are included 20 shillings, therefore multiply the one 20 by the other 20, and there will amount 400, whereby you may knowe, that in 20 pounde are contayned 400 shillinges. Likewayes if you would reduce 30 shillinges into pennies, considering that in 1 shillinge, are 12 pennies, you muste multiply 30 by 12, and it will bee 360: whereby you finde, that in 30 shillinges, are contayned 360 pennies. And thus may you reduce any grosse denomination into à more subtiler, by multiplication, if you know howe many of the lesser doe make the greater: of whiche thinge I will anone giue you à briefe table for ỹ moste accustomed kindes of mony, weightes, measures, and time, and such like, whereby you maye knowe howe often ech Subtile denomination is contained in the Grosser, when you shall neede it for the foresaid kinde of Reduction. And also the same shall serue you, if you woulde reduce any summe of à Subtiler denomination, into

To reduce Subtile

à summe

à ſumme of à groſſer denomination: For in ſuche reduction you muſte conſider (as in the other forme) how many of the ſmaller doe make the greater, and by that number muſte you diuide the other ſumme, and the quotient will declare how many of the greater denomination, are comprehended in that ſumme: as for example: If you would know how many ſhillings are contayned in 3240 pens, conſider that 12 pennies we make 1 ſhillinge: you muſt diuide that 3240 by 12, and youre quotient will be 270, whereby you knowe that ſo many ſhillinges are in 3240 pennies. But and you would knowe farther, how many pundes are in thoſe 270 ſhillinges, ſeynge that euery punde contayneth 20 ſhillinges, diuide that 270 by 20, and it will be 13, and 10 remayninge, whereby you may knowe, that in 3240 pennies, or 270 ſhillings, are 13 pundes & 10 ſhillings. For euermore the remayner muſt bee named by the name or denomination of the ſumme that was diuided, whiche in this place were ſhillings. And thus may you we with anye other kindes of denominations. Denomination to groſſe.

Wherefore to the intente that you maye haue à light knowledge in the commō coines, weightes, meaſures, and ſuche other, I haue

prepared heere a breife table, whiche shall suffise to you at this time, till hereafter at more conuenient opportunitie I maye instructe you more exactly in the same.

Note (gentle reader) these valewes of Englishe coynes as they were when this Author firste published his booke. But in oure tyme, (namelye Anno 1570) they are muche diuerse. Therfore somthinge to pleasure the in this purpose, I haue caused at the ende of the booke, à Table to bee annexed of oure coynes, and their valewes currant in our time, within this realme of Englande.

A table for Englishe coynes. Anno 1540.

Englishe coynes.

A Souerayne.	A quarter Noble.
Halfe a Souerayne.	A Croune.
A Royall.	Halfe a Croune.
Halfe a Royall.	A Crounc.
A quarter Royall.	A Grote.
An olde Noble.	A harpe Grote.
Halfe an olde Noble.	A penny of 2 price.
An Angell.	A dandy platte.
Halfe an Angell.	A penny.
A George Noble.	An halfe penny.
Halfe a George Noble.	A Farthinge.

The

The valewe of Englishe coynes.

The valewe of Englishe coynes.

A Souerayne is the greatest englisch coyne, and containeth 2 Royalles, or 3 Angelles, eyther 9 halfe Crounes, or 4 Crounes and an halfe, that is to say, 22 ſ.6 d.

Hale à Soueraine is equall with à Royall.

A Royall containeth an Angell and a halfe, that is to say: 11 ſ,3 d.

Halfe à Royall containeth 5 ſ,7 d,ob.

A quarter of à Royall containeth 2 shillinges, 9 d,ob,q̃.

An olde Noble, called an Henrye, is worthe 2 Crownes, or à noble and halfe, that is 10 ſ.

Halfe an olde Noble is worth 5 ſ.

An Angell containeth a Crowne and halfe, or 3 halfe Crownes, that is 7 ſ,6 d.

Halfe an Angell is worth 3 ſ,9 d.

A Noble, called a George, is worth 6 ſ,8 d.

Halfe a Noble is worth 3 ſ,4 d.

A quarter of a Noble (whiche in ỹ olde Statutes is called a farthing) containeth 20 d.

A Crowne containeth 5 ſ: ꝛ the halfe Crowne 2 ſ,6 d. How be it there is an other Crowne of 4 ſ,6 d. whiche is knowen by the rose syde: for the rose hath no Crowne ouer it, as in the other Crowne, but it is enuironed on the 4 quarters with 4 floure deluces, whereby you

may best know it. But I wil returne to speake of the value of the coynes, for I intende not now to describe the formes of them. Nowe of golde are there no more common coynes.

In siluer the greatest is a Grote, whiche contayneth 4 pennies. Then is there another Grote called an Harpe, whiche goeth for 3 d. Then next is a Penny of 2 d. And the a Dandipratt, worth 3 halfe pence. Next it a Penny, then halfe a penny, and laste and leaste of all a Farthinge, whose coyne is on the one side a crosse, and on the other side a purculles. This I tell you, because I see many that can not knowe a Farthinge from a small halfe penny.

Nowe haue I tolde you all the Englishe coynes bothe of golde & siluer, but yet of ỹ three moste cõmon valowers of mony spake I nothing: that is to saye, of Pounds, Markes, and shillinges, whiche though they haue no coynes, yet is there no name more in vse then they: of whiche the shillings containeth 12 pennyes or 3 grotes: and the pounde 2 olde Nobles, 3 George Nobles, or 4 Crownes, that is to say, 20 s. A marke, two George Nobles, that is 13 s. 4 d.

Here would I now expresse the valewes of sundry other coynes of diuers countryes, but for

for three causes I nowe refrayne. The firste and chiefest is, because they are not currant by the Statutes of this Realme. An other cause is, by reason they are so vncertaine, that they be neuer long at one rate. And againe they are so different in so manye places, that it were matter ynough for a greate boke, to speake sufficiently of them all. Howe bee it, yet because you shall not bee altogether ignorant of them, I will shew you the valewes of some that are most in vse: and firste of Fraunce.

Frenche coynes.

The moste common money are Deniers, Soulx and Frankes. 12 Deniers make 1 ſ. 20 soulx make 1 Frank. so ẏ as you see, these 3 kindes are like in the rate to pennies, shillings, and pundes with vs, but that this is the difference, that their Denier is but the 9 parte of our pennye, and so their soulx (cõmonly called sowses) go 9 to our shillinge, and 9 of their Franks to an Englishe punde of mony: So that 3 of their Franks make à noble. And by those 3 may you practise, how to reduce French mony into Englishe mony. And as for the rest of their coynes I will omit till an other time, when I intende to shew you the rate of sundry other kindes of mony.

But nowe as for the coynes of Flaunders

Flaunders coynes.

be so changeable, that you muste knowe them frō time to time, els you cā not reduce them into our mony, certainely. But yet bicause ẏ you shall haue an example of their monye to exercise you withall, you shall take those ẏ be most common, as Stiuers bothe single and wuble, Grotes Flemmish, Carolus, and Gyldens: A Flēmish Grote is a little aboue 3 farthinges English. A single Styuer is 1 d, ob, q̄. The wuble Styuer is 3 d, q̄. The siluer Carolus single, 2 d, q̄, q, c. The double siluer Carolus is 4 d, ob, q̄, q. Then is there also the Carolus Gylden, whiche is worth 20 Styuers. And the Flemmishe Noble is worth 3 Carolus Gyldens, and xij. Styuers.

But I will let them passe nowe, exhorting you to practise to reduce those kindes into English mony, according as I haue set forth here followinge: 2160 deniers, make 240 d: or 20 ſ. 3240 deniers, make 360 d, or 30 ſ: 8352 denires, make 928 d, or 3 li. 17 ß, 4 d: 2160 soulx, make 240 shillinges: and so of other in like rate.

But if you will reduce Flemmishe monye iustly, you must reduce it first into the smallest parte of Englishe mony that is in ẏ coyne, as for example. If I woulde reduce 368 double

stuyers

ſtyuers into Engliſhe money, conſidering that a wuble Styuer containeth 3 ð, q̄, you ſhal firſt loke howe many q̄, bee in the wuble Styuer, and you ſhall finde them 13, therefore multiply the ſumme of the ſtyuers by 13, and than haue you their valew in farthings, whiche is 4784. Nowe if you diuide that by 4, then will there appeare ẏ numbre of pence: but better it were to diuide it by 48 (for ſo many farthinges are in 1 ſhilling) and then will the quotient declare the ſumme of the ſhillinges.

Likewayes if you woldē reduce any ſumme of ſingle ſtyuers into Engliſh mony, you muſt multiply the ſumme firſte by 13, and than haue you a certayne ſumme, whiche ſumme if you diuide by 8, then will amounte the ſumme of pennies: or if you diuide it by 96, the ſumme of ſhillinges will appeare.

But this marke in all diuiſion, whan yee wo reduce to bringe one denomination into an other, if there bee any remayner after the diuiſion, that muſt bee named by the denomination of the groſſe ſumme that was diuided: as for example: I woldē bringe 254 q̄ into pence, therefore I we diuide ẏ 254 by 4, for ſo many farthings make 1 pēny, & the quotient is 63, whiche is the ſumme of the pence, and then remayneth

mayneth yet 2, whiche are farthinges ſtill, as one may proue by diuiding. And this muſt bee marked in all Diuiſion, namely whē it is one for Reduction.

Weightes

Thus muche haue I ſayd of Money, nowe will I ſhewe you in lyke ſorte the diſtinction of wayghtes, after the ſtatutes of Englande, where the leaſte portion of waight is commōly à Grayne, meaninge à grayne of corne or wheate, dry, and gathered out of the middle of the eare. Of theſe graynes in time paſſed, 32 wayed iuſt 1 penny of Troy, and thā was but 20 pennies in an Ounce. But nowe are there 46 pennies in an Ounce, ſo that there are not fully 14 graynes in one penny.

A graine.

A penny of Troy.

An Ounce

But nowe of Ounces after Troye rate (whiche is the ſtanderde of Englande) 12 doe make 1 pounde.

Haberdepoiſe weightes.

But commonlye there is vſed an other waight called Haberdepoiſe, in whiche 16 ounces make à pounde. Therfore whan you would reduce ounces into pundes, you muſt conſider whether your waightes be Troy waightes or Haberdepoiſe: and if it bee Troye waight, you muſt diuide youre ounces by 12, to bringe them to pundes: but if it bee Haberdepoyſe, you muſte diuide them by 16. Nowe agayne, there

bee

bee greater waights whiche are called an hũdred, halfe à hundred, and à quarterne, and also halfe à quarterne. &c. A hũdred weight.

Scholer. Why? so there may bee reckened 20 pounde, 40 pounde, 200 pounde, and suche innumerable.

Mayster. All these are numbers of waight, but they haue not common waightes made to their rate, as the other haue. And agayne, these that I did name, are not iuste in number as they seeme by theire name, for an hũdred is not iust 100, but is 112 pounde. And so the halfe hundred is 56: the quarter 28: and the halfe quarter 14. And this is the common waightes vsed in moste thinges ỹ are solde by waight.

Howebeit there are in some thinges other names: as in wolle, 28 pounde is not called à quarterne, but à Todde: and the 14 pounde is not named halfe quarterne, but à Stone: and the 7 pounde, halfe à stone. Other names, bycause they differ in manye places, and agree in fewe, I let them passe. Wolle weightes. Todde. Stone.

But a Sacke of wolle by ỹ Statutes is limitted to be 26 Stone. Sacke.

Nowe in cheese, though it bee soulde by the hundred, and by the stone in some places, yet the verye weightes of it are Cloues, and Cheese weightes.

Weyes

Cloue. Weye. Weyes: so that a Cloue shoulde contayne 7 pounde: and a Weye 32 Cloues, that is 224 pundes. Howbeit some Statute bookes saye, that a Cloue should be 6 pundē: and some saye also, that a Weye wthe contayne 36 Cloues, and that is commonlye vsed for the common Weye, for the common weye is of 256 li. ỹ is 36 cloues, reckening 7 li. to the cloue, and there is 4 li. ouer waight. Let this suffise you at this time, touching waightes.

Measures for liquor Now of waightes are made other measures, bothe for grayne and liquor. For a pounde in waighte maketh a pinte in measure, so that 8 pundē (or 8 pintes) doe make a Gallon: halfe a Gallon is named a Pottell: & halfe a Pottell is called a Quarte, whiche contayneth two pintes. Nowe aboue a Gallon the nexte measure is a Fyrken: then a Tertian, a Kilderkin or halfe Barrell, and a Barrell. And by those measures are solde commonly Ale, Bere, Wine, and Oyle, Butter, and Sope: Salmon, Herings, and Eeles.

A pinte. Gallon. Pottell. Quarte. Fyrkin. Tertian. Kilderkin Barrell.

But as these be vnlike thinges, so the measure of their vessels do differ: for the measures of Ale are as followeth.

Ale measures.

Of Ale. { the Fyrken / ỹ Kilderkē / the Barrell } contai-neth { 8 / 16 / 32 } gallōs.

Of

Di Beer. { the Fyrken / ỳ Kilderken / the Barrell } cõtaineth { 9 / 18 / 36 } Gallõs.

Sope measures, bothe Fyrken, Kilderken, and Barrell, should bee all equall to Ale measures. Moreouer the Statutes doth limitte the wayght of euerye of those three vessels, beynge emptye. Sope measures.

1 Barrell	to weyghe emptye	26	pondes.
Halfe Barrell		13	
1 Fyrken		6½	

Hearings also be solde by the same measures that Ale and Sope be solde by. Hearings.

Hearings also are solde by the tale, 120 to the hundred, ten thousand to the last.

Salmon & Eeles haue à greater measure. Salmon & Eeles.

Salmõ & Eeles	the butte / ỳ barrell / halfe bar. / the firken	holdeth	84 / 42 / 21 / 10½ gallons.

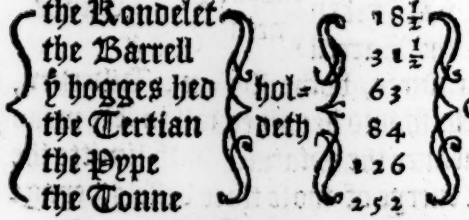
How be it, some Statutes did limitte Eele vessels equall with Hearing vessels.

Now as for wine vessels seldome are smaller then Hogges heddes, which are of 63 gallons: euery Hogges hed is two Barrels: yet there are many other wine vessels, but of them all, see this table, and marke the measures one Wine measures.

to

to an other.

Of wine and oyle.	the Rondelet	holdeth	18½	Gallons.
	the Barrell		31½	
	ye hogges hed		63	
	the Tertian		84	
	the Pype		126	
	the Tonne		252	



Tertians. But you shall marke, that there bee other kindes of Tertians: for there be Tertians (ye is to saye.) Thirdles of Pypes, of Hogges headdes, & of Barrels, as well of other thinges as of wyne.

A Butte. Also of Malueseyes and Secke, &c. ye halfe tonne is not called a Pype, but rather a Butte.

And thus muche haue I thought meete to tell you at this time.

Scholer. And is that alwayes trewe?

Mayster. I haue tolde you how it shoulde bee, but how it is I may not say: howe they doe differ daily from their iust measure, ye Gagiers can tell you better then I. But I wyll let this passe now, and speake briefly of the other measures.

And as of weyghtes there did springe the lyquite measures, (whereof I spake laste) so of the same springeth drye measures: as Peckes, Bushels, Quarters, and suche like, whereby

Dry measures.

are

are measured corne and like graines: also salt, lime, coales and other like. And this is the order and quantitie of them.

A Pecke is the measure of two Gallons. A pecke.
A Bushell containeth foure Peckes. Bushell.

A Quarter holdeth eight Bushells. Quarter.
A Waye containeth six quarters. Waye.

These are the common names and measures, but in diuers places there bee diuers sortes.

The bushell in many places is 2 bushels: but then is the bushell there called a Stryke. And Stryke.
in some places halfe a quarter is called a Cor- Cornoke.
noke. But these diuersities are to many to tell you briefly them all. And againe, sithe they are against the law and Statutes, I counte them vnmeete to be vsed.

But nowe remaineth yet an other kinde of Measure, whereby men meate length & bredth, and thicknes, and those are an Ynche, a foote, and such other: whose names and quantities this table sheweth. Measures to meate lengthe, breadth, & thicknes.

3	Graines of barly in length, make an inche.	An ynche.
12	Ynches make a foote.	Foote.
3	Foote make a Yarde.	Yarde.
3	Foote and 9 Ynches make an Elle.	Elle.
5	Yardes and a halfe, make a Perche.	Perche.
1	Perche in bredth and 40 in length, doe	

make a Rod of lande, whiche some call à rode, some à yarde land, and some à Farthendele.

Acre. 2 Farthendels, make halfe an acre of grou̅d.
4 Farthendels make an Acre.

Here mought I tell you many thinges els touchinge measure, and also howe to reduce straũge measures to our measures, but bicause it can not well bee done without ẏ knowledge of Fractions, whiche as yet you haue not learned, I will let them passe till an other time, when I shall instruct you in Geometrie, wherin I should be enforced els to repeate the same often againe.

The parts of time. A Daye. An houre Weeke. Moneth. Yeare.

Scholer. But yet sir of the partes of time I pray you tell me some what.

Mayster. You knowe that à naturall day hath 24 houres, and euery houre hath 60 minutes. It needeth not to tell you that 7 daies make a weeke, and 4 weekes make a common moneth, and 13 monethes make a yeare, lacking 1 daie and certaine houres and minutes. But of that I shall instruct you hereafter.

Here will I make an ende of Reduction for this time, whiche though it be counted no kinde seueral of Arithmetike, yet you see it is no lesse needefull to be knowen, nor easier to be done, than of any of the other.

Scholer.

Scholer. Marrye sir, it seemeth vnto mee much harder then any other sorte, for it requireth the knowledge of so manye thinges: but nowe sir when you see time, I am readye to learne forth, for as much of Reduction as you haue taught mee, I remember, but and if I we; at any time forget, I shal haue recourse to the tables, whiche you haue set forthe for mee.

Mayster. So doe you, for it will not bee remembred without exercise. Now Progression will I begin.

PROGRESSION.

Although vntill this daye the moste parte of writers haue defined Progression as a compendious kinde of Addition, yet truely it is not so: for progression (as the very nature of ỹ worde wthe informe any man) is a goinge forwarde and procedinge in numbres, and that regularly and orderly, whose place is aptly chosen to bee very neare, or rather nexte after the exposition of the foure principall partes of Arithmetike, for in it after a moste easye manner, are all the

foure former partes exercised and practised: & not onely Addition, as custumablye is done. Whiche custome hathe bene the cause, why it hath so specially bene named a kinde of Addition, and defined to be a quicke and briefe Addition of diuers summes, proceedinge by some certaine and reasonable order.

You shall also vnderstande, that there are infinite kindes of progressions, but for you (as yet) two are sufficient to bee exercised in: of whiche the one I call Arithmeticall, and the other Geometricall.

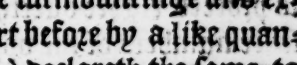
Arithmeticall progression. Arithmeticall progression is a rehearsing or placinge downe of many numbers, numbre after numbre, in such sorte, that betweene euery two next numbres rehearsed or placed downe, ỹ differẽce, diuersity, or exces, be equal & alike.

Scholer. Sir, I thanke you for that you haue bothe opened vnto me what Progression is, truly, and also why it is heere placed. But I praye you wᵗ an example make plaine your definition.

Mayster. Examples cannot wante, seinge all reasonable creatures naturallye vse the order of one kinde of Arithmeticall progression, (whiche therefore is also named Naturall) whan so euer they distinctly wo coũte or numbre

bee any multitude one by one, saying: 1. 2. 3. 4. 5. 6. whereby the proceedinge from number to number, and euery one surmountinge and exceeding his fellowe next before by a like quantitie (whiche heere is 1) declareth the same to be Arithmeticall progression. And for the more plainnes, I set it downe in this manner.

The common excesse. 1 1 1 1 1



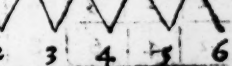
The Progression. 1 2 3 4 5 6

Scholer. This is moste euidente. And I thinke that I am able to tell you nowe of any progression Arithmeticall propounded, what is that common exces or difference whereby it proceedeth, if this order be kepte in it.

Mayster. What say you of 3. 6. 9. 12. 15?

Scholer. They exceede eche other by 3. And that maye I set downe in suche euident order, as you did youre example of Naturall progression, in this wise.

The common excesse. 3 3 3 3



The Progression. 3 6 9 12 15

Mayster. And doe you not also nowe perceaue, that the whole table of Multiplication maye be made by the order of progression Arithmeticall?

rithmeticall? either if ye will begin at the firste number of any of them on the lefte hande, and to proceede righte ouerthwarte: or at any of the firste numbers of the vpper rowe, and goe directly downewarde?

Scholer. I praye you let mee consider the thinge a litle, and I will aunswere you.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

By this triall I perceaue it now very well: for the common excesse or difference betweene any two nexte, is continually as muche as the firste number of euery row, either from the lefte hande ouerthwarte taken, or from anye of the vppermoste ouerthwarte rowes downewarde.

Mayster. Nowe than if of any suche pro-

gression

greſſion you woulde ſpeedely knowe the totall ſumme, muche quicker than by common additions rules: firſte tell howe many numbers there are (which numbers here wee call places or parcels) & if they be odde, write their ſumme wwne by it ſelfe, as in this example, 1, 2, 4, 6, 8, 10, 12, 14, where the numbers are 7, as you may ſee, therefore ſet wwne 7 in a place alone: the adde togither the firſt number and the laſte, as in this example: adde 2 to 14, and that maketh 16, take halfe of it, and multiply by the 7, which you noted for the number of the places, and the ſumme that amounteth, is the ſume of all thoſe figures added together, as in this example: 8 multiplied by 7, make 56: and that is the ſumme of all the figures. To know the totall ſumme of an Arithmeticall progreſſion.

Scholer. That will I worke by an other example. I would know how much this ſume is, 5, 8, 11, 14, 17, 20, 23, 26, 29. I tell the places and they are 9, that I note. Then I put the firſt number 5, and the laſte 29, together, and they make 34, I take the halfe of it, that is, 17, and multiply by 9, and it maketh 153. That you ſay is the ſumme of al the numbers.

Mayſter. So ſhall you finde it if you trie it.

Scholer. How ſhall I trie it?

Mayſter. By your common addition: for

if you adde all the parcels together, you shal see the same summe amounte, if you did worke well. And that manner of Addition trieth all kindes of summing any Progression.

Sc. Then can I summe a progression, if the numbers of the parcels be odde. But what if they be euen? as in this example, 1, 2, 3, 4, 5, 6, 7, 8?

Mayster. When the number of the parcels is euen, then note that also as you did before, and likewaies adde the first summe to the laste, and by the halfe of the number of the places w you multiplie it: as in your example, the parcels are 8, that note I: then adding the first summe to the laste, there amounteth 9, that we I multiplie by the halfe of parcels, that is by 4, and it maketh 36, which is the summe of the 8 parcels.

☞ But if you will take one rule for these both, we thus. Multiply the halfe of the one by the other whole, and the summe will amounte all one. For sometime it chaunceth that the number of the parcels bee odde, so that their halfe can not be taken: and sometime it chaunceth the Addition of the first number and the laste, to bring forth an odde number, so that the halfe of it can not be taken: but they will neuer bee

bothe

bothe odde.

Scholer. Then I perceaue this, if there be no more longing to it.

Mayster. As accustomablye it hath bene taught, this hath ben the chiefe & onely exercise in Progression vsed. But that you maye perceaue how diuers waies and to how greate profit so simple a thinge (as this Arithmeticall progression is) may be considered and vsed, I will here propounde you sixe propositions, of which foure of thē were inuented by a friende of mine, and neuer before this published: and the firste two, were neuer to my knowledge written of, but by three men.

Scholer. This bothe greatlye encourage me to be attentike vnto your wordes, seeing I shall not onely bee instructed at your handes in the common knowen rules of this excellente arte, but besides that, so aboundantly in other new rules informed, as my very entrance shall seeme to passe a great many mens farther studie, and longer continuaunce. Therefore sir, I beseech you, let me knowe your sixe propositions.

Mayster. These they are.

To know the last number without proceding by

continuall

continuall addition, till you come vnto it, so that the common exces, the first number and the number of the places be knowne.

2 The first number of the progression and the laste being knowen, with the common exces, to finde the number of the places.

3 The exces being geuen, and the first or laste, to knowe the quantitye of anye middle numbre, whose place is geuen from the first or laste.

4 The totall summe being geuen, and the firste and laste, to finde out the number of the places.

5 The totall sum of any Arithmetical progression being geuen, and the firste and laste, to finde out the common exces.

6 The totall summe being geuen, and the mutuall exces, with the number of the places, to geue the first or laste number of the same progression.

Many mo considerations could I propound you in these Arithmeticall progressions, but these are sufficiēt to giue you occasiō to thinke, that rules of knowledge & artes are infinitely capable of enlargement.

Scholer. Happy were I, if I did but well vnderstande that whiche is all ready inuented & written. And yet in my simple fantasie these thinges offer them selues (in manner) to bee studied for about progression. therefore I pray you to procede to the rules answeringe to these

propositions

proposítions.

Mayster. I will orderly for euery of these fyre proposítiõs giue you rules, and with euery one an example, vnlesse the plainesse and easines neede no farder exemplefyinge.

For the Solution of the first. Multiplye the excesse by à number lesse by 1 than the number of the places, and the ofcome adde to the firste number, so shall you haue the laste number, whiche is sought for.

As (for example), if there were seuen places in a progression Arithmeticall, whose continuall encrease or mutuall excesse were 4, and the first numbre were 5, and I would know what the last and seuenth numbre is: I multiplye 6, whiche is 1 lesse than 7, (the numbre of the places) by 4, thereof commeth 24, whiche I adde to 5, that maketh 29: and that is the laste numbre, whiche I desired to knowe. And this you may straight way proue, by continuall proceedinge from 5 till the seuenth place, encreasinge euery one by 4, as thus.

5 9 13 17 21 25 29.

For heere, the laste, beynge also the seuenth, is, 29.

Scholer. I perceiue all ready one good property in this rule, whiche in all workes is to be desired

desired: ẏ is, it will ease one from great labour, if a progression were propounded of a hundred or two hundred places, or moe. And also it is very easie to worke, & moste necessary for ẏ totall summe findinge, in a very longe progression.

2 Mayster The seconde rule is this. From ẏ
» laste subtracte the firste, the remainer diuide by
» the common excesse, to the Quotient adde 1, &
» you haue the number of the places, whiche you
» wolde knowe: As in this progression.

6 11 16 21 26 31

If I knowe onely 6 and 31, and that they encreale by 5. than accordinge to the rule, from 31 I subtracte 6, there remayneth 25: whiche 25 I diuide by 5 (the common exces) the quotient commeth forth 5, to whiche I adde 1, that maketh 6: and so many are the places, as you see.

Scholer. This rule is so easie, that I were muche to blame, if I coulde not remembre it.

3 Ma. The thirde proposition may alwayes
» thus be soluted: Multiply ẏ exces by a number
» lesse by 1, than ẏ distance of ẏ place is from the
» firste or ẏ laste number giuen: the offcome adde
» to the firste, if the distaunce be reckened from ẏ
» first, & the first also knowne or subtracte from ẏ
» last: if the distance be from the laste counted, &
» ẏ laste geuen also, and ẏ whiche cometh forth,

either

either in that addition to ỹ first, or subtraction from the laste, is the number sought. As for example, I propounde you this progression.

8 15 22 29 36 43 50 57.

And for the apte considering ỹ manner of this question, I will note ouer euery place his distaunce from the firste: & vnder euery place his distance inclusiuely from the laste, thus.

1 2 3 4 5 6 7 8
8 15 22 29 36 43 50 57
8 7 6 5 4 3 2 1

Now if ỹ exces wherby this progression stãdeth, be knowne to be 7, & the first number giuen, being 8, I would know what number stãdeth vnder 4, ỹ is to say in the fourthe place. I multiply 7 by 3 (whiche is lesse by 1 than the number of the place propounded) that yeldeth 21, to whiche I adde 8, (ỹ first numbre) so commeth 29: which I saye to belonge to ỹ fourthe place, as ye see in ỹ example it also wthe: or if in the thirde place from the laste, you woulde knowe what number in this example shoulde stande, the laste numbre beynge knowne to bee 57, and the common exces 7, than by 2 (whiche is lesse by 1 than the place propounded) I multiply 7, ỹ giueth 14: which I subtract from 57, so remaineth 43: which appertaineth to ỹ third

place

place inclusiuely reckened from the laste, and so my example giueth you.

Scholer. I perceaue right good vse of this rule: for if I had forgotten what the first number were, and remember still but the laste, the common exces, and the number of the places, then mighte I come by the knowledge of my first number againe.

And me thinketh, that it differeth not much from the first proposition, sauinge that whiche you make here a middle number, there was made the laste: and also in this poynt it differeth, that in it the laste was onely sought, and no consideration had in numbringe the places from the laste, as here I marke in youre numbers noted vnder your progression.

Mayster. And thinke you not, the middle numbres of a progression standinge of a hundred or three hundred places or moe, maye as much cumber a mã to come to the knowledge of them by continual encreasing from the first (by the common excesse) or abatinge from the laste continuallye (the common excesse) as the berye finall numbres in a shorter Progression would doe?

Scholer. Yes sir, that I thinke righte well, and therefore I am glad of this newe framed

proposition

propoſition, and the manner of the worḱinge of it.

Mayſter. The rule of the fourthe is this. 4
Adde the firſt and the laſte together, and by the ,,
ofcome diuide the totall ſumme. Double the ,,
Quotient, and that will bee the number of the ,,
places. ,,

Scholer. Than if in a Progreſſion, whoſe ſumme were 207, and the firſt number 12, and the laſte 57, if I adde 57 and 12 together, that maketh 69: and by it I diuide 207, the Quotient will be 3, whiche I double, and ſo I haue 6, and ſo many muſt bee the number of the places, that this progreſſion ſtandeth on.

Mayſter. Whether it be ſo or no, how will you trie?

Scholer. Halfe 6, whiche is 3, beinge multiplyed by 69, muſt make 207, the totall ſumme, if 6 bee the number of the places. For ſo the whole worḱe of youre rule in ſumming any Arithmeticall progreſſion did enforme me. I will than multiply 69 by 3, thus.

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 69
  3
---
207
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It commeth forth iuſtly.

Mayſter. I muſte muche herein commende your promptnes, bothe in memory and in well applyinge youre rule: although in manifeſt

worḱes

wordes it did containe no fudy matter.

Scholer. Sir, I pray you heare me frame one example more.

Mayster. I am well pleased, so that ye be short, for you make me more longer here, then willingly I would haue bene: but I can not perceaue how I could haue omitted any thing as yet, without your greate lacke thereof.

Scholer. If I had receiued 85 pounds of certaine men, but of howe many I haue forgotten, yet I remember that the firste gaue me 7 li, and the laste 27 li, and euery payment after other did rise by a like summe. And the man for whome I receiued this mony, conditioned with me, ℔ of euery payment I shoulde haue twelue pence for my labour: nowe vnleast I can by arte finde the truthe of this case, I am like to lose the most part of my rewarde.

Mayster. I perceaue you can handsomlye frame an example, whithe shoulde concerne your owne gaine: I pray you let me see howe you would we Justice in this poynte.

Scholer. I adde the first and the laste together, that maketh 34: by whiche I diuide 85, thus. Why how now? Sir, here

1		
x 7		17
85	(2	
34		34
8		

is

is á remnãt of 17, in whiche 34 can not be had: so that now I am in the bryars for doublinge of my quotient, and farewell than bothe my Justice and á good lumpe of my gaines.

Mayster. Yee are neuer the farther from the matter, though it fall into á Fraction. For you shall vnderstande, that the fraction whiche of any suche worke proceedeth, is euer halfe of one suche, as the vnits of the Quotient before are. And that you may trie, if you double that whiche so remaineth, for than it will bee equall to your diuisour, as if ye double 17 (the remnant) it maketh 34, and your diuisour also was 34, this noteth the remainder to bee halfe of one.

Scholer. Nowe I am glad of this harde example, For with it I haue á generall rule for the Fraction that may hap in this worke. So that the quotient beinge two and á halfe, I double that, and it maketh 5, therefore shoulde my gaine bee 5 shillinges. And to bee sure, (by your leaue) I will trie it, for I will multiplie halfe of 34, (whiche is the firste and laste number ioyned together) by 5, thus. It is most true (I see) that I should leese nothing by the

17
5
85

former workinge.

5 Mayster. The fifth proposition hath this
" rule appertaininge vnto it: By the fourth rule
" finde the number of ye places, that beinge done,
" from the laste subtract the firste, and the resi-
" due diuide by à number lesse by 1, then the
" number of the places, and the quotient will
shew the exces whiche is sought for.

An example hereof shall be this: If yee had disbursed 685 poundes to à certaine number of men, you neither can tell howe many they were, or howe muche the ones mony exceeded his next before, but you are sure that the exces was equall betweene euery two next: and also you remember ye the firste had 19, and the last 118 poundes, how would you finde bothe ye number of the men and the exces, continually obserued in the succession of their paiments.

Scholer. Your rule doth plainely bid, first to finde the number of the places, whiche I will doe accordinge to the fourthe rule. I adde 19 and 118 together, thus.

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 118
  19
 ---
 137
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By this 137, I diuide 685, thus.

```
 13
 685(5
 133
```

Seinge there is no fraction, but à whole nũber, being 5, I

double

double that, and than muste the number of ẏ places be 10. 118
Now from ẏ laste I subtracte the firste, as 19 from 118, thus: 19
And so remayneth 99. 99

This 99 I diuide by a number lesse by 1 than the numbre of the places, and seinge the places were 10, I diuide 99 by 9, thus. 99 (11
99
ẏ quotient is 11, & so was the excesse, if I haue followed your rule right.

Mayster. You haue wrought euery part of this question bothe well in ordre and truely in the practise of your rules.

Scholer. I will than set it wone also formably, so that the numbre of the places, the excesse and the totall summe maye streighte appeare, as your firste example stoode.

11 11 11 11 11 11 11 11 11 11 The commō exces.
19 30 41 52 63 74 85 96 107 118 The progression.

That the places bee 10, and that from the firste to the laste the common excesse is 11, I perceiue moste euidentlye, but whether the totall summe be 685, I haue not yet proued, whiche I will nowe doe. I adde 19 and 118 togither, that maketh 137: I multiplye that by

halfe the numbre of the places, thus.

137
5
685

All thinges agree moste exactlye, so that I am perfecte ynough in these rules, if I forgette them not agayne.

6 Mayst. Use maketh all thinges perfecte.

„ Youre sixte rule is this. By the numbre of
„ the places diuide the totall summe, doubble the
„ quotient, and that will bee the first and ẏ laste
„ ioyned in one summe. Than by à numbre lesse
„ by 1, than the numbre of the places multiplye
„ the excesse, that ofcome subtracte from the firste
„ dubbled quotient, and the halfe of the residue
„ is the firste numbre. The laste numbre you
„ maye diuersly finde out, as by the firste of oure
„ sixe rules, or by subtractinge this firste numbre
„ from the summe whiche here contayned bothe
„ the firste and the laste ioyntlye, (or thirdly) by
„ continually adding the excesse.

Scholer. I pray you make this somewhat more playne with an example.

Mayster. Yf euerye moneth in the yeare (countinge them nowe as thirtene) you gayned cleerely 40 shillinges more than you did the moneth next goinge before, & at the yeares ende you finde ẏ whole gayne 520 shillinges,

but

but yee remembre not howe muche either the gayne of the firste moneth or the laste was, by this rule it may bee tried out.

Scholer. So that here yee seeme to applye the 13 moneth to thirtene places, the 40 shillinges euery one more then the other nexte before it, to bee the common excesse, and 5720 ℔. to bee the totall Summe.

Mayster. It is trewe: by 13 than I diuide 5720, in this manner.

```
  1
 1 5
 5 7 2 0 (440
 1 5 5 3
   1 1
```

I double this quotient, so haue I 880 for the firste, and the laste summe ioyned togither by 12, whiche is lesse by one than the nũbre of the places, I multiply 40, (the common excesse) so commeth 480.

```
  40
  12
 ----
  80
 40
 ----
 480
```

This 480 I subtracte from 880, so remayneth 400: halfe whereof is the firste Numbre whiche wee desired to knowe: that is 200.

And as for the laste numbre I can giue you it three wayes: As by the first of my six rules, I multiplye the excesse by a numbre lesse by 1 than the numbre of the places: as 40 by 12, that giueth 480, whiche I adde to the firste

beinge 200, so shall the laste be 680.

The same summe commeth forthe, if yee subtract 200 from 880.

And thirdely, if I beginne at 200, and so proceede, encreasing by 40, I shall at the thirtenthe place haue 680, as thus.

200. 240. 280. 320. 360. 400. 440. 480. 520. 660. 600. 640. 680.

Scholer. I thanke you moste hartelye for these fyre rules. Nowe if it bee youre pleasure I would heare and learne somewhat of Progression Geometricall.

Mayster. There are yet very many rules and propositions, which fall into this Arithmeticall progression: but these shall suffise for this time.

And in Geometricall progression I will be more breife, bothe because I haue byn so longe in this parte of Arithmeticall progression, and also for that it woulde require the knowledge of Rootes, and numbres surde, (wherof ye haue yet learned nothing) if I should frame the like propositions in them as I haue done in these. Therefore I will onely teache you two practises, about it, and so ende the considerations

and

und worꝛkes of theſe pꝛogreſſions. Pꝛogreſſion Geometricall is when the numbers increaſe by a like pꝛopoꝛtion, that is, if the ſecond nũ-ber containe the firſte, 2, 3, oꝛ 4 times, and ſo foꝛth: then the thirde containeth the ſecond ſo many times alſo: and ſo the fourth the thirde, and the fifth the fourth: Progreſſion Geometricall.

wherfoꝛe I ſet theſe three examples.

3, 6, 12, 24, 48,
1, 3, 9, 27, 81,
2, 10, 50, 250.

Here in the firſte example you ſee, that euery number containeth the other (that goeth next before him) 2 times: and in the ſeconde example 3 times: in the third example 5 times. Now if you will know how to finde eaſilie the ſumme of any ſuch numbers, we thus. Conſider by what number they be multiplied, whether by 2, 3, 4, 5, oꝛ any other, and by the ſame number we you multiplie the laſte ſumme in the Pꝛogreſſion.

Scholer. I pꝛay you woꝛke it by this example. 2,8,32,128,512,2048, whiche I haue framed by pꝛoceeding from 2, and continuallye multiplying by 4.

Mayſter. Then muſt I multiply the laſte ſumme (whiche is 2048) by 4 alſo, and it will be 8192. Now muſt I bate from this ſumme

the firſte number of the progreſſion, whiche here is 2, then reſteth 8190. whiche ſumme I muſt diuide by 1 leſſe then was the number that I multiplied by. Seing then I multiplied by 4 I muſt diuide by 3, ſo diuiding 8190 by 3, the quotient will be 2730, whiche is the ſumme of all the Progreſſion. And now to proue whether you can doe the ſame, I geue you theſe numbers to adde by this rule. 3, 15, 75, 375, 1875, 9375, 46875.

Scholer. I can not well tell by what number this Progreſſion doth increaſe.

Mayſter. In anye ſuche doubt, doe thus: ☞ Diuide the ſecond number by the firſte, and the quotient will ſhew you the number that engēdzeth the Progreſſion.

Scholer. Then is that number in this example 5, for ſo many times is 3 in 15.

Mayſter. So is it. Nowe worke as I taught you.

Scholer. The laſte number is 46875, whiche I multiply by 5, & it yeldeth 234375, from whiche I bate the firſte number of the Progreſſion, that is 3, & there reſteth 234372, whiche I diuide by 4, for that is one leſſe then 5, & the quotient is 58593, whiche is the whole ſumme of the progreſſion.

Mayſter.

Mayster. Nowe that you knowe the summinge of Geometricall Progression, I will shew you à compendious māner either to proceede by, or to finde out the quantity of à number whose distaunce from the first may be very greate, whiche to doe by continuall multiplication, woulde be very tedious, if the numbers be greate and the places many. An abridgement in progressiō

Scholer. Nothing can pleasure me more then breuitie, if it be plaine.

Mayster. I thinke I am not yet in anye pointe so darke or harde, that you neede to feare any obscuritie nowe. The manner is this: set wnne of your progression foure or fiue of the firste places, and vnder the firste put à cipher, vnder the seconde 1, vnder the thirde 2, &c. as if ye had à progression encreasinge by à fiue folde quantitie: as here, 2, 10, 50, 250, 1250: then vnder 2 I put à cipher, and vnder 10 the figure of

1: vnder 50, 2 10 50 250 1250
2: vnder 250, 0 1 2 3 4
3: vnder 1250,
4: and so forthe if yee will: but to a wise and wary worker, à fewe places were sufficient to proceede by to anye number of places in this sorte, if anye two of youre numbers

progressionall bee multiplied the one by the o-
« « ther, and the ofcome diuided by the first of your
« « progression, the quotient is one of your num-
« « bres progressionall, & belonginge to that place
« « of your vnder numbers, that is equall to that
« « summe, that is made of Addition together of
« « your two numbers whiche stoode vnder these
« « two of youre Progressionall numbers, that
« « were multipli-

ed ỹ one by the other, as in this example.

2	10	50	250	1250	6250	
0	1	2	3	4	5	

If I multiplie 10 by 50, thereof commeth 500, whiche I diuide by 2, (the firste number of the Progression,) and the quotiente is 250: whiche 250, muste stande in the thirde place, because the number whiche standeth vnder 10, is 1, and that vnder 50, is 2: and 2 and 1 maketh 3. Therefore I saye, that 250 belongeth to the thirde place of this progression, as yee see also heere it dothe. Moreouer If I multiply

50
50
00
250
2500

50, into it selfe, thereof commeth 2500: that 2500 I diuide by 2, the Quotient is 1250, whiche muste bee sette in the fourthe place: bicause 2 added to himselfe agayne, maketh 4, and in oure example 1250 occupieth the fourth place.

Scholer. Than for that fifthe place I multiply the Progressionall numbres ouer 2 and 3, one by the other: and for the sixte, I multiplye that ouer 3 in it selfe. &c.

Mayster. Ye muste well remember ẏ these places that we nowe speake of, belonge to the vnder numbers, for the true places of the vpper numbers is euer one place more.

Scholer. That I see the reason of, bicause the vnder numbers begin one after, & againste the first place of my progression standeth a cypher, so that the 250 whiche you sayde before did belonge to the thirde place, I see belongeth to the nũber of 3 amonge your vnder numbres, but from the trewe progressions beginninge, it is the fourthe.

Mayster. You vnderstande mee as I meane. Therefore for youre exercise of bothe the rules here geuen for Geometricall progression, I will aske you a question, muche vsed amonge the common people, (as they haue a great

greate many the like) Yf I woulde sell you a Horse, hauinge 4 shoes, and in euery shoe six nailes, with this condition, that you shall paye for the first naile 1 ob, for the seconde 2 ob, for the thirde 4, and for the fourthe 8, and so forthe dubblinge vntill the laste nayle. Nowe I demaunde of you, howe muche the pryce of the Horse woulde amounte vnto?

Scholer. Seinge the Horse hathe 4 shoes, and in euerye shoe 6 nayles, I perceyue heere will bee 24 places. Yf I coulde nowe haue the laste number, I woulde quicklye dispatche this question. I will therefore with as fewe multiplications as I can deuise, come to the knoweledge of the laste number of this progression. In double I sette forthe 1 2 4 8 16 32 64 than a fewe of my 0 1 2 3 4 5 6 progression, thus.

If I nowe multiplye the numbers ouer 5 and 6, the one by the other, I shall haue the number of the eleuenth place for the vnder nũbers, but of the twelft for the vpper numbers in whiche my progression standeth, and than that of the eleuenth place vnder, if I multiplye in it selfe, I shall haue for the 22 place vnder, but for the 23 of that aboue, whiche I multiply

ply by that ouer 1 of my nether places, and I shall haue the 23 of my nether places, and the 24 of the vpper, whiche is the number I seeke for.

Mayster. Mee thinketh you haue forgotten youre rule for abbridginge youre multiplications: for in it, the ofcome euer of any multiplication, is to bee diuided by the firste of the progression. And you nowe speake of no Diuision.

Scholer. Sir I neede not, as my progression beginneth nowe: for if I shoulde diuide by 1, it maketh no other quotient, than the nūber is, it dothe diuide.

Mayster. It is very well remembred & noted of you, to youre worke than accordinge to youre prescribed manner, whiche I lyke well.

S. I multiply 64 by 32, as heere. And it maketh 2048, whiche is the eleuenth place vnder, but the twelfth aboue, and this, I multiply in it selfe, in this manner. And this is the 22 place vnder, but the 23 aboue. I multiplye this than by 2, as heere,

```
      64
      32
    ----
     128
    192
    ----
    2048
    2048
    2048
    ----
   16384
   8192
  0000
 4096
 -------
 4194304
```

And

And this ofcome 8388608, is my foure and twentith place, whiche I haue founde nowe by 3 multiplications.

4194304
2
8388608

Than we I reſorte to the rule of ſumminge this Progreſſion, where I conſider that the encreaſe of this ſumme proceedeth by multiplication of 2, and therefore I doe multiply the laſte ſumme by 2 alſo, and it yeeldeth 16777216. from whiche I abate the firſte numbre whiche is 1, and then reſteth 16777215, whiche I ſhoulde diuide by 2 leſſe than I did multiply: but ſeinge that it is 1, I neede not to diuide it, for 1 (as I haue before ſayde) withe nother multiplye nor diuide, therefore I take that ſumme 16777215 for the whole ſumme of the halfe pennies, whiche by Reduction I finde to bee 699050 ℔, and 7 ð, ob: that is 34952 li, 10 ℔, 7 ð, ob.

Mayſter. That is well wne, but I thinke you will buye no horſe of the price.

Scholer. No ſir if I bee wiſe. Yet for my aſſurance will I take ſo muche payne, as to come to this laſte 8388608 by continuall multiplication by 2, as in the page followinge you maye beholde my worke till I haue done.

1	1	done.
2	2	Mayſter. Well, are
4	3	yee not almoſte weary?
8	4	Scholer. Well fare
16	5	my ſhorte rule, for in
32	6	troth it hathe more cun-
64	7	ning and more eaſe.
128	8	Mayſter. Well, than
256	9	anſwere mee to this que-
512	10	ſtion.
1024	11	A Lorde deliuered to
2048	12	à Bricklayer à certaine
4096	13	number of lodes of brick,
8192	14	whereof he willed him to
16384	15	make 12 walles, of ſuche
32768	16	ſorte, that the firſte wall
65536	17	ſhoulde receaue 2 thirde-
131072	18	les of the whole number:
262144	19	and the ſeconde 2 thirde-
524288	20	les of that ỹ was lefte. &
1048576	21	ſo euerye other 2 thirdeles
2097152	22	of that ỹ remained: and
4194304	23	ſo did the bricklayer: And
8388608	24	when the 12 walles were
		made, there remaineth

one lode of bricke.

Nowe I aſke you, how many lode went to

euery

euery wall, and howe many lode was in the whole?

Scholer. Why sir, it is impossible for me to tell.

Mayster. Naye, it is verye easie, if you marke it well. Marke well that I saide, that euery wall shoulde receaue 2 thirdels of the summe that was lefte. Nowe take awaye 2 thirdels from any summe, and you must needes graunt that that whiche remaineth is 1 thirdle of the summe laste before: example of 9, from whiche if you take 2 thirdels, there will remaine 3, whiche is one thirdel of 9. Likewaies from 3 bate 2 thirdels, and there will remaine 1.

Scholer. This is true, and nowe I perceaue, that the leaste wall had but two lode of bricke.

Mayster. And by the same reason maye you knowe howe many lode euery wall had, according as this figure folowinge wth shew, and likewaies what ẏ whole summe of bricks was: for if you make 12 summes, multiplying by 3, still from ẏ laste remainer, as you see here on the left side of the table, there will appeare all the remainers of euerie wall: and if you multiplie the laste of those 12 summes

by

by 3 also, than will that bee the summe of the loades whiche were deliuered to the bricklayer.

1	1 2	2
3	1 1	6
9	1 0	18
27	9	54
81	8	162
243	7	486
729	6	1458
2187	5	4374
6561	4	13122
19683	3	39366
59049	2	118098
177147	1	354294
531441		

Againe, if you we double euerye remainer, as you see at the right side of this table, those numbers will shewe the summe of loades that went to eche wall: whereby also you may perceaue, that eche wall was 3 times so greate as the next lesser.

Scholer. Lo, nowe it appeareth easie inough. Now surely I see that Arithmetike is a right excellent arte.

M.i. Mayster.

Mayster. You will say so when you know more of the vse of it: For this is nothinge in comparison to other poyntes that maye bee wrought by it.

Scholer. Than I beseech you sir, cease not to instruct mee farther in this wonderfull cunninge.

THE GOLDEN RVLE.

Mayster.

The rule of Proportion.

BY the order of the science (as men haue taught it) ther should followe nexte the extraction of Rootes of numbre, which because it is somewhat harde for you, yet I will let it passe for a while, and wil teache you the feate of the rule of Proportions, which for his excellencie is called the Golden rule. Whose vse is, by three numbres knowen, to finde out an other vnknowen, which you desire to know: as thus. If you pay for your horse for three monethes 16 shillinges, howe muche shall you pay for 8 monethes.

The Golden rule.

Question of borâdinge.

To knowe this and all suche like questions, you shall consider which two of your 3 numbres be of one denomination, and set those two

the

the one ouer the other, so that the vndermost be it ẏ the question is asked of: as in my question 3 and 8 be bothe of one denomination, for they bothe be monetyes, and because 8 is ẏ number that the question is asked of, I set them one ouer ẏ other, ẏ 8 vndermost, thus, 3 / 8 with suche a croked draught of lines. Then we I set the other number whiche is 16, againste 3, at the right side of the line, thus. 3 — 16 / 8

And now to knowe my question, this must I we: I must multiply the lowermost on the left side, by that on the right side, and ẏ summe that amounteth I must diuide by the highest, on the left side. Or in plainer wordes thus: I shall multiply the number of whiche the question is asked (whiche is called ẏ Thirde number) by the number of an other denomination, (whiche is called the Second) and that summe that amounteth must I diuide by the summe of like denomination, whiche is called ẏ First. Than for the knowledge of this question, I multiply 8 into 16, and there amounteth 128, whiche I diuide by 3, and it yeldeth 42 shillings, and 2 ℔ remaineth, whiche I tourne into pennies, and they be 24 ð, of whiche the third part is 8 ð, so the third part of 128 ℔

The third numbre.
The seco̅d numbre.
The firste numbre.

is 42 s, 8 d: whiche sum I write at ye right hād of the figure against 8, thus. 3 — 16 s.

Hereby I knowe, that if 8 — 42 s, 8 d. three monethes boording doe come to 16 s, that 8 monethes bordinge will come to 42 s, 8 d: and likewise of any other like question.

But heere muste you marke, that the firste number and the thirde be of one denomination, and also the seconde and the fourth, for whiche you seeke: or els bee of suche denominations, that you in working may bringe thē into one. As if a man should aske me this question.

Question of expenses.

Twelue weekes iourneyinge coste me 14 Nobles, howe manye poundes is that in one yeare? Heere you see no two numbres of one denomination, but yet in workinge you maye turne them into like denominations, as thus. Tourne the one yeare into 52 weekes, and the fourthe summe will bee nobles, by the order of the working. Then to knowe this question, multiplye the thirde summe, 52, by the seconde 14, and the summe will bee 728, that diuide by 12, and it will be 60, and 8 remayning: whiche if you tourne into pennyes, they will bee 53 d, and $\frac{1}{3}$ of a penny more: whiche summe of pennies if it bee diuided by 12, will yelde 4 s, 5 d. and the thirde parte of a penny: but this 60

Nobles

Nobles (whiche maketh 20 li) with the 4 f, 5 d and q, and little more: for the summe that aunswereth to ỹ question, and it is the expence of a yeare, and the summes will bee thus. Or els, for to note them as they fall oute, precisely, with the fraction: and to leaue out the grosse termes and inartificiall, of à little more, or à little lesse: thus they must bee witten. And whan you come to the Fractions, you shall by order bee certified, what eight twelfthes or two thirds of a Noble is, or what the valuacion is of any Fraction, bee it of any coyne, measure, waighte, or other quantitye, what so euer, whiche hathe vsuall or determined partes knowen.

	12 — 14	Nobles.
	52 — 60, 4 f, 5 d, q.	
VVeekes.	*Nobles.*	
12	14	
52	60	8/12

And take this for à generall rule, that euermore the Thirde numbre must bee it that the question is ioyned with: and the first, the numbre that is of the same denomination, thẽ must the second needes be that other. A generall rule.

Remembre also, that the place of the Firste numbre is the higeste on the lefte side: and the place of the seconde right agaynst it on ỹ right

ſixe: the place of the Thirde numbre is vnder ỹ firſt, as by thoſe examples you haue ſeene.

Scholer. This I truſt I can we.

Mayſter. But and if the queſtion be aſked thus: In s weekes I ſpende 40 ℔, howe longe will 105 ſhillinges ſerue mee? Thoughe the ordre ſeeme vnlike, yet take you 105 for the Thirde numbre, and 40 beeynge of the ſame denomination, for the Firſt, and then 8 for the Seconde. Then multiply 105 by 8, and it will be 840, whiche if you diuide by 40, it will yelde 21, whiche is the Fourth numbre, & ſheweth howe many weekes 105 ℔ will ſerue, if you ſpende 40 ℔ in eight weekes.

The figure of this queſtiō is this: as if you ſhold ſaye: If 40 ℔ ſerue for 8 weekes, 105 ſerue for 21 weekes.

Shillings.	*VVekes.*
40	8
105	21

Other diuerſities there be of working by this rule, but I had leuer that you wolde learne this one well, then at the beginninge to trouble your minde with manye fourmes of working, ſith this way can doe as muche as all the other, and hereafter you ſhall learne the other more conueniently.

Note. But yet before wee make an ende of this rule

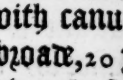
rule, this shall you note, ỹ there is an other order quite contrarye to this that you haue learned. For in this rule hitherto, euermore loke how muche the Thirde number is greater then the Frst, so muche the Fourth number is greater then the Seconde. And contrarie waies: looke how muche the Firste summe is greater then the Thirde, (if it doe chaunce so) so muche is the Seconde summe greater then ỹ Fourth. But there is a cõtrarie order, as this: That the greater the thirde summe is aboue the firste, the lesser the fourth sume is beneth the second: and this rule you maye call the Backer rule, as in example. The Backer rule.

If I haue bought 30 yardes of clothe of 2 yardes breadth, and woulde buy canuas of 3 yardes broade to line it withall, howe many yardes should I neede? Question of byinge clothe.

Scholer. Why, there is none so broade.

Mayster. I we not care for that, I we put this example onely for youre easie vnderstanding: For if I shoulde put the example in other measures, it woulde bee harder to vnderstande. But now to the matter: If you would knowe this question, set your numbres as you did before: but you shall multiplie nowe the first numbre by the seconde, and that thereof

ariseth thereof, you shall diuide by the thirde: whiche thinge if you we here, I meane if you multiplie 30 by 2, it will be 60: whiche summe if you diuide by 3, there will appeare 20: where by I know, that if 30 yardes of clothe of two yardes broade, shoulde be lined with canuas of 3 yardes broade, 20 yardes of canuas would suffice, as this figure sheweth.



And now because ye found fault at my example, how say you, perceaue you this?

Scholer. Yes sir, I suppose.

An other like question.

Mayster. Than answere me to this question: howe many elles of canuas of elle breadth will serue to line 20 yardes of Saye, of three quarters of a yard broade?

Scholer. In good faith sir, I can not tell, for I know not how to bringe the summes to like denominations.

Mayster. Than I will tell you: sithe there is mention here of quarters, and againe euerie one of the measures bothe elles and yardes may be parted into quarters, if you part them so bothe in the breadth and length, and than put forth the question by quarters.

Scholer. Than I shall say thus. Howe many

many quarters of canuas of 5 quarters broade, will line 80 quarters of 3 quarters broade?

Mayster. Now answere to the question.

Scholer. First I will set the wwne in their forme, thus: for 5 is ioyned with the question, and is therefore the thirde number: then is 3 the number of the same denomination, I meane because they be bothe referred to breadth. Nowe I multiply 80 by 3, and it is 240, whiche I diuide by 5, and it yeldeth 48. Then saye I, that 48 quarters of 5 quarters broade, will suffise to line 80 quarters of 3 quarters broade.

Breadth. Length.

3 — 80

5 — 48

Mayster. Turne the quarters againe into elles and yardes.

Scholer. Then I say, that 9 elles and 3 quarters of à yarde of elle broade, will serue to line 20 yardes of 3 quarters broade: as this figure sheweth.

Breadth. Length.

3 — 80

5 — 48

Mayster. This rule is so profitable for all estates of men, that for this rule onely (if there were no more but it) all men were bounde highly to esteeme Arithmetike.

By this rule maye à Captaine in warre

worke many thinges, as I will hereafter instruct you abundantly, onely now I will shew you this one example.

Question of prouision touchinge an army.

If it shoulde chaunce a Capitaine whiche hath 40000 souldiers; to bee so inclosed with his enemy, that hee coulde haue no freshe pursueyaunce of vittailes, and that the vittailes whiche hee hath, woulde serue that army but onely 3 monethes, how many men shoulde hee dimisse, to make the vittaile to suffise the residue, 8 monethes?

Moneths. *Men.*

3 Z 40000
8

Sc. As you taught me, I set the numbers thus, saying: If three monethes suffise 40000, to howe many will 8 monethes suffise?

To knowe this I multiply the first number 3 into the seconde 40000, & it yeldeth 120000, whiche summe I diuide by 8, and there will be in the quotient 15000, whiche if I doe subtract from 40000, the remainer will declare that he must dimisse 25000: as this figure sheweth.

Moneths. *Men.*

3 Z 40000
8 15000

Mayster. Well, sithe you perceaue nowe the vse of this rule, I will shewe other whiche ensue

ensue of the same. And first the Double rule, whiche is so called, bicause there is in it double worckinge, by whiche thinge onely it differeth from this. The Double rule.

Scholer. Then by an example I shal understand it well inough.

Mayster. So shall you, and let this bee the example: If the cariage of 100 pounde weight 30 miles, doe coste 12 d, howe muche will the cariage of 500 weight coste, beyng caried 100 miles? Of cariage.

Sc. I pray you shew me the worcking of it.

M. You must make 2 worckings of it: ỳ first thus. If 100 pounde weyght coste 12 d, howe muche will 500 pounde coste?

Set your figure thus. *Weight.* *Pence.*

And multiply 500 by 12, and therof amounteth 6000, whiche if you diuide by 100, the quotient will be 60, ỳ is the price of 500 for 30 miles.

100 — 12
500 — Z

Then begin the second worcke, saying: if 30 miles coste 60 pence, how muche will 100 miles cost? Set youre figure thus. *Miles.* *Pence.*

Than multiply à 100 by 60, wherof amounteth 6000, whiche beynge diuided by 30, will yelde 200. Than you maye saye, that so manye pennies shall

30 — 60
100 — Z

shall coste the cariage of 500 pounde waighte 100 miles, after the rate of 12 pence for the 100, caried 30 miles.

Scholer. Nowe I perceiue it also.

Question of sowinge.

Mayster. Then aunswere mee to this question, 30 bushels of wheate sowed, yeldeth in one yeare 360, howe manye will 80 bushels yelde in 7 yeare. I meane sowing euery yeare of those seuen, still 80 bushels.

Sc. First I say, that if 30 bushels will yelde 360 in 1 yeare, then 80 bushels will yelde 960 in one yeare. Then for the seconde worke I say: If one yeare yelde 960, then 7 yeare will yelde 6720: as these two figures we shewe.

Seede.	*Encrease.*	*Yeares.*	*Encrease.*
30	360	1	960
80	960	7	6720

Question of corne.

But nowe sir, if I set forthe 30 bushels of corne to an other man for 7 yeare, agreyng so, that hee shall sowe euery yeare the whole encrease of the corne, and I at the ende of those seuen yeares to haue the halfe of the whole encrease: I would knowe howe manye bushels will there amount to my parte, suppsinge the encrease to bee after the rate of the laste question, for 30 bushels in one yeare, 360.

Mayster. In suche a question you must haue so

so many seueral worykings, as there be yeares: as for example. In the first yeare 30 bussheis yelde 360: than to know the yelding of the seconde yeare, I must say: if 30 yelde 360, howe many yeldeth 360? Worke by your rule, & you shall finde 4320. The say for ỹ thirde yeare: If 30 yelde 360, how many will 4320 yelde? you shall haue 51840, & so euery yeare, multiplying ỹ whole encrease by 360, and diuiding it by 30, the encrease of the nexte yeare will amount: as these 7 figures we orderly declare.

a

30 — 360

b

30 — 360
360 — 4320

c

30 — 360
4320 — 51840

d

30 — 360
51840 — 622080

e

30 — 360
622080 — 7464960

f

30 — 360
7464960 — 89579520

g

30 — 360
89579520 — 1074954240

Where I haue set 7 letters for the 7 yeares, of which the first is set without arte, because that is the increase which you we presuppose: and the laste numbre of eche other, wthe shewe the increase of the yeare that it standeth for, which the letters wth declare: so that the increase of the 7 yeare, is 1074954240 busshels: how many quarters that is, and also howe many waies, you may by Reduction some finde.

Question of mowinge.

Now with one question more I will proue you. If 6 mowers doe mowe 45 acres in 5 dayes, how many mowers will mow 300 acres in 6 daies?

Scholer. If 45 acres we require 6 mowers, then 300 acres requireth 40. Now againe: if 5 dayes require 40 mowers, then 6 daies neeth but 33 mowers.

Mayster. Why we you not make mentiō of the 2 that remaineth in the laste diuision? for the laste part of the question is wrought by the Backer rule, where the first number 5, is multiplied into the second that is 40, wherof amounteth 200, whiche if you diuide by the thirde number 6, the quotient will bee 33 as you said, but then will there remaine 2, which can not well bee diuided into 6 partes: howe

be it,

be it, you may vnderſtande by the firſt part of 2, the thirde part of one mans worke, whiche you muſt put to the 33, or els you may ſaye, that 33 workemen will ende all the 300 acres in 6 daies, ſaue two mens worke for one daie, or 2 daies worke for one man. But ſuche broken number called Fractions, you ſhall here after more better perceaue, when I ſhall wholy inſtruct you of them.

The Rule of Fellowſhip.

BVt now will I ſhewe you of the rule of Fellowſhip or Company, whiche hath ſundry operations, accordinge to the diuers numbre of the companie. This rule is ſometime without difference of time, & ſometimes there is in it difference of time. Firſt I will ſpeake of that without difference of time, of whiche let this be an example. Without Tyme.

Foure marchantes of one company made a banke of mony diuerſly, for the firſt laide in 50 li, the ſeconde 50 li, the third 60 li, and the fourth 100 li, whiche ſtocke they occupied ſo long, till it was encreaſed to 3000, li. Now I demaunde of you, what ſhoulde eche man receaue at the parting of this mony. Queſtion of a Banke.

Scholer.

Scholer. I perceaue that this rule is like the other, but yet there is a difference, which I perceaue not.

Mayster. Then will I shewe it to you. First by Addition you shall bringe all the particular summes of the Marchantes into one summe, which shall bee the first sum in youre worckinge by the Golden rule, and the whole summe of the gaines by that stocke shall be the Second summe. Now for ỳ Thirde sũme, you shall set the portion of eche man one after an other, and then worcke by the Golden rule, and the Fourthe summe will shewe you eche mans gaines: as in example.

The parcels of those foure marchants make in one sũme 240 li: set that in the firste place, the gaines in the seconde, & the first mans portiõ of stocke in ỳ 3 place, thus.

240 — 3000
30 — Z

Now multiplie the second by the third, and it will be 90000, which you shall diuide by 242, and there will appeare 375 li. thus.

240 — 3000
30 — 375

And that is ỳ gaines for the first man.

Now for the seconde man, set the 50 li. that he brought, in the thirde place, and worcke as before

before: and his part will be 625 li. as this figure sheweth.

240 — 3000
50 — 625

Likewaies for the thirde man set his mony, whiche was 60 li, and his part of gaines will be 750 li. as here appeareth.

240 — 3000
60 — 750

And so for the fourthe man, if you set his summe whiche is 100 li, his gaines will bee 1250 pounde, as the proofe will declare.

240 — 3000
100 — 1250

Scholer. This I perceaue: but is there anye waye to examine whether I haue well wne or no?

Mayster. That must you we by one common proofe whiche serueth to the Golden rule, and all other insuinge of the same: and that is this: Chaunge the standinges of the numbers, and set the thirde in the first place, the 4. in the seconde place, and the firste in the thirde place, and then worke by the Golden rule, and if you haue wne well, the fourth number now will be the same that was the seconde before. As for example. I will take ỹ laste worke whiche was this.

Note this common proofe.

Proofe.

240 — 3000
100 — 1250

100 — 1250
240 —

Whiche to examine, I alter as I saide, thus.

Nowe if I multiply the second number by the third, and diuide that that amounteth by ỹ first, then will the fourth number bee 3000, whiche was the seconde before, as you see here. whiche is a token, that I haue well done. But as in a single rule one proofe thus is sufficient, so in a rule where many operations bee, you muste tourne euery of them as I haue done with this one.

100 — 1250
240 — 3000

Scho. Then for the proofe of the first worke of this rule, I shoulde turne the numbers thus.

30 — 375
240 —

50 — 625
240 —

And the seconde, thus.

And for the thirde, thus.

60 — 750
240 —

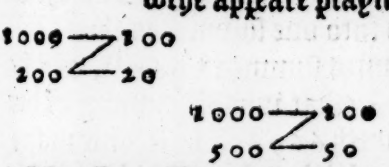
And in eche of them if the working were trewe, the fourth number will be still 3000.

Mayster. Well, now an other example will I put to you, not of gaines, but of losse: for one reason serueth for bothe.

Question If three Marchantes in one shippe and of one

one fellowship, had bought marchandise, so ỳ the first had layd out 200 li, the seconde 300 li, and the thirde 500 li, and it chaunced by tempest that they did cast ouer boarde into ỳ Sea marchandise of the valew of 100 pounde, howe muche should eche man beare in this losse? of losse.

Scholer. If I shall we in this as you did in the other question, then muste I ioyne theire three ꝓtions togither, 200, 300, and 500, which maketh 1000: Than saye I, if 1000 leese 100, thē shall 200 leese 20, and 300 shall leese 30, and 500 shall leese 50, as by these three figures it wthe appeare playne.



Mayster. Thus you perceaue the vse of the rule without time. And that you may as well perceaue the same with diuersitie of time, I ꝓpose this Example. The rule of fellowe ship with Tyme.

Foure Marchantes made a common stocke, whiche at the yeares ende was encreased to 35145 li. Nowe to knowe what shall be eche mannes ꝓtion of gaynes, you must knowe eche mannes stocke and time of continuance. Question of a Banke.

The first man of these foure layde in 669 li,

whiche he did take from the stocke agayne, at the ende of 10 monethes. The seconde mã layde in 810 li. for 8 monethes. The thirde layde in 900 li. for 7 monethes. And the fourthe layde in 1040 li. for 12 monethes.

This question shall you examine as you did the other before, sauinge that where as in the thirde place of the figure you did set eche mans summe alone, heere you shall set the same being multiplyed by the numbre of their time: & likewayes in the firste place of the figure, you shall sette that number whiche amounteth of theire whole summes so multiplied by theire time, and added into one summe, as thus.

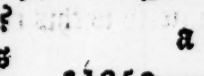
The first mannes summe is 669 li, whiche I multiply by 10 (that was the numbre of his time) and it maketh 6690. The seconde mans summe 810 li, multiplyed by 8 (whiche was his time) maketh 6480. The thirde mannes summe 900 li, multiplyed by 7 (for that was his time) yeldeth 6300. The fourthe mans summe was 1040 li, and his time 12, multiply the one by the other, and it will be 12480.

These foure summes thus multiplyed by their time, must be set orderly in ỹ thirde place of the figure: and in the firste place must bee set the whole summe of all foure, whiche is 31990.

and

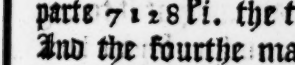
and the gayne muste bee in the seconde place, whiche is 35145. Now to ende the question, I saye firste: If 31950 did gette 35145, what did 6690 gete? Answere, 7359 li, as by this figure appeareth.

a

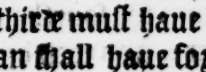


Likewayes the seconde man had to his parte 7128 li. the thirde muste haue 6930 li. And the fourthe man shall haue for his parte 13728 li. as these figures w parcelly declare.

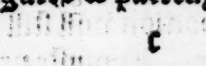
b



c



d



Scholer. This I like very well: but what proffe is there of this worke?

Mayster. The same that I taughte you for the other. How be it, there is vsed both for this worke and the other also this maner of proffe, to adde all the portions togither, and if they agree to the whole summe, then seemeth it well done. But this is no sure rule. An other proofe.

Scho. Yet will I proue it in this example.

The foure parcels are these, whiche if I adde togither, there will amount 35145, and that was the whole summe: so is this rule trewe heere.

	7359
	7128
	6930
	13728
	35145

Mayster. And so will it bee still, when the worke is trewly wrought.

Note the imperfection of this kinde of proofe.

But if you lisle to see it proued false, take 10000li, from the fourth man, and put it to any of the other 3, and then be you sure ẏ you haue not wrought well, and yet will the proofe alowe it, for the Addition will still be all one.

Scholer. It muste needes be so: but what haue I now to learne?

Mayster. There are many other excellente partes behinde, of whiche I will not, as nowe, make mention, bicause that without ẏ knowledge of Fractions, they cānot be duly taught, and muche lesse vnderstanded. Therefore will I propose to you two or three questions more, whereby you maye practise the better the feate of the rule of Felowship, and so make an ende for this time.

But this maye not be forgotten, that in all suche questions, if ẏ mony be of diuers kindes, you muste by Reduction bringe it into one kinde, that is to saye, to the leaste valure that is

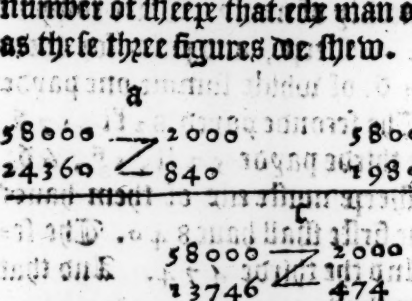
is named in the question. And like waies shall you do, if the time be of diuers kindes, as some yeares, some monethes, weekes and daies, you shall make all moneths, weekes or daies, according as the leaste name of time in the question is: As for example.

Firste in diuersity of mony. Three companions bought 2000 sheepe, and paide for them 241 li, 13 s, 4 d. of whiche summe one payde 101 li, 10 s. The seconde payed 82 li, 17 s, 10 d. And the thirde payde 57 li, 5 s, 6 d: Howe manye sheepe muste eche of them haue? Answere: The firste shall haue 840. The seconde 686. And the thirde 474. And that must you worke thus. Question of sheepe. Solution.

Firste consideringe that youre mony is of diuers denominations, you shall (by Reduction) bringe it all into the smalleste denomination whiche is in it, that is to say, pence, and so will the totall summe bee 58000 pence. Now if you turne eche mans mony into pennies also, the firste mans summe will bee 24360 pence: The seconde mannes summe 19894 d. And the thirde mannes mony will bee 13746 d.

Nowe to knowe howe many sheepe euery man shall haue, set the whole summe of mony

that is 58000 ō, in the first place: and in the seconde place set the number of sheepe, and then orderly in the thirde place set edx mans mony, and then multiplying the third and the seconde summes together, and diuidinge that that amounteth by the firste, there will appeare the number of sheepe that edx man ought to haue: as these three figures we shew.



Scholer. Why we you set the monye in the firste place, seeinge in the question you saye 2000 sheepe cost 58000 ō? & not thus, 58000 ō, coste 2000 sheepe.

Maister. You remember, I taught you at the beginning of this Golden rule, that the firste and third number muste bee of one name and of like thinges: and euermore the number that the question is asked of, must bee set in the thirde place. Now is the question plainely this: If four men bought 2000 sheepe for 58000 pence, howe many sheepe shall edx man haue?

But

But seing in this question there ought more respect to be had to the summe of mony, than to the summe of the persons, (for in the sūmes of mony is there proportion towarde ẏ sheepe, and not in the number of persons) therefore must we turne the question thus.

If 58000 pence bought 2000 sheepe, howe many did 24360 ō buye? Againe, how manie did 19894 ō buy? and how many bought 13746 pence?

Scholer. I perceaue it reasonable, and so shall I worke in all like questions.

Mayster. Euen so. But for easinesse of the worke marke this: When so euer the first and seconde number hath ciphers in their firste places, you may bothe in the multiplication and in the diuision leaue out those ciphers, so that you leaue oute like manie out of bothe summes, as in this question the firste number 58000 hathe three ciphers, and so hathe the seconde that is 2000: therefore caste awaye their ciphers, and so will the first number bee 58, and the seconde 2: set them in their places, and worke according to the rule, and you shall perceaue that it will bee all one, sauinge that this is the shorter and easier way, as these three figures we shewe. Note.

a

58 2
24360 840

b

58 2
19894 686

c

58 2
13746 474

And this you see is bothe easier, and also the more certaine way to knowe the answere to this question.

Scholer. Truthe it is as you say: but sir, mee seemeth I might aske a farther question here, not onely howe many sheepe eche man should haue, but also what euery sheepe cost.

Mayster. That question doeth not onely belonge to this rule, but may also bee discussed by Diuision, especially if the questions number be one onely: as thus. Diuide the totall summe 58000 pence, by 2000 (other 58 by 2, omittinge the ciphers) and the quotient will be 29 pence, that is, 2 s, 5 d. howe be it, by this rule you may we it, and beste when the number of the question doth exceede 1: as if I shoulde aske this question, 2000 sheepe coste 58000 d, howe much did 20 cost? Thy shall I set my figures thus.

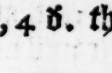
2000 58000
20

And winge after the rule, there will amounte

580

580 pence: that is 2 li, 6 s, 4 d. the price of one score.

But if you will vse that easy waye that I did teache you, you may chaunge the first and seconde numbre, thus.



Yet nowe one question more will I moue (that you may perceaue ỹ vse of all other like) and so make an ende.

Question of Cannons.

There is in a Cathedrall churche 20 Cannons, and 30 Vicars, those maye spende by yeare 2600 li. but euery Cannon muste haue to his parte 5 times so muche as euery Vicar hathe: howe muche is euerye mannes portion saye you?

Scholer. I praye you make the answere your selfe, so shall I perceaue beste the meanes to answere to suche other like.

Mayster. In this question you muste doe as in those that haue diuersitye of partes, for here is diuersitie of portions: Therefore shall you multiplye the numbre of the persons by theire difference of portion (as you did in the other by Time.) Then must you multiplye the 20 (whiche is the numbre of Cannons) by 5, (for that is the numbre of theire portion) so will it bee 100: Then 30, (that is the numbre of

Vicars

Vicars) by 1, (that is the numbre of their portion) and it will be 30: put those two summes togither, and they make 130: then say thus: If 130 spende 2600 li, what may 100 spende? The rule sheweth 2000 li.

Vicars. Agayne for Vicars: If 130 spende 2600 pounde, what may 30 spende? Answer, 600 li. as these figures shewe.

130 — 2600 || 130 — 2600
100 — 2000 || 30 — 600

But if euery Cannon shoulde haue so often times 4 li, as the Vicar should haue 3 li, then should I multiply 20 by 4, (that were 80) and 30 by 3 (that were 90) and then bothe were 170. Then shoulde the figures be set thus.

li.	s.	d.		li.	s.	d.
170	2600		170	2600		
80	1223	10,7	90	1376	9,5.	

But this sorte is to harde for you, by reason of the fractions, therefore I will let it reste to that place. And by this rule you see what the 20 Cannons maye spende, whiche summe if you diuide by 20, you shall see the Cannons portion: and so of ỹ Vicars, if you diuide their summe by 30, the quotient will declare euery Vicars portion.

The

The seconde Dialogue.

The accompting by Counters.

Mayster.

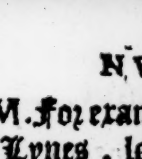
Nowe that you haue learned the common kindes of Arithmetike with the penne, you shall see the same arte in Counters: whiche feate withe not onelye serue for them that can not write and reade, but also for them that can we bothe, but haue not at sometimes theire penne or tables ready with them.

This arte is in two formes commonlye: The one by lines, and the other without lines. In that that hath lines, the lines we stande for the ordre of places: and in that that hathe no lines, there muste be set in their steede so many counters as shall neede, for eche line one, and they shall supply the steede of the lines.

Scholer. By examples I shoulde better perceaue your meaninge.

Mayster.

M. For example of the Lynes, lo heere you see vj. lines, whiche stande for vj. places, so that the nethermoste standeth for the first place, and the next aboue it for the second, and so vpwarde, till you come to the highest, whiche is the sixt line, and standeth for the sixte place.



Nowe what is the value of euery place or line, you maye perceaue by the figures whiche I haue set on them, whiche is according as you learned before that in Numeration of figures by the pen: for the firste place is the place of vnits or ones, and euery counter set in that line betokeneth but one: and the seconde line is the place of 10, for euery counter there standeth for 10, The thirde line the place of hundreds, the fourth of thousands: and so forthe.

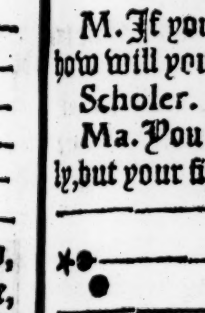
Scholer. Sir, I we perceaue that the same order is heere of lines, as was in the other figures by places, so that you shall not neede longer to stande about Numeration, excepte there be any other difference.

Mayster.

M. If you we vnderſtande it, then how will you ſet 1543?

Scholer. Thus as I ſuppoſe.

Ma. You haue ſet the places truly, but your figures bee not meete for this vſe: for the meeteſt figure in this behalfe is the figure of a counter, round, as you ſee here, where I haue expreſſed that ſame ſumme.

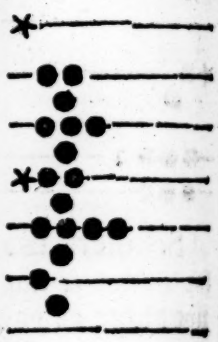


Scholer. So that you haue not one figure for 2, nor 3, nor 4, and ſo forth, but as manye digits as you haue, ſo many counters you ſet in the loweſt line: and for euery 10 you ſet one in the ſeconde line: and ſo of other. But I knowe not by what reaſon you ſet that one counter for 500 betweene two lines.

Mayſter. You ſhall remember this, that whenſoeuer you neede to ſet downe 5, 50, or 500, or 5000, or ſo forth any number whoſe numerator is 5, you ſhall ſet one counter for it in the next ſpace aboue the line that it hath his denomination of: as in this example of that 500, bicauſe the numerator is 5, it muſte be ſet in a voide ſpace: and becauſe the denominator is hundred, I knowe that his place is the

the voide ſpace next aboue hundredes, that is to ſay, aboue the third line.

And farther you ſhall marke, that in all worcking by this ſorte, if you ſhall ſet downe any ſumme betweene 4 and 10, for the firſt part of that numbre, you ſhal ſet downe 5, and then ſo many counters more, as there reſt nũbres aboue 5. And this is trewe bothe of digitts and articles. And for example I will ſet downe this ſume 287965, which ſumme if you marke well, you neede none other examples for to learne the Numeration of this forme.

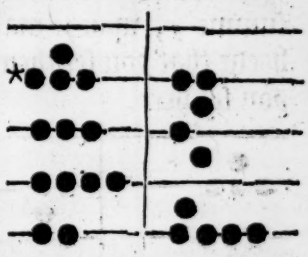


But this ſhall you marke, that as you did in the other kindes of Arithmetike, ſet a pricke in the places of Thouſands, in this worke you ſhall ſet a Starre, as you ſee before.

Scholer. Then I perceaue Numeration: but I pray you, how ſhall I doe in this arte to adde two ſummes or more togither?

Mayster.

The easiest way in this arte, is to adde but two summes at ones togither: howbeit, you may adde more, as I will tell you anone. Therefore when you will adde two summes, you shall first set downe one of them, it forceth not which, and then by it drawe a line crosse the other lines. And afterwarde set downe the other summe, so that that line may be betwene them: as if you woulde adde 2659 to 8342, you muste sette youre sūmes as you see here.

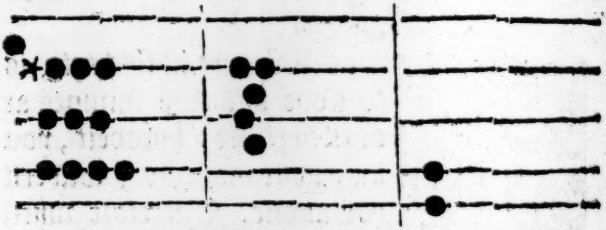


Addition of two summes.

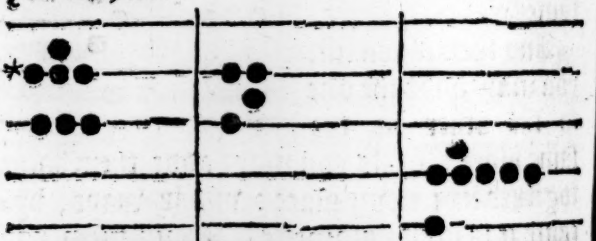
And then if you list, you may adde the one to the other in the same place: or els you maye adde them bothe togither in a newe place: which waye, bycause it is moste plaineste, I will shew you firste.

Therefore will I beginne at the vnits which in the firste summe is but 2, and in the seconde summe 9, that maketh 11. Those doe I take

vp, and for them I set 11 in the newe roome, thus.

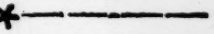


Then doe I take vp all the articles vnder a hundred, whiche in the firste summe are 40, and in the seconde summe 50, that maketh 90: or you may say better, that in the first summe there are 4 articles of 10, and in the seconde summe 5, whiche maketh 9, but then take heede that you set them in their right lines, as you see here.

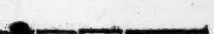


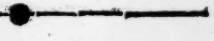
Where I haue taken away 40 from the firste summe, and 50 from the seconde, and in their steede I haue set 90 in the thirde roome, whiche I haue set plainely, that you might well

well perceaue it: howe be it, seinge that 90 with the 10 that was in the thirde rome already, wthe make 100, I might better for those 6 counters set 1 in the thirde line, thus.



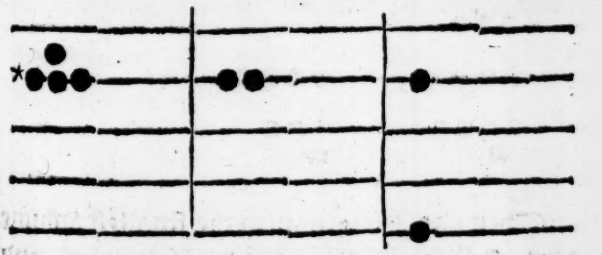
For it is all in one summe as you may see, but it is best neuer to set 5 counters in any line, for ỹ may bee done with one counter in a higher place.





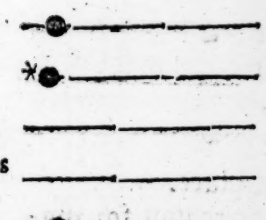
Scholer. I iudge that good reason, for many are vnneedefull where one will serue.

Mayster. Well, then will I adde forthe of hundredes: I finde 3 in the firste summe, and 6 in the seconde, whiche maketh 900, them we I take vp, and set in the thirde rome, where is one hundred already, to whiche I put 900, and it will bee 1000, therfore I set one counter in the fourth line for them all, as you see heere.



Then adde I the thousandes together, whiche

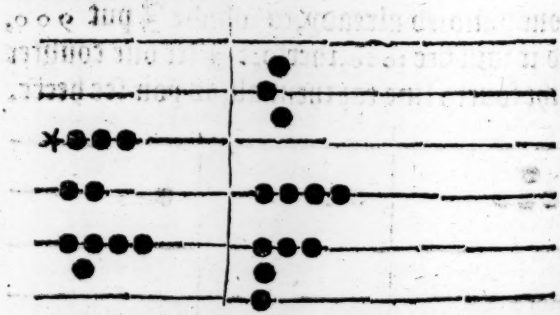
in the first summe are 8000, and in the seconde 2000, that maketh 10000: them we I take vp from those two places, and for them I set one counter in the fifte line, and then appeareth as you see to bee 11001, for so manye dothe amounte of the Addition of 8342 to 2659.



To adde 2 summes togither.

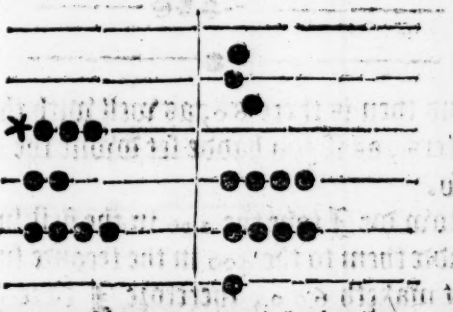
Scholer. Syr, this I doe perceaue: but howe shall I sette one summe to an other, not chaunginge them to a thirde place?

Mayster. Marke well howe I woe it: I wil adde togither 65436 and 3245, whiche firste I set wyne thus.



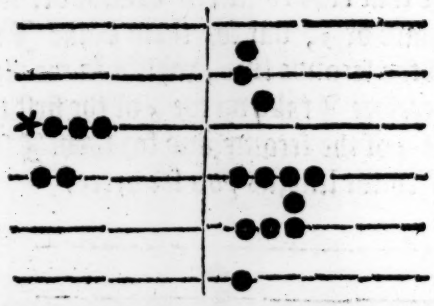
Then doe I begin with the smallest, whiche in the first summe is 5, that we I take vp, and

woulde put to the other 5 in the seconde summe, sauinge that two counters cannot bee set in a boyde place of 5, but for them bothe I muste set 1 in the seconde line, whiche is the place of 10: therefore I take vp the 5 of the first sume, and the 5 of the seconde, and for them I sette 1 in the seconde line, as you see heere.



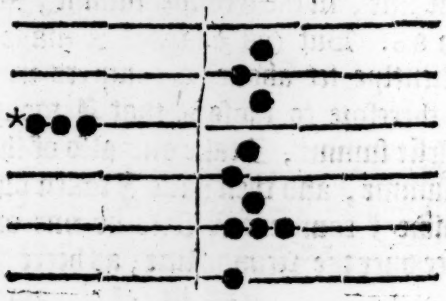
Then we I likewayes take vp the 4 counters of the first summe and second line, (whiche make 40) and adde them to the 4 counters of the same line, in the seconde summe, and it maketh 80. But (as I sayde) I maye not conuenientlye set aboue 4 counters in one line, therefore to those 4 that I tooke vp in the firste summe, I take one also of the seconde summe, and then haue I taken vp 50, for whiche 5 counters I sette downe one in the space ouer the seconde line, as heere dothe

appeare.



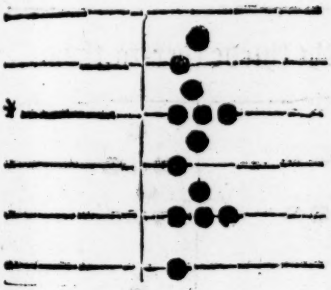
And then is there 80, as well with those 4 counters, as if you hadde set wone the other 4 also.

Now we I take the 200 in the firſt ſumme, and adde them to the 400 in the ſeconde ſumme, and it maketh 600, therefore I take vp the 2 counters in the firſte ſumme, and 3 of them in the ſeconde ſumme, and for them 5, I ſet 1 in the ſpace aboue, thus.



Then

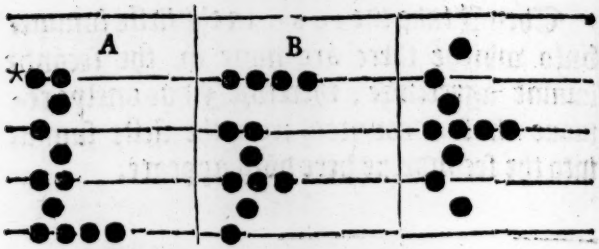
Then I take the 3000 in the firſte ſumme vnto whiche there are none in the ſeconde ſumme agreeinge, therefore I doe onelye remoue thoſe 3 counters from the firſte ſumme into the ſeconde, as here doth appeare.



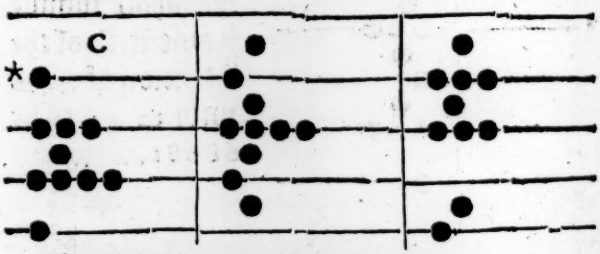
And ſo you ſee the whole ſumme ỳ amoũteth of the Addition of 65436 with 3245, to be 68681.

And if you haue marked theſe two examples well, you neede no farther inſtruction in Addition of 2 onelye ſummes: but if you haue more then two ſummes to adde, you may adde them thus.

Firſte adde two of them, and then adde the thirde and the fourthe, or more if there bee ſo many: as if I woulde adde 2679 with 4286 and 1391. Firſt I adde the two firſte ſummes, thus.



And then I adde the thirde thereto, thus.



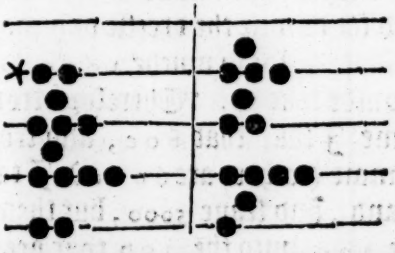
And so of more, if you haue them.

Scholer. Now I thinke beste that you passe forth to Subtraction, except there be any waies to examine this māner of Addition, then I thinke that were good to be knowen nexte.

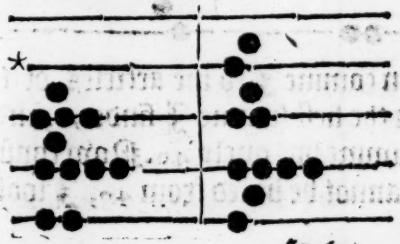
Mayster. There is the same pῖcke here that is in the other Addition by the penne, I meane Subtraction, for that onely is a sure waye: but considering that Subtraction must be first knowen, I will first teache you the arte of Subtraction, and that by thys example.

Subtraction

Woulde ſubtracte 2892 out of 8746. Theſe ſummes muſte I ſet wwne as I did in Addition: but here it is beſte to ſet the leſſer number firſte, thus.

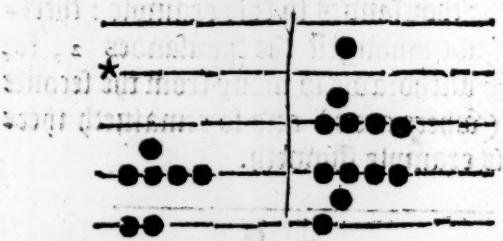


Then ſhall I beginne to ſubtract the greateſt numbers firſt (contrary to the vſe of the penne) that is the thouſandes in this example: therefore I finde amongeſt the thouſandes 2, for whiche I withdrawe ſo many from the ſeconde ſumme (where are 8) and ſo remaineth there 6, as this example ſhoweth.



S.v. Than

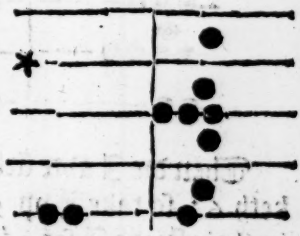
Than we I likewaies with the hundzeds, of whiche in the firſte ſumme I finde 8, and in the ſeconde ſumme but 7, out of whiche I can not take 8, therefore this muſte I doe: I muſte looke howe muche my ſumme differeth from 10, whiche I finde heere to bee 2, then muſt I bate for my ſumme of 800, one thouſande, and ſet downe the exceſſe of hundzedes, that is to ſaye, 2, for ſo muche 1000 is more then I ſhoulde take vp. Therefore from the firſte ſumme I take that 800, and from the ſeconde ſumme (where are 6000) I take vp one thouſand, and leaue 5000, but then ſet I downe the 200, vnto the 700 that are there already, and make them 900, thus.



Then comme I to the articles of tennes, where in the firſt ſumme I finde 90, and in the ſecond ſumme but onely 40. Now conſidering that 90 cannot be bated from 40, I looke how

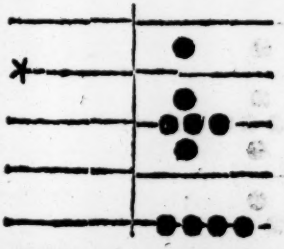
muche

muche that 90 wth differ from the next ſumme aboue it, that is 100, or els (whiche is all to one effecte) I loke howe muche 9 wthe differ from 10, and I finde it to bee 1, then in the ſteede of that 90, I we take from the ſeconde ſumme 100: but conſideringe that it is 10 to muche, I ſet wwne 1 in the nexte line beneth for it, as you ſee heere.



Sauing that heere I haue ſet one counter in the ſpace, in ſteede of 5, in the next line.

And thus haue I ſubtracted all ſaue two, whiche I muſt bate from the 6 in the ſeconde ſume, and there wil remaine 4, thus.



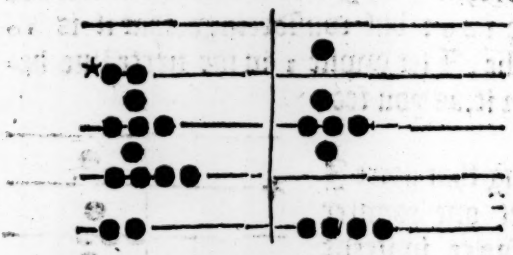
So y̆ if I ſubtract 2892 from 8746, the remayner will bee 5854.

And that this is trulye wroughte, you may proue by Addition: for if you adde to this remayner the ſame ſumme that you did ſubtracte, th an will the former ſumme 8746, a-

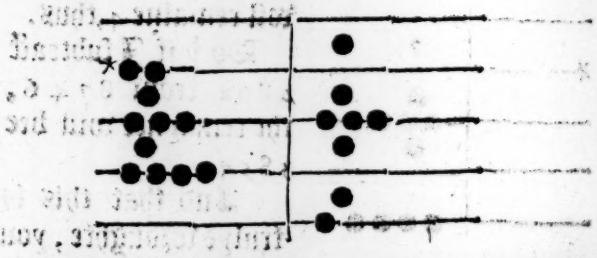
mount

mounte agayne.

A proofe. Scholer. That will I proue: and first I set the summe that was subtracted, whiche was 2892, and then the remayner 5854, thus.

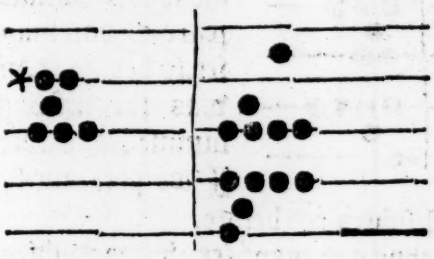


Then we I adde first the 2 to 4, whiche maketh 6: so take I vp 5 of those counters, and in their steede I set 1 in the space, and 1 in the lowest line, as here appeareth.

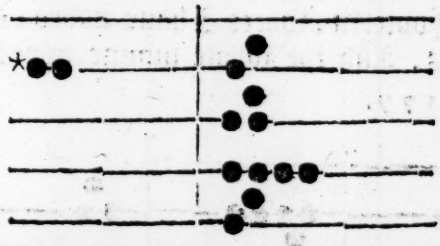


Then w I adde the 9 next aboue to ỹ 5, and it maketh 14, therefore I take vp those 6 counters, and for them I set 1, to the hun-

hundredes in the thirde line, and 4 in the seconde line, thus.

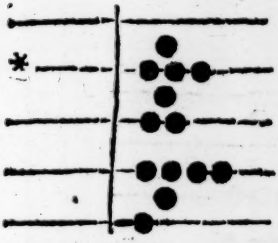


Then we I come to ỹ hundredes, of whiche I finde 8 in the firste summe, and 9 in the seconde, that maketh 1700: therefore I take vp those 9 counters, and in theire steede I set 1 in the fourth line, and 1 in the space nexte beneath, and 2 in the third line, as you see here.



Then is there lefte in the firste summe but onely 2000, whiche I shall take vp frō thence, and set in the same line in the seconde summe,

to



to the one that is there already: and then will the whole summe appeare as you may well see, to bee 8746, whiche was the firste grosse summe, and therefore I we perceaue that I had wel subtracted before.

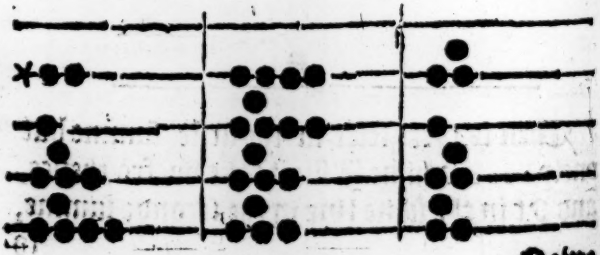
And thus you maye see, howe Subtraction may be tried by Addition.

Scholer. I perceaue the same order here with counters, that I learned before in figures.

Mayster. Then let me see how can you trie Addition by Subtraction.

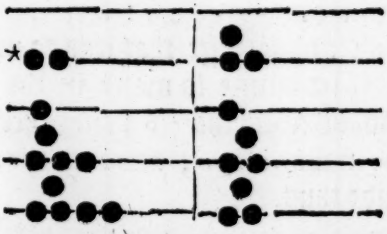
Scholer. First I will set forth this example of Addition, where I haue added 2189, to 4988. And the whole summe appeareth to be 7177.

A proofe of Addition, by Subtraction.

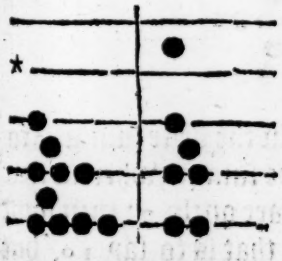


Nowe

Now to trie whether that ſumme bee well added or no, I will ſubtracte one of the firſte two ſummes from the thirde, and if I haue well done, the Remainer will be like that other ſummme, as for example. I will ſubtracte the firſte ſumme from the third, whiche I ſet thus, in their order.

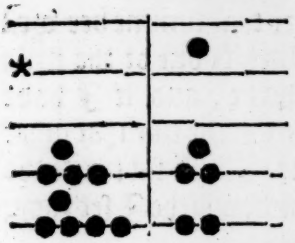


Then we I ſubtract 2000 of the firſt ſumme from the ſeconde ſumme, and then remaineth there 5000, thus.

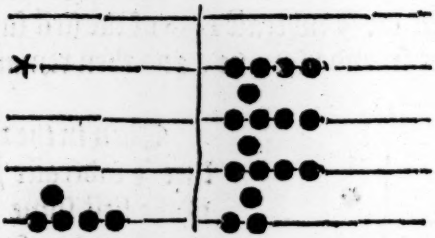


Then in the thirde line I ſubtracte y 100 of the firſt ſume from the ſeconde ſumme, where is onely 100 alſo: and then in the thirde line reſteth nothing, as you may ſee in the page followinge.

Then



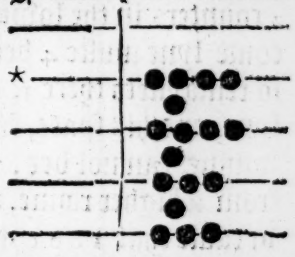
Then in the seconde line with his space ouer him, I finde 80, whiche I shoulde subtracte from the other summe, then seeinge there are but onelye 70, I must take it out of some higher summe, whiche is here onely 5000: therefore I take vp 5000: and seeinge that it is to muche by 4920, I set wwne so many in the seconde roume, whiche with the 70 being there alreadye, doe make 4990, and then the summes doth stande thus.



Yet remaineth there in the firste summe 9 to be bated from the seconde summe, wherein that place of vnits doth appeare onely 7: then must I bate a higher summe, that is to say 10, but seing that 10 is more then 9 (whiche I shoulde

abate

abate) by 1, therfore shall I take vp one counter from the seconde line, and set downe ỹ same in the firste or lowest line, as you see here.

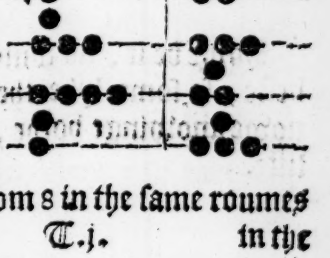


And so haue I ended this woorke, and the summe appereth to be the same whiche was the seconde summe of myne Addition, and therefore I perceaue I haue well done.

Mayster. To stande longer about this, it is but folly, except that this you maye also vnderstand, that many we beginne to subtract with counters, not at the highest summe as I haue taught you, but at the nethermost, as they we vse to adde: & when the sume to be abated in any line appeareth greater then the other, then we they borowe one of the nexte higher rowme, as for example.

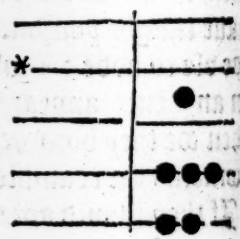
An other way of Addition

If they should abate 1846 from 2378, they sette the summes thus.



Firste they take 6, whiche is in the lower lyne, and his space, from 8 in the same roumes

in the seconde summe, and yet there remayneth 2 counters in the lowest lyne. Then in the seconde lyne muste 4 bee sutracted from 7, and so remayneth there 3. Then 800 in the thirde lyne, and his space, from 300 of the seconde summe, can not bee, therefore we they bate it from a higher rome, that is from 1000: and bycause that 1000 is to muche by 200, therefore must I set downe 200 in the thirde lyne, after I haue taken vp 1000 from the fourthe lyne. Then is there yet 1000 in the fourthe lyne of the firste summe, whiche if I withdrawe from the seconde summe, then dothe all the figures stande in order, thus.



So that (as you see) it differeth not greately whether you beginne Subtraction at the higher lynes, or at the lower.

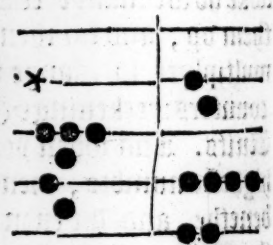
Howbe it, as some men like the one way beste, so some lyke the other: therefore you nowe knowinge bothe, may vse whiche you liste.

Multi-

But nowe touchinge Multiplication: you shall set youre numbres in two rowmes (as you did in those two other kindes) but so that the multiplyer be set in the firste roume, then shall you begin with the highest numbres of the seconde rowme, and multiply them first, after this sorte. Take that ouermoste line in youre firste worke, as if it were the lowest lyne, settinge on it some moueable marke (as you liste) and loke howe manye counters bee in him, take them vp, and for them sette downe the whole multiplyer so manye times as you toke vppe counters: reckeninge (I saye) that line for the Units. And when you haue so done with the highest numbre, then come to the nexte lyne benethe, and we euen so with it, and so with the nexte, till you haue done all. And if there bee anye numbre in a space, then for it shall you take the multiplyer 5 tymes: and then muste you recken that lyne for the Units, whiche is nexte benethe that space. Or else after a shorter waye, you shall take onelye halfe the multiplyer, but then shall you take the

lyne nexte aboue that space for the line of vnits. But in suche worʒkinge, if by chaunce youre multiplier bee an odde numbʒe, so that you can not take the halfe of it iustelye, then muste you take the greater halfe, & sette downe that, as if that it were the iuste halfe: and farther you shall set one Counter in the space beneath that line, whiche you recken for the line of vnits, or elſe onelye remoue forwarde the same that is to be multiplyed.

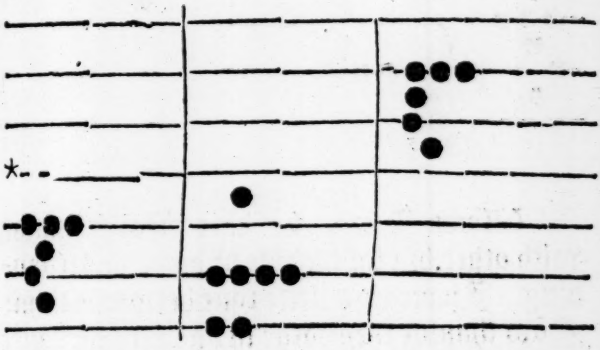
Scholer. If you set forth an example hereto, I thinke I shall perceaue you.



Mayster. Take this exaumple: I woulde multiply 1542 by 365, therefore I set the numbʒes thus.

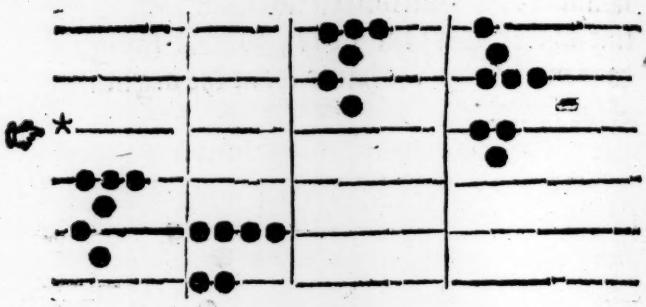
Then firste I beginne at the 1000 in the higheste roume, as if it were the firste place, and I take it vp, settinge downe for it so often (that is ones) the multiplyer, whiche is 365, thus as you see heere: where, for the one counter taken vp from the fourthe line, I haue set downe other 6, whiche make the

ſumme of the multiplier, reckening that fourth line as if it were the firſte, whiche thinge I haue marked by the hande ſet at the beginning of the ſame.

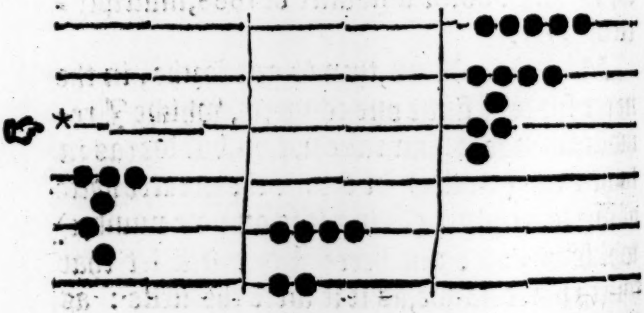


Scholer. I perceaue this well, for in deede this ſumme ỹ you haue ſet wwne, is 365000: for ſo muche dothe amounte of 1000, multiplyed by 365.

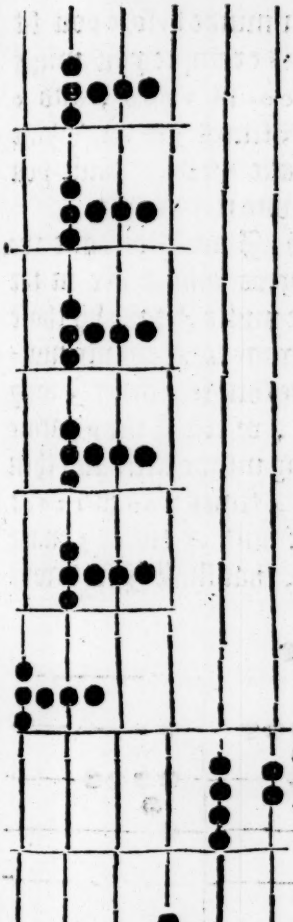
Mayſter. Well, then to goe forthe, in the nexte ſpace I finde one counter, whiche I remoue forwarde, but take not vp, but we (as in ſuche caſe I muſte) ſet wwne the greater halfe of my multiplyer (ſeeing it is an odde numbre) whiche is 182, and heere I doe ſtill let that fourth place ſtande, as if it were the firſte: as in the page following you ſhall ſee.



Where I haue set this Multiplication with other, but for the ease of your vnderstanding, I haue set a little lyne betweene them: Now shoulde they bothe in one summe stande thus.



Howe

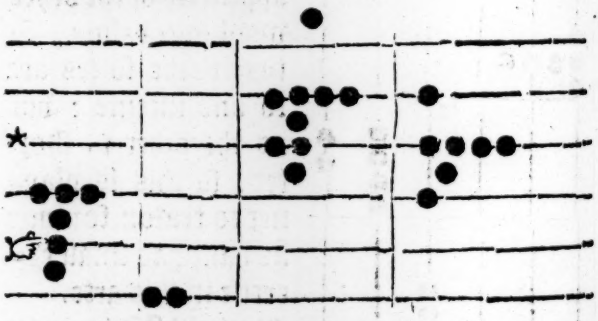


How be it, an other fourme to multiplye ſuche coũters in ſpace, is this: Firſte to remoue the finger to the lyne next beneth that ſpace, and thẽ to take vp that Counter, and to ſet downe the multiplyer fiue tymes: as here you ſee. An other forme of Multiplication.

Whiche ſummes if you we adde togither into one ſumme, you ſhall perceaue that yt wyll be the ſame that appeareth of the other woꝛkinge befoꝛe, ſo that bothe ſoꝛtes are to one intente: but as the other is ſhoꝛter, ſo this is playner to reaſon, foꝛ ſuche as haue had ſmall exercyſe in this arte. Notwithſtãding you

maye adde them in your minde before you set them downe: as in this example you might haue sayde, 5 times 300 is 1500, and 5 times 60 is 300: Also 5 times 5 is 25, which all put together, doe make 1825, which you may at one time set downe if you liste.

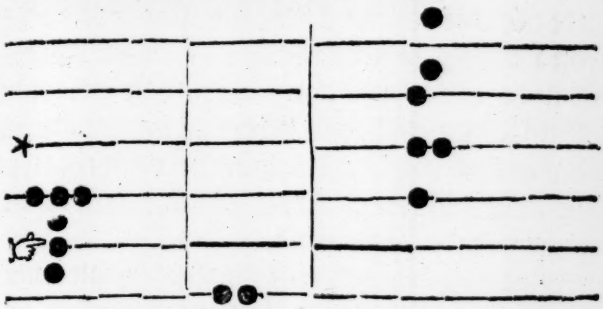
But now to go forth, I muste remoue the hande to the nexte counters which are in the seconde line, and there muste I take vp those 4 counters, settinge downe for them my multiplier 4 times: whiche thinge, other I may doe at 4 times seuerally, or else I may gather that whole summe in my minde first, and then set it downe: as to say, 4 times 300 is 1200: 4 times 60 are 240: and 4 times 5 make 20, that is in all 1460, that shall I set downe also, as here you see.



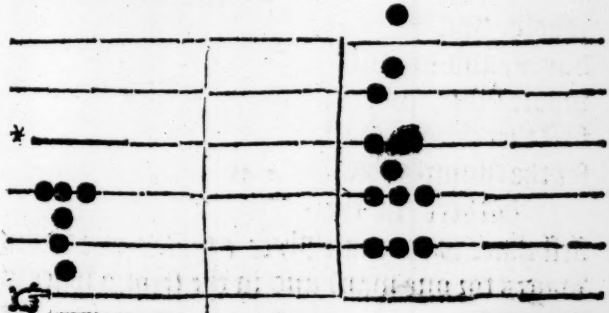
Which if I ioyne in one sum with the for-

mer

mer numbers, it will appeare thus.



Then to ende this Multiplication, I remoue the finger to the lowest line, where are onelye 2, them we I take vppe, and in their steede we I set downe twise 365, that is 730, for which I set one in the space aboue the third line for 500, and 2 more in the third line with that one that is there all ready, and the rest in their order, and so haue I ended the whole summe, thus.



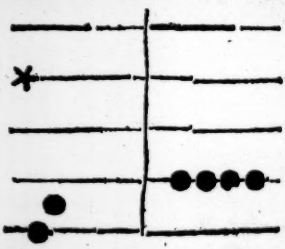
Whereby you see, that 1542 (which is the number of yeares sithe Christes incarnation) beynge multiplyed by 365 (which is the number of dayes in one yeare) doth amounte vnto 562830, which declareth the number of dayes sithe Christes incarnation vnto the ende of 1542 yeares, (beside 385 dayes and 12 houres for leape yeares.)

The sum of the dayes sith Christes incarnation.

Scholer. Now will I proue by an other example, as this: 40 labourers (after 6ð the day for ech man) haue wrought 28 dayes: I woulde knowe what their wages dothe amount vnto.

A questiō of wages.

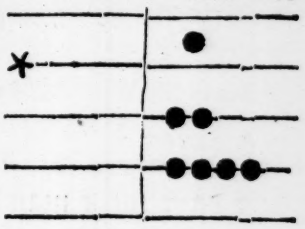
In thys case muste I woorke doubelye: firste I muste multiplye the number of the labourers by the wages of a manne for one day, so will the charge of one daye amounte. Then secondarily shall I multiply the charge of one daye by the whole number of dayes, and so will ẏ whole sume appeare: firste therefore I shall set the summes thus.



Where in the firste place is the multiplyer (that is one dayes wages for one man) and in the second space is set

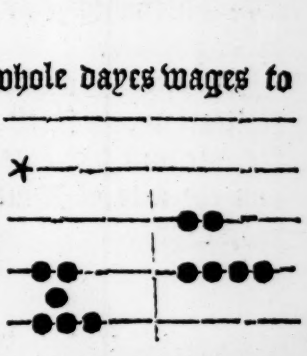
ſet ẏ numbꝛe of ẏ woꝛkemen to be multiplied.

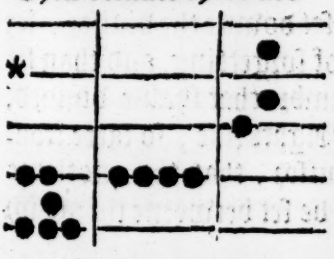
Than ſaye I: 6 times 4 (reckeninge that ſeconde line as the lyne of Units) maketh 24, foꝛ whiche ſumme I ſhoulde ſet 2 counters in the thirde line, and 4 in the ſeconde, theꝛefoꝛe doe I ſet 2 in the thirde line, and let the 4 ſtande ſtill in the ſeconde lyne, thus.



So appeareth the whole dayes wages to be 240 ð, that is 20 ₤.

Then doe I multiplye agayne the ſame ſumme by the numbꝛe of dayes, and firſte I ſet the numbꝛes thus. Then bicauſe there are

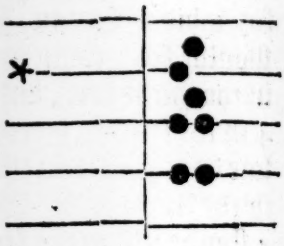




counters in diuers lines, I ſhall begin with the highelte, & take them vp, ſetting foꝛ them the multiplier ſo many times as I toke vp counters, ẏ is twiſe, then will the ſũme ſtande thus.

Then

Then come I to the seconde line, and take vp those 4 counters, setting for them the multiplyer fower times, so will the whole summe appeare thus.



So is the whole wages of 40 workemen for 28 dayes (after 6 d. ech daye for a man) 6720 d. that is 560 s. or 28 pounde.

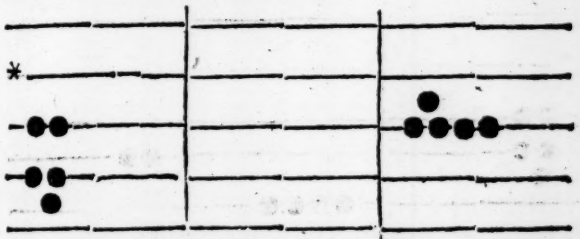
Mayster. Nowe if you would proue multiplication, the surelte waye is by Diuision: therfore will I ouerpasse it, till I haue taught you the arte of Diuision, whiche you shall worke thus.

DIVISION.

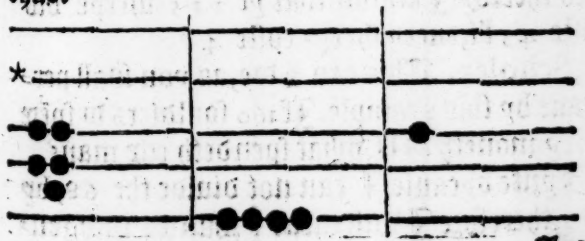
THyrde set downe the diuisor, for feare of forgetting, and than set the numbre that shalbe diuided, at the righte side, so farre from the diuisor, that the quotiente may be set betweene thē: as for example.

If 225 sheepe coste 45 li, what did enerye sheepe

sheepe coste? To knowe this, I shoulde diuide the whole summe, that is 45 li, by 225, but that can not bee, therefore muste I firste reduce that 45 li, into a lesser denomination, as into shillings: then I multiply 45 by 20, and it is 900: that summe shall I diuide by the numbre of sheepe, whiche is 225, these two numbres therefore I set thus.

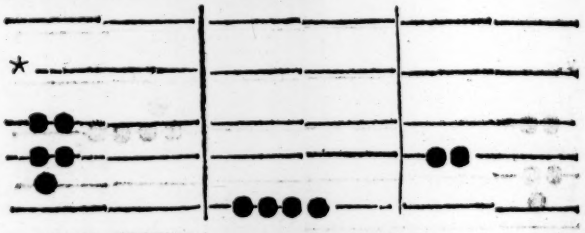


Then beginne I at the highest line of the diuidente, and seeke howe often I maye haue the diuisour therein, and that may I doe foure times: then saye I, foure times 2 are 8, whiche if I take from 9, there resteth but 1, thus.



And

And bycause I founde the diuisour 4 times in the diuidente, I haue sette, as you see, 4 in the middle roome, whiche is the place of the quotient: but now must I take the reste of the diuisour as often out of the remayner, therefore come I to the seconde lyne of the diuisor, sayinge: 2 foure tymes make 8, take 8 from 10, and there resteth 2, thus.



Then come I to the lowest numbre, whiche is 5, and multiply it 4 times, so is it 20, that take I from 20, & there remayneth nothinge, so that I see my quotient to bee 4, whiche are in valewe shillinges, for so was the diuident: and thereby I knowe that yf 225 sheepe did coste 45 li, euery sheepe coste 4 s.

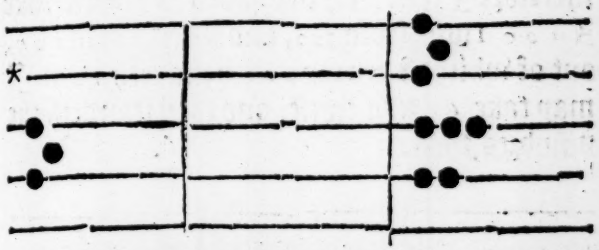
Example of wages.

Scholer. This can I doe, as you shall perceaue by this example. If 160 souldiers do spēde euery moneth 68 li, what spendeth eche man?

Fyrste bycause I can not diuide the 68, by 160, therefore I will turne ẏ poundes into pennies

nies by multiplication, so shall ther be 16329 d.

Nowe muste I diuide this summe by the number of souldiors, therefore I set them in order, thus.

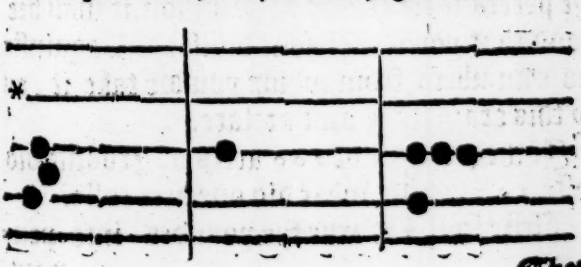


Than beginne I at the highest place of the diuident, seekinge my diuisour there, whiche I finde ones, therefore sette I 1 in the nether lyne.

Mayster. Not in the nether lyne of the whole summe, but in the nether lyne of that worke whiche is the thirde lyne.

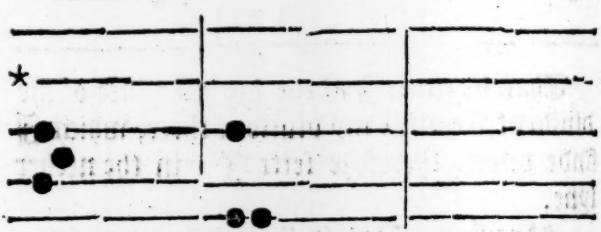
Scholer. So standeth it with reason.

Mayster. Then thus we they stande.



Then

Then seeke I againe the reste, howe often I maye finde my diuisour: and I see that in the 300 I might finde 100 three times, but than the 60 will not bee so often founde in 20, therefore I take 2 for my quotient: then take I 100 twise from 300, and there resteth 100, out of whiche with the 20 (ỳ maketh 120) I may take 60 also twise, and then standeth the numbres, thus.



Where I haue set the quotient 2 in the lowest line: So is euerye soldiers portion 102 d: that is 8 s, 6 d.

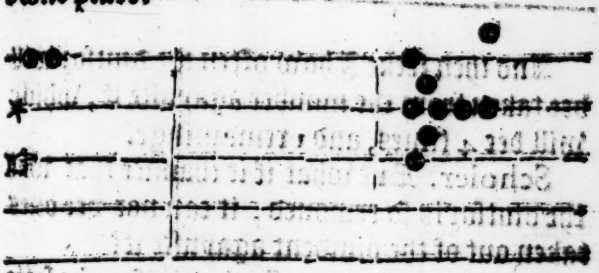
Mayster. But yet because you shall iustlye perceaue the reason of Diuision, it shall bee good that you we set your diuisor still againste those numbers from whiche you we take it, as by this example I will declare.

Example of purchase.

If the purchase of 200 acres of ground did coste 290 pounde, what did one acre coste?

Firste will I tourne the poundes into pennyes,

nies, so will there bee 69600 pence. Then in setting downe these numbers, I shall doe thus. Firste set the diuident on the right hande as it oughte, and then the diuisor on the left hande agaynst those numbers from which I intende to take him firste, as here you see, where I haue set the diuisor two lines higher thā is his owne place.



Scholer. This is like the order of Diuision by the penne.

Mayster. Ceuthe you say: and nowe must I set the quotient of this worke in the thirde line, for that is the line of Unites in respecte to the diuisour in this worke.

Then I seeke howe often the diuisor maye be founde in the diuidente, and that I finde 3 times, then set I 3 in the thirde line for the quotiente, and take awaye that 60000 from the diuident, and farther I doe set the diuisour

one line lower, as you see here.

And then seeke I how often the diuisor will bee taken from the number agaynste it, which will bee 4 times, and 1 remayninge.

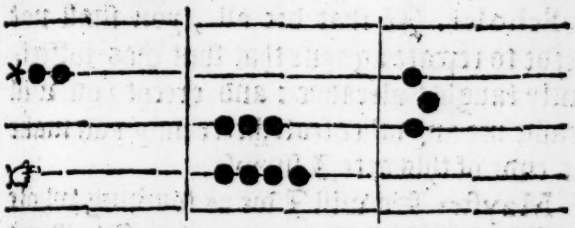
Scholer. But what if it chaunce that whe the diuisor is so remoued, it can not bee ones taken out of the diuident agaynste it?

Mayster. Then muste the diuisour be sette in an other lyne lower.

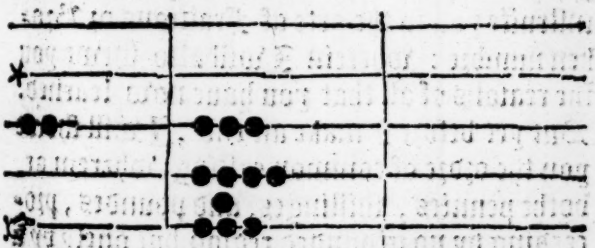
Scholer. So was it in Diuision by ỹ pen, and therefore was there à ciphre set in the quotient: but how shall that be noted here?

Mayst. Here needeth no token, for the lines we represent the places: onely looke that you set your quotient in that place, which standeth for vnits in respect of the diuisour. But to we to returne to the example. I finde the diuisor 4 tymes in the diuident, and 1 remayning, for 4 tymes 2 makes, which I take from 9 & there resteth

resteth 1, as this figure folowinge sheweth: and in the middle space for the quotient I see 4 in the seconde line, whiche is in this worke the place of vnits.



Then remoue I the diuisour to the next lower line, and seeke how ofte I may haue it in the diuident, whiche I maye doe heere 8 times iuste, and nothing remayne, as in this fourme.



Where you maye see, that the whole quotient is 248 d, that is 20 s, whereby I knowe that so muche cost the purchase of one acre.

Scholer. Now resteth the proues of Multiplication, and also of Diuision.

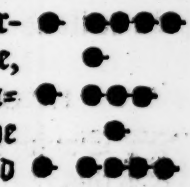
Mayster. Their beste proues are eche one by the other: for multiplication is proued by Diuision, and Diuision by multiplication, as in the worke by the pen you learned.

Scholer. If that bee all, you shall not neede to repeate agayne that that was sufficiently taughte alreadye: and except you will teache me any other feate, heere may you make an ende of this arte, I suppose.

Mayster. So will I we as touching whole numbre, and as for broken numbre, I will not trouble youre witte with it, till you haue practised this so well, that you be full perfecte, so that you neede not to doubt in any poynt that I haue taught you, and then maye I boldelye instructe you in the arte of Fractions or Broken numbre: wherein I will also shewe you the reasons of all that you haue now learned. But yet before I make an ende, I will shewe you the order of common casting, wherein are bothe pennies, shillinges, and poundes, proceeding by no grounded reason, but onely by a receaued forme, and that diuerslye of diuerse men: for Marchantes vse one fourme, and Auditours an other.

The reasons of all the former rules.

But firste for Marchauntes fourme, marke this example here, in whiche I haue expressed this summe 198 li, 19 s, 11 d, So that you may see, that the lowest line serueth for pennies, the nexte aboue for shillinges, the thirde for poundes, and the fourthe for scores of poundes.



And farther you may see, that the space betwene pence and shillinges maye receaue but one counter (as all other spaces likewaies we) and that one standeth in that place for 6 d.

Likewaies betweene the shillinges and the poundes, one counter standeth for 10 s.

And betweene the poundes and 20 li. one counter standeth for 10 poundes.

But beside those you maye see at the lefte side of shillinges, that one counter standeth alone, and betokeneth 5 s.

So againste the poundes, that one counter standeth for 5 li. And againste the 20 poundes, the one counter standeth for 5 score poundes, that is 100 li, so that euerye side counter

is 5 times so muche as one of them agaynste whiche he standeth.

AVDITOVRS ACCOMPT.

Auditours accompt.

NOw for the accompt of Auditours, take this example.

Where I haue expressed the same summe 198 li. 19 s, 11 d.

But here you see the pennies stande towards the righte hande, and the other increasinge orderly towarde the lefte hande.

Agayne you maye see that Auditours will make 2 lines (yea and moze) for pennies, shillings, and all other valewes, if their summes extende thereto. Also you see that they set one counter at the righte ende of ech rowe, whiche so set there, standeth for 5 of that roume: and on the left corner of the row it standeth for 10 of the same rowe.

But now if you would ade other subtracte after anye of bothe those sortes, if you marke the order of the other feate whiche I taughte you, you may eassly we the same here without muche

muche teachinge: for in Addition you muste firste set downe one summe, and to the same set the other orderly, and like manner if you haue many: but in Subtraction you muste set downe firste the greatest summe, and from it must you abate that other, euerye denomination from his due place.

Scholer. I doe not doubt but with a little practile I shall attaine these bothe: but howe shall I multiply and diuide after these formes?

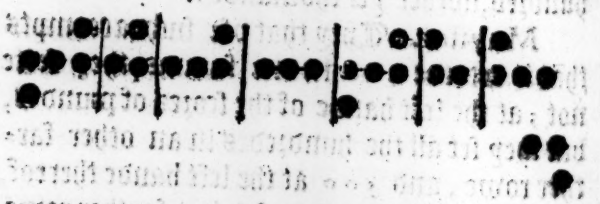
Mayster. You can not duely doe none of bothe by these sortes, therefore in suche case you muste resorte to your other artes.

Scholer. Sir, yet I see not by these sorts how to expresse hundredes, if they exceede one hundred, nother yet thousandes.

Mayster. They that vse suche accompts that it exceede 100 in one summe, they sette not; at the left hande of the scores of pounds, but they set all the hundredes in an other farther rowe, and 500 at the left hande thereof, and the thousandes they set in a farther rowe yet, and at the lefte side thereof they sette the 5000, and in the space ouer they sette the 10000, and in a higher row 20000, whiche all I haue expressed in this example, whiche is 97869 li, 12 s, 9 d, ob, q. Ninety seuen

thousand, eight hundred, three score and nyne poonde, twelue shillings and nine pence halfe-peny farthing, for I had not tolde you before, where, nother how you shoulde set downe farthinges, which (as you see here) must be set in à voyde space sidelinge benethe the pennies: for q̃ one counter, for ob, 2 counters: for ob, q̃, 3 counters: and more there cã not bee: for 4 farthinges doe make 1 d̃, which must bee in his dewe place.

And if you desire the same summe after Auditours manner: Lo here it is.



But in this thing you shall take this for sufficient, and ẏ rest you shall obserue as you maye see by the working of ede sorte: for the diuers wittes of men haue inuented diuerse and sundry wayes, almoste vnnumerable.

The

Vt one feate I shall teache you, whiche not onely for ỹ strangenes & secretneſſe is muche pleasaunt, but also for the good commoditie of it, right worthy to bee well marked. This feate hathe ben vsed aboue 2000 yeares at the leaſte, and yet was it neuer commonlye knowen, especially in Englishe it was neuer taught yet. This is the arte of numbringe on the hand, with diuers gestures of the fingers, expressing any summe conceiued in the minde. And firſte to beginne.

If you will expreſſe any ſumme vnder 100, you shall expreſſe it with your left hande: and from 100 vnto 1000, you shall expreſſe it with your right hande, as here orderly by this Table folowinge you may perceaue.

(.˙.)

Here followeth the Table of the arte of the hande.

1
2
3
4
5
6
7
8
9
10
20
30
40
50
60
70
80
90
100
200
300
400
500
600
700
800
900
1000
2000
3000
4000
5000
6000
7000
8000
9000

1 In whiche (as you may see) 1 is expressed by the little finger of the left hand, closely and harde crooked. 1

2 Is declared by like bowinge of the wedding finger (whiche is the next to the little finger) togither with the little finger. 2

3 Is signified by the middle finger, bowed in like manner with those other two. 3

4 Is declared by the bowing of the middle finger, and the ringe finger or wedding finger, with the other all stretched forth. 4

5 Is representeth by the middle finger onelye bowed. 5

And 6 by the weddinge finger onely crooked: and this you maye marke in these a certayne order. But nowe 7, 8, and 9, are expressed with the bowinge of the same fingers, as are 1, 2 and 3, but after an other fourme. 6

For 7 is declared by the bowing of the little finger as is 1, saue that for 1 the finger is clasped in, harde and rounde, but for to expresse 7, you shall bow the middle ioynte of the little finger onelye, and holde the other ioyntes straighte. 7

Scholer. If you will gene mee leaue to expresse it after my rude manner, thus I vnderstande your meaninge: that one is expressed by

by crokinge in the little finger, like the head of a bishops bagle: and 7 is declared by the same finger bowed like a gibbet.

Mayster. So I perceiue you vnderstande it.

8 Then to expresse 8, you shall bowe after the same manner bothe the little finger and the ring finger.

9 And if you bowe likewise with them the middle finger, then withe it betoken 9.

10 Now to expresse 10, you shall bowe youre forefinger rounde, and set the ende of it on the highest ioynte of the thombe.

20 And for to expresse 20, you must set youre fingers straight, and the end of your thombe to the particiō of the formost and middle finger.

30 30 Is represented by the ioyninge together of the heads of ẏ formoste finger & the thombe.

40 40 Is declared by settinge of the thombe crosseways on the formost finger.

50 50 Is signified by right stretchinge forthe of the fingers iointly, and applying of ẏ thombes ende to the partition of the middle finger, and the ringe finger or wedding finger.

60 60 Is fourmed by bendinge of the thombe croked, and crossing it with the forefinger,

70 70 Is expressed by the bowing of the formost finger, and setting the ende of the thombe betweene

betwene the 2 formoſt or highest ioyntes of it.

80 Is expreſſed by ſettinge of the formoſt finger croſſewayes on the thombe, ſo that 8o differeth thus from 40: that for 80, the forefinger is ſet croſſe wayes on the thombe, and for 40 the thombe is ſet croſſe ouer the forefinger. 80

90 Is ſignified by bendinge the forefinger, and ſetting the end of it in ỹ innermoſt ioynte of the thombe, that is euen at the fote of it. 90

And thus are all the numbres ended vnder 100.

Scholer. In deed theſe bee all the numbres from 1 to 10, & then all the tenthes within 100, but this teached mee not howe to expreſſe 11, 12, 13. &c. 21, 22, 23. &c. and ſuche like. 11, 12, 13, 21, 22, 25.

Mayſter. You can little vnderſtand, if you can not doe that without teachinge. What is 11? is it not 10 and 1? then expreſſe 10 as you were taughte, and 1 alſo, and that is 11: and for 12 expreſſe 10 and 2: for 23 ſet 20 and 3: and ſo for 68, you muſt make 60, and thereto 8: and ſo of all other ſortes.

But now if you would repreſent 100, other any numbre about it, you muſte doe that with the right hande, after this manner.

You muſte expreſſe 100 in the righte hande with the little finger, ſo bowed as you did expreſſe 1 in the lefte hande. 100

And

And as you expzelled 2 in the lefte hande, the same fashion in the righte hande dothe declare 200.

200

300 The fourme of 3 in the right hande standeth foz 300.

400 The fourme of 4, foz 400.

500 Likewayes the fourme of 5, foz 500.

600 The fourme of 6, foz 600. And to bee shozter: loke howe you did expzelle single vnities and tenthes in the lefte hande, so must you expzelle vnities and tenthes of hundzedes, in the right hande.

Scholer. I vnderstande you thus: that if I would repzesent 900, I muste so fourme the fingers of my righte hande to expzelle 9. And as in my lefte hande I expzelled 10, so in my right hande must I expzelle 1000.

900

1000

And so the fourme of euery tenth in the left hande, seruethe to expzelle like numbze of thousandes, so the fourme of 40 standeth foz 4000.

4000 The fourme of 80 foz 8000.

8000 And the fourme of 90 (whiche is the greatest) foz 9000, and abbue that I can not expzelle any numbze.

9000

Mayster. No, not with one finger, howbe it, with diuers fingers you maye expzelle 9999, and all at one time, and that lacketh but

1 of

1 of 10000. So that vnder ten thousande you may by your fingers expresse any summe. And this shall suffise for Numeration on the fingers. And as for Addition, Subtraction, Multiplication, and Division (whiche yet were neuer taught by any man as farre as I we know) I will instruct you after the treatise of Fractions: and now for this time farewell, and looke that you ceasse not to practise that you haue learned.

Scholer. Sir, with moste harty minde I thanke you, bothe for your good learning and also your good counsell, whiche (God willinge) I truste for to follow.

FINIS.

THE SECOND PART OF ARITHMETIKE, TOVchinge Fractions, briefly sette forthe.

SCHOLER.

All bee it I perceaue youre manifolde busines wherto occupye, or rather oppresse you, that you cannot as yet completelye ende that treatise of Fractions Arithmeticall, whiche you haue prepared, wherein not onelye sundrye woorkes of Geometry, Musike, and Astronomy bee largely set forthe, but also diuers conclusions & naturall woorkes, touchinge mixtures of metalls, and compositions of medicines, with other straunge examples, yet in the meane seaso I cannot stay my earnest desire, but importunely craue of you some briefe preparation towarde the vse of Fractions, whereby at the leaste I maye bee able to vnderstande the common woorkes of them, and the vulgare vse of those rules, whiche without them can not

not well be wrought.

Mayster. If my leyser were as greate as my will is good, you should not neede to vse any importunate crauinge for the attaininge of that thinge, whereby I maye bee perswaded that I shall anye waies profite the common wealth, or helpe the honest studies of any good members in the same: wherefore, while mine attendaunce will permitte mee to walke and talke, I am well willinge to helpe you as I maye.

Therefore, firste to beginne with explication of this name Fraction, what take you it to bee?

Scholer. Mary sir, I thinke à Fraction, (as I haue harde it often named) to bee à broken numbre, that is to saye, to bee no whole number, but àparte of a number. What à Fraction is.

Mayster. A Fraction in deede is à broken number, and so consequently the parte of an other number: but that must bee vnderstanded of such an other number, as can not be diuided into any other partes then Fractions: for although I may take the thirde parte of 60, or the fourth part of it, and so of other partes diuerslye, yet these partes bee not properly, nor ought not to be called Fractions, because they

may be expressed by whole nũbers: for ỳ thirde parte of it is 20: the fourthe part is 15: the twelfth part is 5, and so forth of other partes, whiche all be whole numbers.

What a fraction is properly.

Wherefore properlye a Fraction expresseth the partes or part onely of an vnite, that is to say, that the number whiche is the whole or entiere summe of any Fraction, may not be greater then one: and therefore it followeth, that no one fraction alone can be so great, ỳ it shall make 1, as by exaumples I will declare as soone as I haue taught you to know the forme how a fraction is expressed or represented in writing.

NVMERATION.

But first to begin with the expressinge of a Fraction, whiche is the numeration of it, you must vnderstand that a fraction is represented by 2 numbers, set one ouer the other, and a line drawen betwene thẽ, as thus, $\frac{1}{3}$ $\frac{3}{4}$ $\frac{2}{5}$, $\frac{10}{17}$, whiche foure fractions you muste pronounce thus: $\frac{1}{3}$, one third part: $\frac{3}{4}$, three quarter: $\frac{2}{5}$, two fifte partes: $\frac{10}{17}$, ten seuenteene partes.

Scholer.

Scholer. I vnderstande the forme of theire expression and pronunciation, but there meaninge or valuation seemeth more obscure: yet I thinke that by the two firste fractions I vnderstande the valuation of the two latter fractions, and so consequently of other.

Mayster. Ualew them then, that I maye perceaue your taking of them.

Scholer. $\frac{2}{5}$ betokeneth two fifte partes, that is to say, if onebe diuided into 5 partes, that fraction wthe expresse ij. of those fifthe partes: $\frac{10}{17}$ wthe signifie, that if one bee diuided into xvij. partes, I muste take x. of them. And this I gather of the ij. firste examples: for $\frac{1}{3}$, that is one third parte, wthe easily declare, that if any one thinge be diuided into iij. partes, I muste take but one of them: so $\frac{3}{4}$, that is iij. quarters, wthe declare that one beyng diuided into iiij. quarters, I muste take (for this fraction) iij, of those quarters.

If there be no more difficulty in theire Numeration, then I praye you goe forewarde to their Addition and Subtraction, and so to the other kindes of worke: for I vnderstande that the same kindes of workes bee in Fractions, that be in whole numbers.

Mayster. There are the same kinde of

workes in bothe, albeit the ordre of them is diuers, as I will anone declare: but yet more in numeration before wee leaue it. You muste vnderstande, that those two numbres whiche expresse à fraction, haue seuerall names. The ouermost whiche is aboue the line, is called the Numerator, and the other benethe the line, is called the Denominator.

Numerator and Denominator.

Scholer. And what is the reason of theire diuers names? For in mine opinion bothe bee Numerators, seynge both they doe expresse the numeration of the Fraction.

Mayster. You are deceyued: for one onelye (whiche is the ouermost) dothe expresse the numeration: and the Denominator dothe declare the numbre of partes into whiche the vnit is diuided, as in this example, when I say: Diuide a pounde weight of golde betwene iiij. men, so that the firste man shall haue $\frac{2}{15}$, the seconde $\frac{3}{15}$, the thirde $\frac{4}{15}$, and the fourthe $\frac{6}{15}$. Now do you perceaue ẏ by the Denominator (whiche is one in all foure fractions) it is intended, that the pounde waight shoulde bee diuided into so many partes, I meane 15, and by the iiij. seuerall numerators is limited the diuers portion that eche man shall haue, that is, that when the whole is parted into 15, the first man

man shall haue 2 of those 15 partes: the second man three of them: the thirde man 4: and the fourth man, 6. And so may you see the seuerall offices (as it were) of those two numbres, I meane of the Numerator and the Denominator.

And hereby you perceaue, that a man can haue no more partes of any thing then it was diuided into: nother yet aptly so many: so that it were vnaptly sayd: You shall haue $\frac{1}{15}$ $\frac{1}{15}$, that is 15. fifteene partes of anye thinge, seeinge it were better sayde: You shall haue the whole thinge.

Scholer. So withe it appeare reasonablye: for the labour is vayne, to diuide anye thinge, and then to apply the Diuision to no vse. And muche lesse reasonable were it to say, $\frac{16}{15}$: for if the whole be diuided into 15 partes onelye, it is not possible to take 16 of them, that is to say, more then altogither.

Mayster. This is trew, touchinge the proper and apt vse of the name of a fraction: yet improperly and after a vulgare acceptation (for easines in worke) bothe those formes bee called Fractions, because they bee written like fractions, although they be none in deede. for $\frac{16}{15}$, and generallye all suche other where the

Numerator & denominator bee equall, are not fractions: but ỳ whole thing, ẅ all his partes. And so $\frac{12}{12}$ is not to be called a fraction, but a mixt numbre, of a whole numbre and a fraction: for it is as muche, as $1\frac{4}{12}$, that is one whole one, and 4 twelue partes, as shall bee declared in Reduction. Therefore they we abuse the names, that call them fractions, where the Numerator is either equall or greater then the Denominator.

Scholer. But is there anye needefull cause why they should so abuse the name?

Mayster. There is cause why they shall sometimes, for easines in worke, write some numbres after that sorte, like fractions: but they needed not to call them fractions, but as they bee whole numbres, or mixte numbres (that is whole numbres with fractiõs) expressed like fractions.

Nowe muste you vnderstande, that as no fraction properly can bee greater then 1, so in smalenes vnder one the nature of fractions dothe extende infinitely: as the nature of hole numbres is to increase aboue one infinitelye, so that not onely one, maye bee diuided into infinite fractions or partes, but also euerye fraction may be diuided into infinite fractions

or

or partes, which commonly bee called Fractions of Fractions, and they bee expressed diuersely: As for example $\frac{3}{4}\ \frac{2}{3}\ \frac{1}{2}$, that is three quarters of two third partes, of one halfe part. Whereby is signified, that if one bee diuided into two halues, and the one halfe into three partes, and two of those three partes, be diuided ioyntly into foure quarters, this Fraction of Fraction doth represent three of those quarters. Fractions of fractions.

Scholer. I pray you let mee proue by an example in common money, whether I we rightly vnderstande you or no. One Crowne, which I take for an vnit, doth containe 60 pennies, therefore the halfe of it, is 30 pence: $\frac{2}{3}$ of that halfe is 20 pence, whereof $\frac{3}{4}$ is 15 pence, so than 15 pence is $\frac{3}{4}\ \frac{2}{3}\ \frac{1}{2}$ of a crowne. And so 5 pence, is $\frac{3}{4}\ \frac{2}{3}\ \frac{1}{2}$ of a shilling.

Mayster. You perceaue this well inough, but how happened that you founde no doubte in the forme of wryting these Fractions, seing the two latter fractions haue no lyne betwene theyr numbers, as the first hath?

Scholer. Because I had forgotten (as schollers oft tymes we) that that was tolde mee before: but I praye you, expresse the reason thereof.

Mayster. This forme is but voluntary, and therefore hath none other reason then the will of the diuiser, whiche forme many we folow. Some other we make lines betweene euery fraction, and adde wordes of distinction, after this sorte, $\frac{3}{4}$ of $\frac{2}{3}$, of $\frac{1}{2}$, whiche forme is good also,

Some other expresse them thus in slope forme, to distinct them from seuerall Fractions of one whole number: for if they were

$\frac{3}{4}$
$\frac{2}{3}$
$\frac{1}{2}$

☞ set in one right line, thus, $\frac{3}{4}\ \frac{2}{3}\ \frac{1}{2}$, then ought it to be pronounced, three quarters, and two third partes and an halfe, which maketh almoste two whole vnits, lackinge but one xij. parte. And so is it nothinge agreable with the other Fraction of fractions, wherefore it is a greate ouersight in certaine learned men, which doe expresse them so confusedly with suche seuerall fractions, that à man can not knowe the one from the other.

Therefore some men (as Stifelius) we expresse without à lyne numbers of proportion, being applyed to Addition or Subtraction: bicause they muste bee taken as two: where the line in fractions maketh them to bee taken for one: for of the Numeratour and Denomina-

tour

tour is made one number.

Scholer. Than I perceaue there be three seuerall varieties in fractions: First when one onely fraction is set for one number, as $\frac{4}{5}$, that is foure fifth parts. The seconde, is when there bee set two or more seuerall fractions of one number, as $\frac{4}{9}\ \frac{2}{5}$, that is iiij. ninthe partes, and two fifth partes. The thirde sort is Fractions of fractions, as $\frac{4}{9}\frac{2}{5}$, that is iiij. ninth partes of two fifth partes. Three seuerall varieties.

Mayster. You haue saide well if you will vnderstand well your owne wordes.

Scholer. If it shall please you, I will by an example in the partes of an olde Englishe Angell expresse my meaning.

Mayster. Let me heare.

Scholer. The olde Englishe Angell did containe vij, s. bj d. that is 90 d. Nowe $\frac{4}{5}$ of it, is 72 d. And of the same 90 d. if I take $\frac{4}{9}\frac{2}{5}$, that is foure ninthe partes, and two fifte partes, $\frac{4}{9}$ is 40, and $\frac{2}{5}$ is 36, whiche bothe make 76: but if I take $\frac{4}{9}\frac{2}{5}$, that is four ninthe partes of two fift partes, seing $\frac{2}{5}$ is but 36, then $\frac{4}{9}$ of 36 will yelde but 16: for $\frac{1}{9}$ of 36 is but 4, and that taken foure times, maketh 16.

Mayster. This is plainely expressed and

truelye, and hereby (I doubt not) but you doe perceaue, that as greate a difference as is betweene 16 and 76, so muche difference is betweene these two fractions $\frac{4}{5}\frac{2}{5}$, and $\frac{7}{5}\frac{2}{5}$.

And nowe that you vnderstande these varieties, I will proceede to the rest of the woorkes: firste admonishing you that there is an other order to bee followed in Fractions then there was in whole numbres, for in whole numbres this was the order: Numeration, Addition, Subtraction, Multiplication, Diuision, and Reduction, but in fractions (to followe the same aptnes in proceeding from the easiest woorkes to the harder) we must vse this order of the woorkes, Numeration, Multiplication, Diuision, Reduction, Addition, and Subtraction.

Theordre of workes in Fractions.

Scholer. That Multiplication and Diuision shoulde go together, and Subtraction to follow Addition, naturall order doeth perswade: but why Multiplication should be firste in order here nexte to Numeration, and Reduction in the middle, I desire to vnderstand the reason.

Mayster. As in the arte of whole numbres order would reasonably beginne with the easiest, and so go forewarde by degrees to the

hardest,

hardeſte, euen ſo reaſon teacheth in Fractions the like order. And conſideringe that Addition or Subtraction of Fractions can very ſeldome be wrought without Multiplication and Reduction: and contrary wayes, Multiplication and Reduction may be wrought without this forme of Addition or Subtraction, therefore was it orderly required, that Multiplication and Reduction ſhoulde goe before Addition and Subtraction. And the ſame reaſon ſerueth for the placinge of Multiplication before Reduction.

Scholer. Then, if Multiplication bee the eaſieſt, I pray you declare the forme of it, firſte by rule, and then by example.

Mayſter. Your requeſt is good.

MVLTIPLICATION.

Therfore when anye two Fractions be propoſed to be multiplyed together, the Numerator of the one muſt be multiplyed by the Numerator of the other: and the ſumme that amounteth therof, muſte be ſet for a newe numerator: likewayes the Denominator of the one muſt bee multiplyed by the denominator of

the

the other, and that that amounteth shall bee set for the common denominator: and this newe thirde Fraction expresseth the produce of the multiplication of the two first Fractions propoued, whereof take this example, $\frac{3}{5}$ multiplied by $\frac{5}{12}$, dothe make $\frac{15}{60}$.

Scholer. I perceaue then, that 3 beynge the Numeratour of the firste Fraction, is multiplyed by 5, beynge the numeratour of the seconde Fraction, whereof amounteth 15, the Numerator of the thirde fraction. And so like wayes, 5 beynge denominator of the firste Fraction, is multiplyed by 12 the Denominator of the seconde fraction, whereof amounteth 60. the newe denominator: so that I perceaue howe the worke is done, but I doe not perceaue howe $\frac{15}{60}$ is greater then $\frac{3}{5}$: For if I shall vse my former māner of examination by the partes of some coyne, I see that $\frac{3}{5}$ of a crowne, is 36 d, and $\frac{5}{12}$ of a crowne, is 25 d, whereof the one multiplyed by the other, dothe make 900 d, whiche is 15 crownes: but by youre multiplication there amounteth $\frac{15}{60}$, whiche is but 15 d, and that is muche lesser then any of bothe the first fractions.

Mayster. That differēce is betweene multiplication in whole numbers, and multiplication

cation in broken numbres, that in whole numbres the sume that amounteth, is greater then bothe the other whereof it came: but in fractions it is contrary wayes: for the summe that amounteth is lesser then anye of the other two fractions, whereof it came.

Scholer. I desire muche to vnderstande the reason thereof.

Mayster. Although I purposed to reserue the reasons of workes Arithmeticall for the perfect booke of Arithmetike, yet I will shewe you this, bicause of the straungenesse of the worke.

You see in whole numbres, that of two numbres beinge multiplyed together, is made the thirde number: whiche thirde numbre wthe beare the same proportion to the numbre multiplyed, that the multiplyer wth beare to an vnit. And so in Fractions, the thirde numbre whiche amounteth of multiplication, beareth the same proportion to eche of the twoe firste fractions, that the other of those two fractions wthe beare to an vnit.

Scholer. Sir, I vnderstande youre wordes thus: When 40 is multiplyed by 12, there bothe amounte 480, whiche 480 bothe contayne 40 so many times in it, as 12 wthe

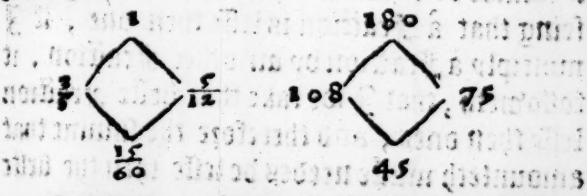
contayne

contayne 40 so manye times in it, as 12 wthe contayne vnits: that is to saye, twelue times. And so it appeareth, that 480 wthe contayne twelue so many times also, as 40 dothe contayne Units, that is 40 times. But nowe I see not how the thirde numbre in this example of Fractions can contayne any of the two former (as it happened in whole numbres) seyng it is lesser then eyther of them.

Mayster. No meruayle if you cannot see that thinge which is not possible to bee seene of any man, howe the thirde numbre in multiplication of Fractions should bee greater than any of the two former Fractions, but yet this maye you see (whiche I saye) that the thirde numbre in Fractions so multiplyed, dothe beare the same proporcion to anye of the two former Fractions, that ỳ other of those ij Fractions wthe beare to an Unit, as in youre example $\frac{3}{5}$ beyng multiplyed by $\frac{5}{12}$, dothe make $\frac{15}{60}$. Now saye I, that $\frac{15}{60}$ wthe beare the same proporcion to $\frac{3}{5}$, that $\frac{5}{12}$ dothe beare to an vnitte, as you may in youre owne forme of examination by coyne trye it. For in an olde Angell are 180 halfe pence, whiche I set for the intire vnit, whose partes (according to ỳ Fractions aforesayde) are these: for $\frac{15}{60}$ sette 45 ob.

for

for $\frac{12}{5}$ take 108 ob. & for $\frac{5}{12}$ put 75 ob. Nowe dothe 45 beare the same proportion to 108 that 75 dothe beare to 180: for 45 is $\frac{5}{12}$ of 108, and so is 75 also $\frac{5}{12}$ of 180. And for easier applying of edr comparison, consider this forme of settinge all these numbers before your eyes.



But these reasons may bee better reserued till an other time, when the knowledge of proportions in dewe order shall be taught. Yet in the meane season I will shewe you howe it commeth to passe, that in Fractions the thirde summe muste nedes bee lesser then any of the other two.

Consider thus, that when a Fraction is proponed, as in the former example $\frac{3}{5}$, if it bee multiplyed by more the one, it will make more the one entiere nũbre. As if I multiply this $\frac{3}{5}$ by 5, that is to say, if I take it fiue tymes, it will make three entiere vnits: example in a crowne, $\frac{3}{5}$ of it maketh 3 5, whiche if I take

fyue

fiue times, it will amounte to 15 shillinges, that is three entiere crownes: so if I take the same $\frac{3}{5}$ but twise, it will yelde 6 s, that is one entiere crowne and $\frac{1}{5}$. Nowe if I take it but ones, it cannot bee more than it was before, that is 3 s. And if I take it lesse thē once, it cannot be so muche as it was before. Then seing that à Fraction is lesse then one, if I multiply á Fraction by an other Fraction, it followeth, that I we take that firste Fraction lesse then ones; and therefore the summe that amounteth muste needes be lesse then the firste Fraction.

Scholer. Sir, I thanke you muche for this reason: And I truste, I we perceaue the thinge, as by example of this same fraction $\frac{3}{5}$ I will expresse. If I take $\frac{3}{5}$ of a crowne ones, that is to say, if I multiply $\frac{3}{5}$ by 1, it will bee as it was before, but 3 s. so if I we multiply it by $\frac{1}{2}$, that is, if I take it but halfe one time, then will it be but halfe so muche: likewaies if I multiply it by $\frac{1}{3}$, that is, if I take but the thirde parte of ones, it will yelde but 12 pence, that is the thirde parte of the firste Fraction.

And so to make an ende. If I take it but the twelfthe part of ones, that is, if I we multiply

tiply it by $\frac{1}{12}$, it will yelde but the twelfthe parte of the firste Fraction, whiche is but 3 pence. And so it followeth, that if $\frac{1}{12}$ make 3 pence, than $\frac{5}{12}$ muste needes make fiue tymes so muche, that is 15 pence. whiche was the summe that hath giuen the occasion of all this doubte.

Mayster. Then I perceaue you haue sufficient vnderstanding in this sort of Multiplication for this tyme, wherefore I will omitte that I might saye more of Multiplication, till wee come to Reduction, and will passe to the other workes, and firste to Dyuision, whose place foloweth Multiplication, bothe by naturall order, and also in easinesse of worke.

DIVISION.

When so euer two Fractions be proponed, that the one should be diuided by ẏ other, I must set downe first the Fraction that shall be diuided (whiche is called the Diuidend) & then after it the other, whiche is the Diuisor. Then

shall I multiply the numerator of the diuided by the denominator of the diuisour, and that whiche amounteth, I must put for a newe numerator. Agayne, I shall multiply the denominator of the diuidend by the numerator of the diuisor, and the numbre that amounteth thereof, I must put for the new denominator. And this thirde fraction is the quotient of the saide diuision.

Scholer. This seemeth easie in forme, as by example, thus: If I would diuide $\frac{4}{5}$ by $\frac{2}{3}$ firste I must multiply 5 (being the numerator of the diuidend) by 6; whiche is the denominator of the diuisor, and thereof riseth 30: then I multiply 8 (being the denominatour of the diuidend) by 2, beinge numerator in the diuisor, and so riseth 16, the whiche I muste make in a thirde fraction, thus, $\frac{16}{30}$.

Mayster. Yee seemeth you are quicker in vnderstanding now, then you were when I taughte you the arte of whole numbres: but that is no maruaile, for the more knowledge that any man getteth, the readyer shall he fynde his wit, and quicker in vnderstanding: but yet of ij. thinges I will admonishe you, whiche you myght haue obserued here for ease of woorke and lyghtnes of vnderstanding the nature of

the

the Quotient.

When soeuer you we diuide one fraction by an other, other they be bothe equal together, other els ỹ one is greater then the other: if they be equall, their quotient shall be suche, that the numerator and the Denominator of it shal be equall also. And if the two firste fractions bee vnequall, their quotient shall declare the same by the vnequalitie of the numeratour and denominator, as in these exaumples folowinge shall appeare.

First of equall Fractions: $\frac{4}{5}$ and $\frac{12}{15}$ bee equall together: and if the one bee diuided by the other, the quotient will bee $\frac{100}{100}$, as you may perceaue by that rule afore sayde.

Nowe in vnequall fractions, as $\frac{4}{5}$ and $\frac{3}{10}$ the quotient will be $\frac{40}{15}$: where the numerator is greater then the denominator.

Scholer. I see it is so, but I see not the reason why it should be so.

Mayster. The reason is this. When any fraction is diuided by an other, the quotiente declareth what proportion the diuidende beareth to the diuisor. So $\frac{1}{2}$ diuided by $\frac{1}{4}$, maketh 2, whiche musse bee founded, not two, but twise: declaringe that $\frac{1}{4}$, is contayned twise in $\frac{1}{2}$.

Note how to knowe the proportion betweene two nũbers.

And note this, that the Numerator in the Quotient, representeth the Diuidend, and the Denominator representeth the Diuisor. And this is alwayes true, whether the greater fraction be diuided by the lesser, or the lesser by the greater. But this proportion will not bee exactly knowen, till you haue learned the arte of Proportions: notwithstandinge somewhat of it will I declare in the nexte rule of Reduction. But nowe for the easy remembraunce of the Quotient in Diuision, assone as you haue set wthe your ij. fractions, the one againste the other, then make a straight line for ỳ quotient: and assone as you haue multiplyed the Numerator of the diuidend, by the Denominator of the diuisor, set the numbre that amounteth, ouer the sayde line, and then multiply the other two numbres, and set theire totall vnder the same line.

Scholer. I perceaue you would not haue me trust to memory till I were better experte, leaste oftentimes I happen by misse remembraunce to bee abused. This example I take for that declaration:

If I would diuide $\frac{2}{3}$ by $\frac{3}{4}$, I must set the numbers one againste the other, (as here wthe appeare) & then make

$$\frac{2}{3} : \frac{3}{4}$$

an

an other line for the Quotient in some good distaunce, where I maye sette the numbres of the Quotient, as sone as any of them is multiplyed: So then as sone as I haue multiplied 2 by 4, whiche maketh 8, I shall set that 8 ouer that line, thus, 8
and then multiply 3 by 3, whiche yeldeth 9: and that 9 must I set vnder the same line, and then will the whole Quotient appeare thus $\frac{8}{9}$. Wherby appeareth (as I remembre your wordes) that $\frac{2}{3}$ is in proportion to $\frac{3}{4}$, as 8 is to 9: but howe may I perceaue that? Note.

Mayster. Although you shall better perceaue it by the rule of Reduction, yet this example may bee declared in common coyne, as in a common shillinge of xij. pence, of whiche $\frac{2}{3}$ maketh 8 ō, and $\frac{3}{4}$ wthe make 9 pence, and so you maye easilye see that theire proportions be agree. And if you had taken this exaumple before, when you toke the example of $\frac{5}{2}$ and $\frac{3}{2}$, your Quotient would appeare (as this wth) more easier to vnderstande, where as that quotient being $\frac{30}{16}$, is not an easie proportion for you to perceaue, beynge yet little acquaynted with proportions: wherefore to giue you some taste, I will enter to the rule of Reduction, in whiche also I will declare other workes,

bothe of Multiplication and also of Diuision, whiche now I must for a time omitte, as thinges that doe neede the helpe of Reduction.

REDVCTION.

Fiue varieties of Reductiō.

HEreforе will I now declare the diuersities of Reduction of Fractions, whiche commonly hathe fiue varieties.

1. First, when there bee sundrye Fractions of one entiere Unit, they muste be reduced to one denomination, and also into one Fraction.

2. Secondarily, when there bee proponed fractions of Fractions, they must be reduced likewayes into one fraction, for otherwayes they cannot be brought into one denomination.

3. Thirdely, when an Improper Fraction is proponed, that is to saye, a fraction in forme, whiche in dede is greater then an Unit, it must be reduced into apte forme, expressing the Unit or Units of it, and the proper fraction distinctlye. And sometimes also it shall bee nedefull to conuerte suche a mixte numbre of Units with Fractions into the forme of a Fraction, that is into an Improper Fraction, whiche

formes

formes I esteeme but as one, bicause they worke on one kinde of numbre.

4. Fourthly, there happeneth sometimes fractions to be written in greate numbres, which myght be written in lesser numbres, therefore is there a meane to reduce such greate numbres into their smalest tearmes.

5. Fiftly, when any fraction betokeneth the partes of a whole thinge, which hath by common partition certayne parts, but none of like denomination with that fraction, then may you reduce the sayde Fraction into an other, whose Denomination shall expresse the common partes of that whole thing.

Scholer. This distinction in doctrine delyteth mee much, but more with hope then present frute, for as yet I we not understande scarsely the varietyes, and much lesse the practise and vse of their workes.

Mayster. Reduction is an orderly alteration of numbres out of one forme into an other, which is neuer wne orderly but for some needefull vse, as in euery of the sayde 5 seuerall varieties I will distinctly declare.

Fyrste therefore, when two or more seuerall fractions of any Unite bee proponed, as for example, $\frac{3}{16}$ and $\frac{5}{8}$: because it is harde to tell The firste sorte of Reductiō

what portion of the entiere number these two Fractions we expresse, therefore was Reduction diuised, to be a meane whereby these seuerall Fractions myght be brought into one Denomination and Fraction.

Howe to reduce Fractions of diuers Denominations into one Denomination.

And in these Fractions this is the arte for bringinge them to one Denomination.

Multiply first the Denominators together, & the totall thereof you shall set twise downe vnder two seuerall lynes for two new Denominatours, or rather for one common Denominatour: Then multiply the Numerator of the first Fraction, by the Denominator of the seconde, and set the totall thereof for the Numerator ouer the first lyne. Lykewise multiply the Numerator of the seconde Fraction by the Denominatour of the first, and set that totall ouer the seconde lyne for the Numeratour of that Fraction, and so are those two firste Fractions of seuerall Denominations, brought to one Denomination.

Scholer. If I vnderstand you, as I thinke I do, my example shall declare the same. The Fractions which you proponed were these, $\frac{1}{16}$, and $\frac{5}{6}$, whose Denominatours (beyng 16 and 6) I multiplye together, and there amounteth 96, which I set vnder ij. lynes, thus. $\frac{96}{96}$

Then

Then I multiply the Numerator of the firſte Fraction by the Denominator of the ſeconde, ſayinge: 3 into 6 maketh 18, that ſet I ouer the firſte lyne for à newe Numeratour, and it will bee thus. $\frac{18}{96}$

Likewayes I multiply the Numeratour of the ſecond Fraction by the Denominatour of the firſte, ſaying: 4 times 16 maketh 64, that I ſet for the ſeconde Numerator, and the Fraction will appeare thus: $\frac{64}{96}$

ſo that bothe Fractions brought to one Denomination, muſt ſtande thus: $\frac{18}{96}$

and $\frac{64}{96}$

Mayſter. You haue done well.

Scholer. I beſeeke you, let me examine it after my accuſtomed fourme, by common partes of coyne.

Mayſter. Go to.

Scholer. A new Angell accompted at eyght ſhillinges, contayneth 96 pens, whereof $\frac{1}{16}$ that is the xvj. parte, is ſixe pence, and $\frac{3}{16}$ is 18 pence, that is $\frac{18}{96}$. Agayne $\frac{1}{6}$ of the ſame Angell, is 16 pence, ſo that $\frac{4}{6}$ maketh 64 d. that is $\frac{64}{96}$. And ſo I finde the ſummes to agree with the other before.

Note the Reductiō of three Fractions (or more) to one.

Mayster. So haue you nowe the arte to bring suche two Fractions into one Denomination. And if there be more thē ij, then must you multiply all the Denominators together, and set the totall thereof so many times wone as there bee Fractions, and then to get for ech one à newe Numerator. Multiply the Numerator of the first, by the Denominator of the second, and the totall thereof multiply by the denominator of the third, and so forthe if there be more. Likewise multiply the Numeratour of the seconde, by the Denominator of the first, and the totall thereof by the Denominator of the thirde. And in the same sorte multiply the Numerator of the thirde into ỹ Denominator of the first: & the totall thereof into the Denominator of the seconde, and so forthe, if there were mo. So these 3 Fractions $\frac{2}{5}$ $\frac{3}{4}$ $\frac{2}{3}$ dothe make by Reduction these other 3 Fractions of one Denomination $\frac{24}{60}$ $\frac{45}{60}$ $\frac{40}{60}$. All whiche you may bringe into one Fraction by adding the Numerators together, and putting that totall for the common Numerator, reserringe still that same common Denominator. And those 3 Fractions make one improper Fraction, thus. $\frac{109}{60}$

Scholer. All this I perceane, and also that this

this laste Fraction is more then an Unit, and therfore you did call it an Improper Fraction.

Mayster. There bee certayne other formes of worckinge in this Reduction, whiche I will briefly touche also, to giue you an occasion to exercise your witte therein.

The firste varietie is this. When you haue made and written downe youre common Denominator (as I haue taughte before) then to get a Numerator for the firste, we thus. Diuide the commō Denominator by the Denominator of the firste Fraction, and the quotient multiplyed by the Numerator of the same, yelderh a newe Numerator for the firste newe Fraction. So likewise we with the seconde and the thirde, and with all the residue, if there be more. The firste variety of this Reduction.

Scholer. That will I proue in youre laste example of these 3 Fractions $\frac{2}{5}$ $\frac{3}{4}$ $\frac{2}{3}$. When the Denominatours be multiplyed, they make 60. for 5 into 4 maketh 20, and 20 by 3, yeldeth 60, that I sette downe 3 times, thus. 60 60 60: then to haue a Numerator for ỹ firste, I must diuide 60 by 5, (the Denominator of the firste) & the quotient is 12, whiche I must multiply by 2 (the Numerator of the firste) and that maketh 24, and so haue I for the firste Fraction $\frac{24}{60}$.

Likewise

The seconde variety.

Likewise for the seconde fraction: I diuide 60 by 4, and there commeth 15, whiche I multiplye by 3, and so haue I 45 and the seconde fraction $\frac{45}{60}$. Then for the thirde in like sorte will come. $\frac{40}{60}$.

Mayster. An other way is this. If it happen so that the lesser Denominator can by any multiplication make the greater, then note the multiplier, and by it multiply the Numeratour ouer that lesser Denominatour, and for the lesser Denominatour put the greater, as thus in these two Fractions, $\frac{1}{12}$ $\frac{2}{3}$, three beynge the lesser Denominatour multiplyed by 4, will make 12, whiche is the greater Denominatour: therefore by the same 4, I we multiply 2, whiche is Numerator ouer 3, and that maketh 8: vnder whiche I put 12 beynge the greater Denominator, whiche is also made by Multiplication of 4 into 3, & so haue I these ij. Fractions $\frac{3}{12}$, $\frac{8}{12}$: thus shortlye reduced without alteringe the one Fraction.

Scholer. This I vnderstande.

The third variety.

Mayster. Then marke this thirde waye: If the denominators we not happen so, that one by multiplication maye make the other, then loke whether they bothe maye bee parted

of any other one number, as in $\frac{5}{12}$ and $\frac{7}{18}$, although the lesser taken but twise bee to greate to make 18, yet they bothe may bee partes vnto 36: therefore loke howe many times 12 is in 36, and that quotient beynge multiplied by the numeratour ouer 12, the totall shall bee put in steede of the Numeratour ouer 12, and for 12 put 36, thus, $\frac{15}{36}$. So likewayes loke how often is 18 in 36, and bycause it is twise, therefore by 2 multiply 7, whiche is ouer 18, and it will bee 14, set that for the numerator, and in steede of 18 put 36, and then shall youre fractions reduced, stande thus, $\frac{15}{36}$ $\frac{14}{36}$ in steede of $\frac{5}{12}$ and $\frac{7}{18}$.

And if you will proue whether you haue wrought well or no, that maye bee proued by Reduction of them againe to their former denominations, whiche arte shall bee taught in the fourthe kinde of Reduction, where greater termes of fractions bee reduced into smaller in nũbre, but no smaller in proportion. And if in suche Reduction the same termes or numbers come agayne that were before, than is ỹ worke good, else not. Proofe.

Scholer. Sir, I heare your wordes, but I we not vnderstande many of them, whiche it may please you to declare.

Mayster.

Mayster. With a good will, when conuenient place serueth, but that muste bee in the sayd iiij. kynde of Reduction. In the meane season I will declare the second forme of Reduction, whiche teacheth howe to reduce Fractions of Fractions into one Fraction, and so to one Denomination.

Reductiō of fractiōs of fractiōns into one fraction and Denomination.

When fractions of fractions bee proponed, you shall multiply the Numerators of eche into other, and set the totall for the new Numeratour, than multiplye all the Denominatours, lykewayes, and take theire totall for the newe Denominator, and so are they speedelye reduced.

Scholer. If that be all, then I vnderstande it allready, as by this example I will declare. These bee the Fractions, $\frac{2}{3}$ $\frac{2}{3}$ $\frac{6}{7}$ $\frac{7}{8}$, whiche I would reduce to one Denomination. Therefore begin I with the Numeratours, and multiply them all togither, sayinge: 2 into 2 maketh 6, and 6 by 6 maketh 36, whiche multiplyed by 7, yeldeth 252, that I set ouer a lyne for the Numeratour, thus:

252

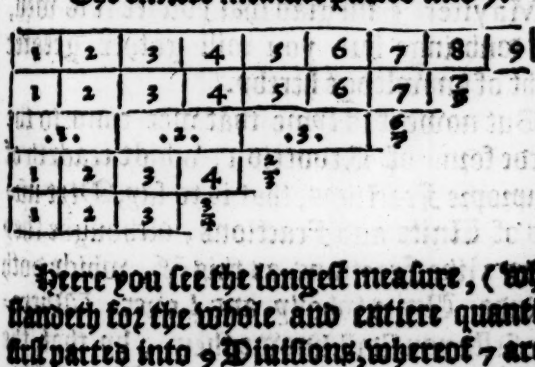
Then I multiply the denominatours, 4 by 3 maketh 12, and that by 7 bringeth 84, whiche multiplyed by 9, yeldeth 756, the new Denominatour.

minatour. And so the whole reduced fraction is this, which is $\frac{252}{756}$ to harde a Fraction for mee to vnderstand yet.

Mayster. You thinke so, and no marualle, but anone you shall learne to iudge it easilye, for this Fraction is no more in deede then $\frac{1}{3}$, although it be in greater termes, and therefore more straunger and more obscure.

And this sufficieth for this Reduction, saue that I will shew you by a figure of measure, the iuste rate and reason of this kinde of Fractions, and also the dewe vnderstandinge of their Reduction.

The entiere measure parted into 9.



Here you see the longest measure, (which standeth for the whole and entiere quantitie) first parted into 9 Diuisions, whereof 7 are severed by the seconde measure: and thereof a-

gayne

gayne are parted out 6. And that 6 beynge distincte into 3 partes, 2 of them are partes by the fourthe measure, of whiche fourth measure, beynge diuided into 4 partes, the lowest measure wythe contayne $\frac{1}{24}$, so that the same $\frac{3}{4}$ must be named, not $\frac{3}{4}$ of the whole measure, but in deede is $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{7}{5}$: or as I would rather expresse it, $\frac{3}{4}$ $\frac{2}{3}$ $\frac{6}{7}$ $\frac{7}{5}$.

Scholer. This example is so sensible, that I cannot chose but see it. And farther more I see also, that the same fraction is equall to $\frac{3}{5}$ of the entiere measure, as the lines whiche runne vp and downe we expressely set foorth. Also I see heere, that $\frac{2}{3}$ $\frac{6}{7}$ $\frac{7}{5}$ is equall to $\frac{4}{5}$. And farther yet, that $\frac{6}{7}$ $\frac{7}{5}$ is equall to $\frac{6}{5}$.

Mayster. I am glad that you see it so well, not doubtinge but you will gather greater lyght of knowledge hereby.

The third forme of Reductiō.

But nowe it is tyme that wee come to the thirde forme of Reduction, whiche teacheth of Improper Fractions, that is to say, Mixt numbers of Units and Fractions, although they appeare like Fractions, as this $\frac{26}{5}$, whiche doth include 5 Units wholly, and $\frac{1}{5}$ ouer. Wherefore first you shall knowe them, by that the Numerator one is greater than the Denominatour.

Scholer.

Scholer. In deede sir that appeareth reasonable, that if the Numeratour we expresse more partes to be taken of any Unit, than the Denominatour doth signifie, that Unit to be diuided into, it must nedes follow, that such a fraction importeth more then the hole, that is to say, the whole with certayne partes ouer. But what Reduction is there in it?

Mayster. There be two seuerall kindes of Reduction, cõcerning such Fractions. Sometymes it shall be needefull to conuert those fractions into the Units and the proper fraction that will remayne. And sometymes contrary wayes, it shall be mete to reduce mixt numbres, that is Units, writtẽ with fractions into the forme of one simple Fraction, and so be there two wayes.

Scholer. What is the meane of the first way to turne improper Fractions into Units with their proper Fractions?

Reductiõ of improper fractions into vnits with their proper Fractions.

Mayster. That is thus. Your Numerator being greater then the Denominator, must be diuided by the same denominator, & the Quotient thereof expresseth the Units, the remayner shall be put for the Numerator of the fraction that resteth, and the denominator must be the same that was before.

Scholer. For example, I take $\frac{17}{5}$. And diuiding 17 by 5, the quotient will be 3, and there will remayne 2.

Mayster. That muſt you write thus, $3\frac{2}{5}$, where (you ſee) I haue written 3 without anye lyne, as entier numbers ought to be written, and the 2 that remayned, I haue ſet ouer the former denominator with a lyne, as a Proper fraction. And this number doth ſignifie now 3 Units, and $\frac{2}{5}$ of one.

Scholer. Then if I would by Units here vnderſtand crownes, ſo it were 3 crownes, and $\frac{2}{5}$, that is 2 ſ.

Mayſter. Euen ſo, and therefore $\frac{17}{5}$ did ſignifie the ſame. But this happeneth ſometimes, that whē the Reduction is ſo wrought, there remayneth nothinge. And then is it not a mixt numbre, but a ſimple entier number, repreſented like a Fraction.

Scholer. As $\frac{15}{5}$ will make 3 iuſte, and $\frac{12}{2}$ wyll make euen 6. This I will remember. But now, what is the ſeconde fourme of Reduction, that you ſpake of for theſe ſortes of Fractions?

Reductiō of whole numbers,

Mayſter. Whenſoeuer you haue anye of theſe two ſortes of numbers, that is to ſaye, whole numbers without fractions, or whole

numbers

numbres with fractions, and you would turne them into the forme of a Fraction, you muste multiply the whole numbre by that denominatour whiche you will haue to remayne still, and to the totall thereof adde the numeratour whiche you haue alreadye, and all that shall you set for the new numeratour, keepinge still the former denominatour: as if you haue $6\frac{3}{4}$, whiche you would conuerte into an Improper Fraction, you muste multiplye 6 by 4, whereof commeth 24, and thereto adde the numeratour whiche is 3, and so haue you $\frac{27}{4}$, for the Numeratour, and 4 still for the Denominatour. other alone, or ioyned with fractions into fractions.

Scholer. Then is $\frac{27}{4}$ equall to $6\frac{3}{4}$.

Mayster. Euen iuste, and so backewarde (as appeareth by the former Reduction) $6\frac{3}{4}$ maketh $\frac{27}{4}$. And thus one of these Reductions may be the proofe of the others worke.

Scholer. This I perceaue: but nowe if you would turne whole nūbres without fractions into any fraction, I see not howe that may be done, bicause there is no denominatour to make the Multiplication by.

Mayster. That was well marked: but this you knowe, that no man intendeth to turne any whole numbre into a fraction, but he hath

in his minde that denominatour by whiche the Multiplication muste bee made: for the proofe whereof I sette downe 7, whiche is a whole numbre. And if you will haue this numbre conuerted into any certaine Fraction, will me so we it.

Scholer. I praye you reduce 7 into a Fraction.

Mayster. Then you care not what the fraction be, so it be some fraction.

Scholer. No, I passe not for the sorte of the Fraction.

Mayster. Then howe can you thinke that you require mee to we anye thinge certayne, when you leaue me to we as I liste? And seynge you stande at that staye, whether thinke you that I muste firste intende in minde what fraction I will make of it, before I can we it in deede?

Scholer. Els you should doe ignorantlye.

Mayster. Then I will limit my selfe (seyng you will not) to tourne it into quarters. And therefore I multiply 7 by 4 (whiche is the denomination of quarters) and there amounteth 28 to bee set for the Numeratour, and the 4 must bee set for the Denominatour, and the Fraction will be thus $\frac{28}{4}$.

Scholer.

Scholer. In deede I perceaue this to bee reasonable, for without muche triall I vnderstande that $\frac{1}{4}$ of any thing, weth make 7. And so then if I woulde tourne 8 into fifte partes, it will make $\frac{40}{5}$, whiche is all one with eighte. for 8 crownes tourned into fifte partes, (that is into shillinges) will make 40 shillinges, that is $\frac{40}{7}$ of a crowne.

Mayster. Seynge you vnderstande nowe these three kindes of Reduction, I will declare vnto you the fourthe kinde, that is when Fractions be written in greater Tearmes thē they neede, howe they maye bee broughte to lesser Tearmes.

The fourth kinde of Reduction.

Scholer. To write any Fraction in greater Tearmes than needeth, seemeth to bee a faulte, and so this rule seemeth to amende that faulte.

Scholer. It were a faulte to we any thinge without neede, whiche after must be redressed: but in this case it is not so: nother did I saye absolutely (as you we) that it needeth not to expresse those Fractions in so great Tearmes, but that the Fractions we not neede, I meane for theire valewe to be vnderstanded: but yet it maye bee needefull for the ease of those workes whereto they be applyed. as for exaumple: In

the firste kynde of Reduction this was youre owne example: $\frac{4}{16}$ and $\frac{4}{6}$, whiche when you would reduce, you were fayne to tourne them firste into one Denomination, and so appeared they thus, $\frac{18}{96}$ and $\frac{64}{96}$, where the Fractions (for theire owne vnderstanding) neded not to be tourned out of smaller termes into greater, but yet the easinesse of workinge needed it.

Scholer. Sir, I vnderstande nowe, not onely the difference of this neede (for the Fractions might better bee vnderstanded as Fractions seuerall, eche in his valewe, when they were in lesser Tearmes, although they coulde not so well be reduced) but also I vnderstande what you meane by greater Tearmes and lesser Tearmes; whereof before I was in doubt, for I see you call the Numeratour and Denominatour, the Termes of the Fraction.

Tearmes of fractions.

Mayster. I am glad you vnderstande it so well. Nowe than when you would valewe any Fractions (bycause that may best bee done when the Tearmes are smallest) you shall reduce them to the smalest that you can, whiche thinge you maye we thus: Diuide the greateste of any suche two Tearmes by the lesser, and if any thinge remayne by that remayner, diuide the laste Diuisour: and if any thinge remayne now,

Reductiō of fractions into their smalest termes

now, by that, diuide the laste Diuisour (whiche was before the remayner of the first Diuision) and so continewe still, till nothing we remaine in the Diuision: and then marke youre laste Diuisour, for it is the number that will easily reduce youre Fraction, if you diuide bothe the Numeratour and the Denominatour by the same number, and put for the Numeratour the quotient of his Diuision, and for the Denominatour also his quotient, that riseth by his Diuision.

Scholer. I take for example $\frac{18}{96}$, and because 96 is the greater number, I diuide it by 18, and the quotient is 5, and there resteth 6: what shall I we with this quotient?

Mayster. Nothinge in this worke, but now seing there remaineth somewhat, by that remayner muste you diuide the laste diuisour.

Scholer. If I shall diuide 18 (whiche was the laste diuisour) by 6, that was the remayner, so is the quotient 3, and nothing resteth.

Mayster. As for the quotient, I omitte him yet: but because there wth remayne nothinge, therefore is 6 (whiche was youre laste diuisor) that number by whiche you may reduce the fraction proponed.

Scholer. Then as you taught me, I muste

diuide the numeratour 18 by 6, and the quotient is 3, whiche I must put for the numerator ouer à lyne, thus. and then by the sayde 6, muste I diuide also the denominator 96, and the quotient will be 16, whiche I muste take for the denominatour, and so is the fraction $\frac{3}{16}$. And so me thinketh this rule doth proue the worke of the first Reduction.

$$\frac{3}{}$$

Mayster. That is true, if the first Reduction were made of Fractions in theyr leaste tearmes, and els not, without some helpe, as the second numbre in that place wyll declare.

Scholer. The second numbre was $\frac{4}{6}$, whiche was turned into $\frac{96}{96}$, by that rule. Nowe yf I shall by this rule reduce it agayne into the leaste termes, I muste diuide 96 by 64, and there resteth 32, by whiche 32 I diuide 64, and there remayneth nothinge: wherefore I muste take that 32 for the diuisour, to reduce the saide fraction. Then doe I diuide 64 by 32, and the quotient is 2, whiche I set for my Numeratour. Agayne, I dyuide 96 by 32, and the Quotient wyll be 3, and so haue I but $\frac{2}{3}$.

Mayster. Muse not at the matter, for you haue done well ynoughe: but you thinke you

you haue not the fraction that you loked for, that is $\frac{4}{6}$, yet haue you one equall to it, as by the partes of a shilling you may proue.

Scholer. Truth it is, for eche of them wyll bringe forth a pence, so that, $\frac{8}{12}$ and $\frac{4}{6}$ & $\frac{2}{3}$, bee all three equall. And now I perceaue, that because $\frac{4}{6}$ was not wrytten in the leaste termes that it myght bee, therefore thys Reduction brought forth not it, but that other whiche is wrytten in the leaste tearmes. Now vnderstand I this rule well. But is there any other way to worke this Reduction?

Mayster. Yes, but first note this, that if you finde no such Diuisor to reduce the Fraction tyll you come to 1, because 1 wyll make no Diuision, therefore that Fraction is already in his leaste Tearmes, as by $\frac{21}{130}$ you may proue, and so of $\frac{85}{98}$, and many other like. But nowe to your question, if you can without that Diuision by memory espye the greatest number that maye diuide exactlye bothe Tearmes of your Fraction proponed, then neede you not to vse that Diuision, as in this Fraction $\frac{48}{72}$, I see that 12, is the greatest number that can diuide them bothe: and therefore without anye worke, by memory onely, I turne that into $\frac{4}{6}$, but this abilitye in knowledge is gotten by

An other way for to worke Reductiõ.

exercise.

An other waye. Yet one other waye of easie Reduction in this kinde there is, when your Fraction hathe any Cyphers in ỳ first places of both Termes, then may you by castinge away the Ciphers, make a briefe Reduction, as thus $\frac{300}{400}$, heere take away the Ciphers, and it will be $\frac{3}{4}$, which is the same in valew with $\frac{300}{400}$.

Sc. And so if I haue $\frac{400}{650}$ it will be $\frac{40}{65}$.

Mayster. You are deceyued, for you take away more ciphers from the numerator, then you we take from the denominator, which you may not we.

Scholer. I confesse my faulte, which came of to much haste. I was more gladder of the rule then wise in vnge it: but nowe I vnderstande it, I trust.

The fifte kinde of Reductiō. Mayster. Then may I go in hande with the fift or laste kinde of Reduction, which teacheth how to turne any fraction proponed into any other denomination that you liste: or into any partes of common coyne, waightes, or measures, or such like.

For declaration whereof, firste you shall marke, whether your fraction be a simple fraction, other els a fraction of sundry partes. I meane of more tearmes then two. And if your

fraction

fraction bee a fraction of fractions, or otherwayes compounde, you muste reduce it to one simple fraction. And then marke well the denomination of that other fraction into whiche you woulde turne this, for by that denominatour you muste multiplye the numeratour of youre firste fraction, and the totall producte thereof shall you diuide by the denominator of youre firste fraction, and that quotient shall be the Numeratour to the denominatour proposed: as for example, I haue this fraction $\frac{3}{5}$, whiche I would turne into tenth partes, therfore I multiply this 10 by 3, that is the numeratour of my fraction, and there riseth 30, whiche I diuide by 5, and the quotient is 6, whiche must be the numeratour to 10, and so $\frac{3}{5}$ will be $\frac{6}{10}$.

Scholer. This is easy ynough to we.

Mayster. Then shall you see an other exaumple of the same Fraction that is not so easy: as if I woulde tourne $\frac{3}{5}$ into viij. partes. proue you that worke.

Scholer. I must multiply 8 by 3, and there amounteth 24: whiche I diuide by 5, and the quotient is 4, then is the newe Fraction $\frac{4}{8}$.

Mayster. And see you nothinge doubtfull in this worke?

Scholer.

Scholer. I see, that when 24 was diuided by 5, there remayned 4, whiche I did not passe of, bycause ye speake nothinge of any remayner, but onely of the quotient.

Mayster. By likelyhode you remembre what I sayde to you in Diuision of whole numbres, that you shoulde not passe of the remayner there, but onely note it as a summe that coulde not bee diuided without knowe-
☞ ledge of Fractions. Wherefore nowe marke this, that in all Diuisions of whole numbres, when there is any remayner, you shall set it ouer a line as a Numeratour, and set the Diuisor for the Denominatour, and that Fraction wthe make the Diuision complete, and is parte of the quotient: as if I woulde diuide 48 by 5, the quotient will be $9\frac{3}{5}$: so in youre former worke when 24 was diuided by $\frac{2}{3}$, the quotient shoulde be $48\frac{2}{3}$, and so the newe Fraction should be thus. $\frac{24\ \frac{2}{3}}{12}$ that is $\frac{2}{3}$ of the entiere numbre, and $\frac{2}{3}$ $\frac{1}{12}$: whiche you may proue by example of some coyne.

Scholer. Then I take a Crowne, whole $\frac{2}{3}$ is 3 s. Now if I would proue whether 3 s. bee $\frac{2}{3}$, I shall haue a $\frac{4\ \frac{2}{3}}{8}$ combrouse worke to me.

Mayster.

Mayster. In deede for whole pennies youre example is troublesome: yet tourninge the Crowne into halfepennies, it is easye ynoughe.

Scholer. Nowe will I doe it.

Mayster. First let me tell you an easy way how to finde any numbre that will easilye bee diuided into suche partes as you desire, whiche way is this. Set wwne the partes that you desire, and then by one of them multiply all the other, the totall whereof shall containe all the partes proponed. As if I woulde haue a numbre that maye bee diuided into 4, 5, 6, and 7 partes, by 4 multiply 5, and there riseth 20: then multiply 20 by 6, and it will make 120: whiche multiplyed by 7, will yelde 840: and so of any other numbres.

Scholer. Then in oure former exaumple where is mention but of 5 partes and 8 partes, I shall onely multiply 5 by 8, whiche maketh 40, and that numbre will serue.

Mayster. So will it.

Scholer. Then what is $\frac{3}{7}$ of 40?

Mayster. Wyne by the same rule whiche you confesse easy ynoughe: 3 times 40, is 120, whiche beynge diuided by 5, maketh $24\frac{5}{5}$: and that is iuste.

Nowe

Now to knowe whether it be equall to 24, firste I see by the same rule, that $\frac{4}{5}$ is 20, and $\frac{1}{5}$ is 5, of whiche 5 I must take $\frac{4}{7}$: and that by the same rule is 4. So that I see nowe, that is equall to $\frac{3}{5}$.

Maister. And by the way note this forme of fraction howe it is written, that is to saye, bothe the Numerator and his Fraction aboue the lyne: although I knowe it maye be written other wayes, as thus $\frac{4}{5}$ and $\frac{5}{7}$ $\frac{1}{8}$, but I accompte the other waye more apte a greate deale.

And so may you expresse by an other waye, then is before mentioned, all Fractions of Fractions, as thus. That is $\frac{1}{4}$ of $\frac{5}{8}$, and so of other, but I remitte these fourmes to the arbitriment of euerye wise artes man, to vse as he thinketh most apt and ready.

But nowe one example more for this rule, and then shall wee ende it. If I haue $\frac{7}{12}$ of a Soueraigne (accompting the Soueraigne 20 shillinges) howe manye shillinges is that $\frac{7}{12}$?

Scholer. I muste multiplye 7 by 20, and that maketh 140, whiche I shall diuide by 12, and the quotient will be 9 $\frac{2}{12}$: or elsse in lesse termes,

times, $\frac{1}{3}$.

Mayster. That is 9 ℔, and one third part of a shilling, that is 4 ♄. as by this same rule you may proue. And this for this tyme shall suffise for Reduction, saue that I muste nowe treate a little touchinge Multiplication and Diuision, and so go forwarde.

MVLTIPLICATION.

IN Multiplication it happeneth sometyme, that there bee whole numbers to bee multiplyed with Fractions: And may bee in two sortes, for other the whole numbre is seuerall from the fraction, and is the multiplyer, or els, the whole number is ioyned with one or bothe of the Fractions, and so maketh a mixt number thereof. If it be in the firste sorte, then needeth there no Reduction, but onelye multiply the Numerator of the Fraction by that whole number, and the totall thereof set for the newe Numerator.

Reduction of whole numbers into fractions.

Scholer. I vnderstande you thus. If I haue $\frac{6}{2}$ to be multiplyed by 16, then muste I multiply that 16 with 6, whiche is the Numerator,

meratoꝛ, whereof commeth 96, and that muſte I ſet foꝛ the newe Numeratoꝛ, keepinge ſtyll 23 foꝛ the Denominatoꝛ, and ſo the Fraction will bee $\frac{96}{23}$ that is $4\frac{4}{23}$.

Mayſter. And in thys ſoꝛte of woꝛke you maye abbꝛidge the labour, thus. If it happen the denominatoꝛ to be ſuch a numbꝛe, as may euenly be dyuided by the ſayde whole numbꝛe pꝛopoſed, then diuide it thereby, and ſet the quotient of that Diuiſion foꝛ the foꝛmer Denominatoꝛ: but reſerue ſtyll the Numeratoꝛ, and ſo is the Multiplication done.

Scholer. Then I fayne this example, $\frac{7}{10}$ to be multiplyed by 5. And becauſe 5 will iuſtlye diuide 20, thereforte I take the quotient of that dyuiſion whiche is 4, and ſet in ſtede of 20, and ſo the Fraction will bee $\frac{7}{4}$, that is $1\frac{3}{4}$.

Mayſter. Whiche is all one with $\frac{35}{20}$ that would haue folowed of the other ſoꝛt of woꝛk.

Sch. I pꝛeceaue it very well.

Howe to multiplye mixt nũbers.

Mayſter. Nowe then foꝛ the other ſoꝛte where the numbꝛe is myxt, take this waye: firſte to reduce the ſayde whole numbꝛe and fraction into one fraction Impꝛopꝛe (as I ſhewed you in Reduction) and then multiplye theym together, as if they were pꝛopꝛe fractions.

fractions.

Scholer. $13\frac{2}{5}$ beinge set to bee multiplyed by $\frac{5}{8}$, firste I muste reduce the myxte number by multiplying 13 by 5, and that maketh 65, whereto I muste adde the Numerator 3, and so the fraction will be $\frac{68}{5}$, whiche nowe I shall multiplye after the accustomed forme, and it will bee $\frac{340}{40}$.

Mayster. You haue wne well: and so may you see, that although moste part of the formes of Multiplication may bee wrought wythout Reduction, yet some can not, as namelye Myxed numbers.

And yet one note more wyll I tell you of Multiplication, before wee leaue it: That is, when so euer you woulde multiply any fraction by 2, whiche commonly is called Duplation, you may we it not onely by doubling the Numerator, but also by partinge the Denominator into halfe, if he be euen. Duplatio

Scholer. Then if I woulde double $\frac{5}{12}$, I may chose whether I will make it, $\frac{10}{12}$, or els $\frac{5}{6}$, And in deede I see that all is one, but that the diuiding of the Denominator seemeth the better waye to make smaller tearmes of the fraction, and so they shall neede the lesse Reduction.

Mayster. It is so: and now I shall not neede to tell you that Multiplication is proued by Diuision, and Diuision likewaies by multiplication, but the like worckes that I shewed you in Multiplication, will I shewe you in Diuision also.

DIVISION.

Diuision to diuide a whole numbre by à Fraction.

Hen anye whole number shall be diuided by à Fraction, you musse multiplie the saide whole numbre with the Denominator of the fraction, and set the totall thereof for the new Numerator, and for the Denominator, set the Numeratour of the Fraction.

Scholer. Then 20 diuided by $\frac{3}{4}$ will make $\frac{80}{3}$.

To diuide à Fraction by a whole numbre.

Mayster. Euen so. But if you woulde diuide the fraction by the whole number, then multiplie the Denominatour by the same whole number, and set the totall for the Denominator, without changing the Numerator.

Scholer. Then to diuide $\frac{20}{23}$ by 4, it will be $\frac{20}{92}$.

An other

Mayster. You say well. And by the same

example

example you giue mee occasion to remember an other breife way to we the same: for if you had diuided the sayd Numerator by 4, and set the quotient for the Numerator, keepinge still the olde Denominator, it woulde haue beene not onelye as well one, but also in a Fraction of lesser termes. breefeway.

Scholer. I gesse it to be euen so, by a like worke that you taught me in Multiplication. And for proofe thereof $\frac{20}{23}$ beyng the diuidende, and 4 the Diuisour, I diuide the Numerator 20 by 4, and the quotient is 5, whiche I set for 20 ouer 23, thus $\frac{5}{23}$. And I see that it is all one with $\frac{20}{92}$, as by diuidinge bothe these tearmes by 4, and so reducinge them to their leaste Denomination, I maye easilye proue.

Mayster. You conceaue it well. And if there be mixt numbres (other one or bothe) you muste firste reduce that mixte numbre into an improper Fraction. And then worke as you haue learned.

Scholer. That was sufficiently taughte in Multiplication. Therefore I pray you go forwarde to some other thinge.

Mayster. Then take this note yet for Diuision. If the Denominators be lyke, then di-

uide the Numeratoꝛs as if it were in whole numbꝛes, and the quotient whether it be Fraction, whole numbꝛe oꝛ mixt, is a good quotient foꝛ that Diuision. And generally, if one of the Numeratoꝛs maye iuſtely diuide the other, by that quotient multiplye the Denominatour of the leſſer Numeratoꝛ, and ſet it that dothe amounte, in the roome of the ſame denominatoꝛ, and then foꝛ a Numeratoꝛ to it, ſet the denominatoꝛ of the other fraction.

Scholer. Then if I woulde diuide $\frac{3}{4}$ by $\frac{12}{17}$ I ſee that 3 will diuide 12, and the quotient will be 4, by whiche I muſte multiply the other 4 that is the Denominatoꝛ vnder 3, and then it is 16, whiche I ſet foꝛ the Denominatoꝛ 4, and ouer it in ſteede of the 3, I muſt ſet 17, the other Denominatoꝛ, and ſo is it thus, $\frac{17}{16}$.

Mayſter. And ſo is $\frac{17}{16}$ in ſteede of $\frac{51}{48}$, whiche woulde haue ryſen by the common woꝛke.

Mediatiõ. And now foꝛ Mediation (whiche is to diuide by 2) marke this: If the Numeratoꝛ bee euen, ſette the halfe of it in his place without the Diuiſour, and ſo haue you wone. and if the Numeratoꝛ bee not euen, then double the Denominatoꝛ.

Scholer.

Scholer. That is, if I woulde mediate $\frac{6}{11}$, I may make the quotient $\frac{3}{11}$. And if I woulde mediate $\frac{7}{11}$ I must make it $\frac{7}{22}$.

Mayster. Nowe truste I that you haue sufficient knowledge in Reduction, Multiplication, and Diuision: and therefore will I goe in hande with Addition and Subtraction, whiche now will appeare easie ynoughe.

ADDITION.

Hen soeuer you haue anye fractiõs to be added, you muste consider whether they bee of one denomination or not. And if they bee of one Denomination, then adde the Numerators togither, and set that amounteth, for the Numerator ouer the common Denominator, and so haue you wne. The reason is, bicause that suche differ little in Addition or Subtraction from the worke of vulgare denominations, where the Denominators bee no nombres: as 3 pence & 5 pence, make 8 pence: where the Denomination is not altered. But

To adde Fractions of one denomination. 1.

2. and if the Fractions bee not of one Denomination, or anye of them be Myxt of wole numbres and fractions, then muste you firste reduce them to one Denomination, and after adde them. And if they bee many, then adde firste two of them, and to the summe that wthe amounte of the Addition, adde the thirde, and then the fourthe, and so forthe, if you haue so manye.

3. To adde Fractions of diuerse Denominations.

4.

Scholer. This seemeth easie ynoughe, nowe that I haue already learned to multiplye and to Reduce, without whiche two, I coulde neuer haue wroughte this. And therefore nowe I see good reason, whye you did place Multiplication and Reduction before Addition.

Mayster. It is well considered, but yet refuse not to expresse youre vnderstandinge of it, by an example.

Scholer. Then woulde I adde firste $\frac{7}{12}$ with $\frac{5}{18}$, and bycause the Denominators are like (and so needeth no Reduction) I adde 7 to 5, whiche maketh 12, and then is my summe $\frac{12}{18}$, that is in smaller numbres $\frac{2}{3}$.

And if I haue many numbres to be added, as heere $\frac{3}{8}$ $\frac{4}{5}$ $\frac{9}{10}$, firste I muste reduce them (bycause they haue diuers denominatours) into

to one Denomination, and then will they bee thus.

$\frac{150}{400}$ $\frac{220}{400}$ $\frac{360}{400}$

or in lesser Tearmes $\frac{15}{40}$ $\frac{22}{40}$ $\frac{36}{40}$, which by Addition we make $\frac{33}{40}$, that is $2\frac{3}{40}$.

Ma. Now may wee goe to Subtraction.

SVBTRACTION.

Vbtraction hathe the same preceptes that Addition had, for if the Denominators bee like, then muste you subtracte the one numeratour from the other, and the rest is to bee set ouer the common Denominator, and so youre Subtraction is ended: but and if you haue many Fractions to be subtracted out of many, then muste you reduce them to one Denomination, and into two seuerall Fractions, that is, all that must be subtracted into one Fraction, and the residue into an other Fraction, and then worke as I sayde before.

Scholer. For the firste exaumple I take $\frac{15}{12}$ to bee subtracted out of $\frac{17}{12}$, and the reste will be $\frac{2}{12}$ or $\frac{1}{6}$.

For an other exaumple I take $\frac{3}{4}$ to be sub-

trahed out of $\frac{7}{8}$, which I muste firste reduce, & it will be thus, $\frac{24}{32}$ and $\frac{28}{32}$.

Then we I subtracte 24 out of 28, and there resteth 4, which I set ouer the common Denominator for a Remayner, thus. $\frac{4}{32}$, that is $\frac{1}{8}$.

Nowe for the thirde example, I take $\frac{1}{4}$ and $\frac{5}{8}$ to bee subtrahed from $\frac{7}{8}$ and $\frac{9}{10}$. And bycause their Denominators be diuers, I we reduce them thus, $\frac{1440}{1920}$ $\frac{1600}{1920}$ $\frac{1680}{1920}$ $\frac{1728}{1920}$.

Then we I adde the two firste, and they make $\frac{3040}{1920}$. Also I adde the two laste, and they yelde $\frac{3408}{1920}$. Then we I subtracte 3040 out of 3408, and there resteth 368, so is the remayner $\frac{368}{1920}$, that is in smaller Tearmes, $\frac{23}{120}$. And thus haue I done with Subtraction, except you haue any more to teache mee.

Mayster. Proue one example more of two Fractions of diuers Denominations.

Scholer. I take these two Fractions, $\frac{7}{8}$ and $\frac{9}{24}$, which being reduced, will stande thus, $\frac{168}{192}$, and $\frac{72}{192}$. Nowe would I subtracte 168 out of 72, but I can not.

Mayster. Then maye you perceaue that you misloke the Fractions: for you can neuer subtract the greater out of the lesser, although you may adde, multiply or diuide the greater

with

with the lesser. And albeit that $\frac{7}{8}$ hathe bothe his Tearmes lesser then $\frac{9}{24}$, yet is $\frac{9}{24}$ the lesser Fraction: for generally if you multiply the Numerators and Denominators of two fractions crosse waies, that Fraction is the greatest, of whose Numerator commeth the greatest summe, as in this example: 7 multiplied by 24, maketh 168: and 9 beinge multiplied by 8, yeldeth but 72, therefore is the first Fraction $\frac{7}{8}$ the greatest of these two, so canne you not subtract it out of a lesser Fraction. The greatest of two fractions.

But and you shoulde subtracte à Fraction out of à whole number, what would you we?

Scholer. Marry I would reduce ỳ whole number into à Fraction of the same Denomination that my Fraction is, and then worke by Subtraction.

Mayster. So may you we, but it is easier mudy, if your fraction be à proper fraction, that is to say, lesse then an Unit, to take an Unit from the whole numbre, and then turne it into an Improper Fraction, and so worke youre Subtraction. As if I woulde subtracte $\frac{3}{5}$ from 4, I maye take one from 4 and tourne it into $\frac{5}{5}$, from whiche if I bate $\frac{3}{5}$, there will remaine $3\frac{2}{5}$. And if the first fraction bee an Improper fra-

ction, then maye I take so manye Units from the whole number, that they may make an improper fraction greater then that first, and then worke the Subtraction: As if there be propouned $\frac{10}{3}$ to bee subtraied from 6. bicause $\frac{10}{3}$ is more then 3, & not so much as 4, I musse take 4 from 6, and turne them into thirdes, thus. $\frac{12}{3}$ then abate $\frac{10}{3}$, and there resteth $\frac{2}{3}$, so the whole remayner is $2\frac{2}{3}$. And thus will I make an ende of the workes of common Fractions for this tyme, not doubtinge, but you can apply them bothe vnto the rules of Progression, and also vnto the Golden rule, without anye other teachinge then you haue learned before, which might seme tedious to repeate, saue that in some speciall diuersities, which be peculier to Fractions, I can not ouerpasse, but instruct you somewhat by the way.

THE GOLDEN RVLE.

Herfore as touching the Golden rule for the placing of the iij. numbres propouned in the question, whereby to finde the thirde, & for the forme of their worke, with other like notes,

I

I referre you to that whiche you haue already learned.

But this easy forme of worlinge by fractions shall you note, ỹ if your three numbres bee fractions, for an apte worke and certaine, multiply the Numerator of the first numbre in the question, by the Denominator of the seconde, and all that agayne multiply by the Denominator of the thirde numbre, and the totall thereof shall you keepe for to bee the Diuisor. Then multiplye the Denominator of the firste numbre by the Numerator of the seconde, and the whole thereof by the Numerator of the thirde, and the totall thereof shall bee your diuidende. Now diuide this diuidend by ỹ diuisor whiche you found out before, and that numbre shalbe the iiij. numbre of the question whiche you seeke for: As in this example, If $\frac{3}{4}$ of a yarde of Uelluet coste $\frac{2}{3}$ of a Soueraigne, (esteemed at 20 shillinges) what shall $\frac{5}{6}$ coste?

A question of Veluet.

Scholer. If it please you to let mee make the answere, I would firste place these three numbres, as I learned in whole numbres, thus. $\frac{3}{4} \quad \frac{2}{3}$ / $\frac{5}{6}$

And then accordinge to youre newe rule, I must multiply 3 beynge Numerator in the firste numbre, by three the Denominator of the

the seconde, and thereof commeth 9, whiche I multiply agayne by 6, the Denominatour of the thirde numbre, and so haue I 54, whiche I keepe for the Diuisor, then multiplye I 4, the Denominator of the firste, by 2, the Numeratour of the seconde, and there riseth 8, whiche agayne I multiply by 5, the Numeratour of the thirde, and it maketh 40: then muste I diuide 40 by 54, and it will be $\frac{40}{54}$ that is $\frac{20}{27}$, in lesser tearmes, and then the figure will stande thus.

But what that is in mony, I can not tell, excepte I shall worke it by Reduction, as you taughte mee.

$\frac{3}{4}$ $\frac{2}{3}$
$\frac{5}{6}$ $\frac{20}{27}$

Mayster. It forceth not nowe, you maye reduce it when you liste, but it were disorderly done here to mingle diuers woorkes togither, where wee we not seeke ỹ valewe of the thing in common mony, but in an apte nũbre, whiche you haue well wne. And therefore will I yet shewe you an other like waye of easinesse in worke, how you may chaunge youre iij. Fractions into 3 whole numbres, by whiche you shall worke as if the question were proponed in whole numbres. The firste numbre you shall finde

finde as I taught you: nowe to finde the Diuisor for the seconde numbre, take the Numerator of the seconde fraction: and for the thirde numbre take that, that riseth of the Multiplication of the Denominator of the firste, by the Numerator of the thirde, and then worke your question.

Scholer. For exaumple hereof, I put this question, If $\frac{11}{12}$ of 1 li waight of siluer, bee worth $\frac{12}{4}$ of a Soueraigne, what is $\frac{1}{2}$ of 1 li. weight worthe? For the aunswere, firste I place the Frations in ordre, thus.

A questiō of Siluer.

$\frac{11}{12} \quad \frac{12}{4}$
$\frac{1}{2}$

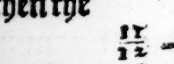
Then to turne these Fractions into whole numbres, I multiplye 11 whiche is the Numerator of the firste, by 4, (the denominator of the seconde) and there commeth 44, whiche I multiply by 2 the Denominator of the thirde, and so amounteth 88, whiche I set for the Diuisour in the firste place. Then in the seconde place I set 12, whiche is Numeratour in the seconde Fraction, and in the thirde place I set the summe that amounteth of 12, beynge the Denominatour in the firste number, multiplyed by 1, beynge numerator in the thirde number, & so the figure will stande as heere you see.

88 — 12
12

Then

Then to worke it forth, I multiply 12 by 12, and there amounteth 144, whiche I diuide by 88, and the quotient will be $1\frac{56}{88}$ or in lesser tearmes, $1\frac{7}{11}$, and then the figure will stande thus.

$$\begin{array}{ccc} \frac{11}{12} & & \frac{11}{4} \\ & Z & \\ \frac{1}{2} & & 1\frac{7}{11} \end{array}$$



Ma. These ij. formes nowe you vnderstande well inoughe: And as for any other, at this time I will not repeat, onely this shall you marke for the proofe of this rule, whether youre worke bee well wrought or no. Multiply the firste numbre by the fourthe, and note what amounteth: then multiply the seconde by the thirde, and marke what amounteth also. Nowe if those two numbres so amounting be equall, then is your worke well done, else you haue erred. And this shal suffice for the former rule, but in the Backer rule, this shall you note for ease of worke, that you multiply the Numerator of the firste by the numerator of the second, and the whole thereof by the Denominator of the thirde, and that amounteth thereof, shall be ye Diuidende. Then multiplye the Denominator of the firste by the Denominator of the seconde, and that whole by the Numerator of the third, and that riseth thereof shall bee the Diuisor. Example of this: I did lende my friende $\frac{2}{4}$ of a Porte-

The profe of the Golden rule.

The Backer rule.

guise

guise vij. monethes, vppon promise that hee should we as mucke for mee againe: and when I shoulde borowe of him he coulde lend me but $\frac{5}{12}$ of a Porteguise, nowe I demaunde howe longe time must I keepe his monye in iuste recompence of my lone, accomptinge 13 monethes in the yeare? A question of lone.

Scholer. The firste number must be the firste monye borowed, that is $\frac{3}{4}$ of the Portеguise: the seconde number the 7 monethes, that is $\frac{7}{13}$ of a yeare: and the third numbre the mony that was lent in recompence, that is $\frac{5}{12}$ of a Portеguise: then I set the numbres thus.

$$\frac{3}{4} \quad \frac{7}{13}$$
$$\frac{5}{12}$$

Then (as you taught me) I multiplie 3 (beinge Numerator in the firste number) by 7 the Numeratour of the seconde number, and it maketh 21, whiche I multiplie by 12 the Denominator of the thirde, and so haue I 252 for the diuidend: then I multiplie 4 the denominator of the firste, by 13 the denominator of the second, and it yeldeth 52, whiche I multiply agayne by 5, the Numeratour of the thirde, and it will make 260, that is the diuisor. Then must I diuide 252, by 260, so it will bee in the smallest fraction, $\frac{252}{260}$ of a yeare.

Mayster.

Mayster. And this we you see some ease in worckinge, better then to multiply and diuide tediously so many Fractions. An other question yet will I propose, to the intent you may see thereby the reason of the statute of assise of breade and ale, which in all Statute bookes in Frenche, Latine, and Englishe, is muche corrupted for wante of knowledge in this arte: for the right vnderstanding whereof I propone this question.

Statute of Assise of bread and ale.

Questiō. When ỹ price of à quarter of wheate is 2 ſ. the farthinge white loke shall weye 68 ſ: then I demaunde, what shall suche à loafe weye, when the quarter of wheate is solde for 3 ſ?

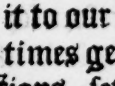
Scholer. This question must be wrought as it is proponed in whole numbers and not in Fractions.

Mayster. You seeme to saye reasonable, how be it, in that Statute of Assise, the rate is made by the proportion of parts in à pounde weight Troye, els could it not bee a Statute of any longe continuaunce, seing the shillings we chaunge often, as all other monies we: but this Statute beinge well vnderstanded, is à continuall rule for euer, as I wyll anon declare by à new table of Assise, conuertinge the shillings into vnces and partes of vnces.

Therfore

Therefore here by à shilling you muste vnderstand $\frac{1}{20}$ of à pound weight, and so by pence $\frac{1}{240}$ of an vnce, wherefore althoughe you might worke this question proponed by whole numbre well ynough, for that time when the statute was made, yet to apply it to our time, and to make it to serue for all times generallye, it is best to worke it by fractions, settinge for ij. shillinge $\frac{2}{20}$: and for 68 shillinges, $\frac{68}{20}$: and so for three shillinges $\frac{3}{20}$, and then will the figure of the question stand thus. In which question, bicause all the denominators be like, you shall worke onely with the numerators.

Note what a shillinge is.



Scholer. Then I shall multiply 68 by 2, whereof commeth 136, which if I diuide by 3, the Quotient will be $45\frac{1}{3}$: but howe shall I make à Fraction of that to stande with the other?

Mayster. Haue you so soone forgotten what was taught you so lately? This is his fourme.

$$\frac{45\frac{1}{3}.}{20}$$

Sch. I remember it now, and than it signifieth 45 twenty partes, and the thirde deale of one twentye parte.

Mayster. So is it, and that maketh in

shillings, 45 shillinges, fowre pence: whereby you may note one great errour in the Statute bookes, which haue constantly 48 shillinges in that Assise. And by this rule, if you examine the statute, you shall finde many summes false, wherefore for the true vnderstanding of that statute & such like as I haue made mencion of it, and somewhat recognised it, so doe I wish that all gentlemen and other students of the lawes, woulde not neglecte this arte of Arithmetike as vnneedefull to their Studies. Wherefore to encourage them thereto, and to gratifie bothe them and all other in generall, I will heere exhibite à Table of that parte of the statute in two columes, and in à thirde columne I will adde ỳ correction of those errours which haue crept into it.

Here followeth the Table.

The price of a quarter of wheate.		The weight of a farthinge white lofe by the ſtatute bookes.			The correction by iuſt Aſſiſe.		
ſ.	d.	li.	ſ.	d.	li.	ſ.	d.
1	0	6	16	0	6	16	0
1	6	4	10	8	4	10	8
2	0	3	8	0	3	8	0
2	6	2	14	4 1/5	2	14	4 2/5
3	0	2	8	0	2	5	4
3	6	2	2	0	1	10	1 0 2/7
4	0	1	16	0	1	14	0
4	6	1	10	0	1	10	2 2/7
5	0	1	8	2 1/2	1	7	2 2/5
5	6	1	4	8 1/2	1	4	8 8/11
6	0	1	2	8	1	2	8
6	6	0	19	11	1	0	11 1/11
7	0	0	19	1	0	19	5 1/7
7	6	0	18	1 1/2	0	18	1 1/5
8	0	0	17	0	0	17	0
8	6	0	16	0	0	16	0
9	0	0	15	0 1/2	0	15	1 1/3
9	6	0	14	4 1/2	0	14	3 11/19
10	0	0	13	7 1/2	0	13	7 1/5
10	6	0	12	11 1/2	0	12	11 1/7
11	0	0	12	4 1/2	0	12	4 4/11
11	6	0	11	10	0	11	9 2/23
12	0	0	11	4	0	11	4

In the common bokes there is no farther rate of assise made, than vnto 12 s the quarter of wheate: but in an auncient copi of 200 yeares ould (which I haue) ther is added the rate of assise vnto 20 s the quarter, but yet was ỳ assise also eyther wrong cast at ỳ first pening, or els corrupt sith that time, for lacke of iuste knowledge in the rule of proportion, which I wil adde here also, to gratifye suche as be studious in thys sorte of knowledge, and desire to vnderstande truth exactly.

The price of a quarter of wheate.		The weight of the farthinge white lofe by the statute bokes.			The correctiō of the errours.		
s.	d.	s.	d.		s.	d.	
12	6	11	0		10	10	12/21
13	0	15	0	1/2	10	5	7/11
13	6	10	1	1/2	10	0	3/5
14	0	9	7		9	8	4/5
14	6	9	2	1/2	9	4	16/29
15	0	9	1	1/2	9	0	4/9
15	6	9	1	1/4	8	9	9/11
16	0	9	0		8	6	0
16	6	8	6		8	2	10/11
17	0	8	3		8	0	0
17	6	7	10		7	9	9/11
18	0	7	6		7	6	2/3
18	6	7	3		7	4	7/11
19	0	7	1		7	1	1/19
19	6	5	10		6	11	3/11
20	0	5	6		6	9	3/5

These ij. tables I haue set seuerall, because no man should thinke that I would either adde or take away from any law those partes whiche might of right seeme eyther superfluous other diminute, but yet I may not be so curious as to neglect manifest errors, whiche is not onely my part, but euery good Subiectes duety with sobriety to correct. And for auoyding of offence I haue rather done it in this priuate boke, rather then in any booke of the statutes selfe, trusting that all men will take it in good parte.

Sc. I woulde wisshe so, but I dare not hope so, sith neuer good man that would reforme errour, could escape ẏ venimous tonges of enuious detractors, whiche because they eyther cānot or lisse not to any good them selfes, we delite to barke at ẏ wings of other, but I beseke you to stay nothing for their peruerse behauiour.

Ma. I consider many thinges ẏ some maye obiect, wherunto I am not vnprouided of iuste aūswers, but I wil not seeme so hasty to make the aūswers before I heare their obiectiōs, but as I trust that men are of a better nature, and more grateful now thē some hath bin in times passed, as I haue done in ẏ statute of Assise for bread in rate of shillings, so wil I set forth the like table in pounds & ounces, and the partes

thereof, ẏ it may be eaſily applied to all times: but I meane not by this to alter any worde of ẏ Statute (beyng ſo good an ordinaũce & of ſo great continuaũce) but onelye to make it as a kind of expoſitiõ & declaratiõ of ẏ ſaid Statute, truſting ẏ therby the ſtatute may be better vnderſtand, & conſequently better put in execution. And heere you ſhall note, ẏ I haue accompted the ſhillings after ẏ rate of lx. ſ. to ẏ pũd weight, becauſe I eſteme it ẏ moſt apt rate for our time. Wherefore if in ẏ firſte columne you finde the price of wheat, directly againſt it in ẏ ſecond columne you may finde ẏ weight of the farthing white loke, in this oure time, & if you wuble ẏ numbre (as I haue done in the thirde columne) thẽ haue you the weight of the halfe pẽny white loke, & ſo in the fourth colũne is ſet the weight of ẏ pẽny white loke. It needeth not to tell you that, ẏ the ſight wthe teſtify, howe ẏ euery colũne is parted into thre ſmaller pillers, whereof ẏ firſt colum hath theſe thre titles, pũdes, ſhillings, & pennies: ẏ other three columnes haue edr of thẽ theſe thre titles, pounds, vnces, & penny weights. And as in ẏ firſt colũne xij. pence make a ſhilling, & xx. ſ. maketh à pũde, ſo in ẏ other iij. columes xx. pẽny weight maketh an vnce, and xij. vnces we make à pounde.

A pounde weight.

The

The weight of the farthinge white lofe.

li.	vnc.	ō. w.
6	9 1/2	2
4	6 1/4	3
3	4 3/4	1
2	8 1/2	2 4/5
2	3	4
1	11 1/4	1 2/5
1	8 1/2	3
1	6	2 2/5
1	4 1/4	1 2/5
1	2 3/4	1 1/5
1	1 1/2	2
1	0 1/2	1 1/5
0	11 1/2	3 1/5
0	10 3/4	2 3/5
0	10	4
0	9 1/2	2
0	9	1 1/5
0	8 1/2	1 1/10
0	8	3 1/5
0	7 3/4	0 3/5
0	7 1/4	3 4/11
0	7	1 2/11
0	6 3/4	1
0	6 1/2	0 2/5
0	6 1/4	0 1/5
0	6	0 8/11

li.	s.	ō.
0	3	0
0	4	6
0	6	0
0	7	6
0	9	0
0	10	6
0	12	0
0	13	6
0	15	0
0	16	6
0	18	0
0	19	6
1	1	0
1	2	6
1	4	0
1	5	6
1	7	0
1	8	6
1	10	0
1	11	6
1	13	0
1	14	6
1	16	0
1	17	6
1	19	0
2	0	6

The price of a quarter of wheate.

The weight of the penny white lofe.

li.	vnc.	ð. w.
27	¼	3
18	1 ½	2
13	7	4
10	10 ½	1 ⅕
9	0 ¾	1
7	9 ¼	0 ⅕
6	9 ½	2
6	0 ½	0 ⅖
5	5 ¼	0 ⅗
4	11 ¼	1 10/11
4	6 ¼	3
4	2	4 4/5
3	10 ½	2 ⅘
3	7 ½	0 ⅖
3	4 ¾	1
3	2 ¼	3
3	0 ¼	0 ⅕
2	10 ¼	2 1/9
2	8 ½	2 ⅘
2	7	1 ⅕
2	5 ½	3 ⅕
2	4 ¼	2 ½ 1/3
2	3	4
2	2	2 ⅖
2	1	2 ⅓
2	0	2 8/9

The weight of the halfpenny white lofe.

li.	vnc.	ð. w.
13	7	4
9	0 ¾	1
6	9 ½	2
5	5 8/20	0 ⅗
4	0 ¼	3
3	10 ½	2 ⅘
3	4 ¾	1
3	0 ¼	0 ⅓
2	8 ½	2 ⅘
2	5 ½	3 5/11
2	3	4
2	1	2 2/15
1	11 ½	1 ⅖
1	9 ¾	0 ⅗
1	8 ¼	3
1	7	4
1	6	2 ⅔
1	5	3 17/19
1	4 ¼	1 ⅖
1	3 ½	0 6/7
1	2 ¾	1 8/11
1	2	3 19/23
1	1 ½	2
1	1	1 3/5
1	0 ½	1 3/13
1	0	1 7

Scholer. Sir, I we thanke you moſt hartely for this, not onelye in myne owne name and in the name of all ſtudentes, but alſo in the name of the whole Commons, to whome the reſtitution of this Aſſyſe (I truſte) ſhall bringe reſtitution of the weighte in breade, whiche longe time hath ben abuſed. And if you kuowe any like thinges more, wherein you woulde vouchſafe to declare the errours and ſet forthe the truthe, you can not but obtayne greate thanks of all good harted men that loue the common wealth.

Mayſter. I haue ſundrye thinges to declare, but I haue reſerued them for a priuate booke by it ſelfe, yet not withſtanding bicauſe the ſtatute of the rate of meaſuring of grounde is ſo common that it toucheth all men, and yet no more common thē neceſſull, but ſo muche corrupt, that it is ſo farre out of all good rate, not onely in the Engliſhe bookes of ſtatutes commonlye printed, but alſo in the Latine bookes, and in the Frenche alſo, for I haue reade of eche ſorte, and conferred them diligently, I will giue you à Table for the reſtitution of thoſe errours, as may ſuffiſe for this preſent tyme. And firſte will I propoſe one queſtion to you touching the vſe of that Sta-

tute, whereby you maye perceaue the order how to examine the whole Statute, and euery parcell thereof, and the question is this.

A questiō of measure of grounde.

When the Acre of grounde doth contayne foure perches in breadthe, then muste it contayne 40 perches in lengthe: then we I demaunde of you, how muche shall the lengthe of an Acre be, when there is in the breadthe of it 13 perches? But before you shall aunswere to this question, I will declare vnto you an other Statute, whiche is the grounde of the former Statute. And that Statute is this. It is ordeined, that 3 Barlye cornes, drie and rounde, shall make vp the measure of an ynche: xij. ynches shall make a foote, and iij. foote shall make a yarde, (the common Englishe bookes haue an elne) fiue yardes and a halfe shall make a perche, and forty perches in lengthe, and iiij. in breadthe, shall make an Acre. This is that Statute: whereby you may perceaue, that the intent of the Statute is, that one Acre should containe 160 square perches. Now let me heare you aunswere to the question.

A Statute of measures.

An Acre.

Scholer. As I perceaue by the wordes of that Statute, a perche to be $\frac{1}{160}$ of an Acre, so coulde I make those numbres all in Fractions

tions, and so worke the question: but seeynge I maye we it also in whole numbres, I take ỳ forme for the moste easye, therfore thus I set the question in forme. Then w I multiply 40 by 4, and it maketh 160, whiche I diuide by 13, and the quotient is $12\frac{4}{13}$.

4 Z 40
13

Mayster. Nowe tourne that $\frac{4}{13}$ into the common partes of a perche, as they bee named in the former Statute: howe be it, it shall be beste to take one of the leaste partes in Denomination for auoydinge of muche labour, as feete, whereof the perche contayneth $16\frac{1}{2}$.

Scholer. Then to tourne $\frac{4}{13}$ into feete, I Multiplye $16\frac{1}{2}$ by 4, and it maketh 66, whiche I muste diuide by 13, and the quotient is $5\frac{1}{13}$.

Mayster. So I finde that if the acre houlde in breadthe xiij. perches, it shall contayne in lengthe 12 perches, 5 foote, and $\frac{1}{11}$ of a foote, whiche is not fully an ynche, for the ynche is $\frac{1}{12}$ of a foote. But heere all the statute bookes in Latine and Englishe (that I haue seene) we note it to be 13 perches, 5 foote and 1 inche: whiche maketh aboue 13 perches to manye in the acre, so that I woulde haue thoughte the

Note this error.

erroure

erroure to haue crepte into the prynted bokes by the greate negligence that prynters in oure time we vse, saue that in wrytten copies of great antiquitye, I we finde the same. Yet haue I one Frenche coppie, whiche hathe xij. perches ½, and one fote, and that milleth very little of the truthe.

Scholer. Then I see it is true that I haue often heardе saye, that the truest copies of the statutes, be the Frenche copies.

Mayster. That is often trewe, but not generally, as I haue by conference tried diuersly: but in this statute the Frenche boke is moste corrupt in all other places lightlye.

But nowe to performe my promise, I will set forthe the Table for measuringe of an Acre of grounde onelye by suche partes as the Statute wthe mention, bicause at this time I doe of purpose write it for the better vnderstanding of the Statute, and hereafter with other thinges I intende to set forthe this same more at large.

In this Table following I haue not done, as in the other statute before compared by restitution with the faultes crepte into the Statute, but onelye haue wrytten that true measure, whiche the equitye of the Statute dothe

pretende

pretende. For it were to vyle to iudge of so noble Princes and worthye Councelloures as haue auctorised and set forthe this statute, that they would make one acre in any forme greater then an other, but euery one to be iuste and equall with eche other, whiche is the grounde also of my worke, and herebye maye all men perceaue howe needefull Arithmetieke is vnto the Studentes of the lawe. But now I thinke beste to make an ende of these matters for this presente tyme, sithe the Table hathe in it none obscuritye, that I shoulde neede to declare.

The breadth.	The lengthe of the acre.		The breadth.	The lengthe of the acre.	
yerdz.	yerdz.	feete.	yerdz.	yerdz.	feete.
10	16	0	28	5	11 11/14
11	14	9	29	5	8 21/58
12	13	5 1/2	30	5	5 1/2
13	12	5 1/13	31	5	2 47/62
14	11	7 1/14	32	5	0
15	10	11	33	4	14
16	10	0	34	4	11 17/17
17	9	6 27/34	35	4	9 3/7
18	8	14 2/3	36	4	7 1/3
19	8	6 18/19	37	4	5 13/17
20	8	0	38	4	3 9/19
21	7	10 3/14	39	4	1 9/13
22	7	4 1/2	40	4	0
23	6	15 18/23	41	3	14 72/82
24	6	11	42	3	13 15/42
25	6	6 2/5	43	3	11 27/86
26	6	2 7/13	44	3	10 1/2
27	5	15 5/18	45	3	9 1/5

Scholer.

Scholer. In deede Sir, I vnderstand the Table (as I thinke) by those other which you set forthe before. For in the firste columne is set the perches of the breadthe of anye Acre, and then in the ij. columnes followinge appeareth howe many perches and howe manye wte the same Acre muste haue for his lengthe.

Mayster. You take it well: how be it to speake exactly of breadthe and length, the firste columne dothe sometime betoken the breadthe, and sometime the length, for properly the longest side of any square dothe limit his length, and the shorter side doth betoken the breadthe: yet it is no greate abuse in suche tables, where a man can not well chaunge the title, to let the name remayne, although the proportions of the numbres we chaunge: for still by the firste colūne, is expressed the measure of the one side, and by the two other pillers in one columne, is set forth the measure of the other side. And this shall be sufficient nowe for the vse of the Golden rule.

Nowe somewhat wyll I touche certaine other rules, which for their seuerall names maye seeme diuerse rules and distincte from

from this, but in deede they are but braundes of it: yet bicause they haue not onely seuerall worckinges in appearaunce, but also pleasante in vse, I will giue you à taste of eche of them. As for the rule of felowshippe, bothe single and double, with tyme and without tyme, I shall neede to say little more than I haue already sayde in teachinge the workes of whole numbres, yet an example or two will wee haue to refreshe the remembrance of the same, and to declare certayne proper vses and applications of it, as this for one.

A question of vnequall societie.

Foure men gette à booty or prise in time of warre, the prise is in valew of mony 8190 li, and bicause the mē be not of like degree, therefore their shares maye not bee equall, but the chiefest person will haue of the bootye the thirde parte, and the tenthe parte ouer: the seconde will haue à quarter and the tenthe parte ouer: the third will haue the sixt parte: and so there is left for the fourthe man à verye small portion, but suche is his lot, (whether hee be pleased or wroth) he must bee contente with one xx. parte of the praye. Nowe I demaunde of you, what shall euerye man haue to his share?

Scholer. You muste be fayne to answere

to

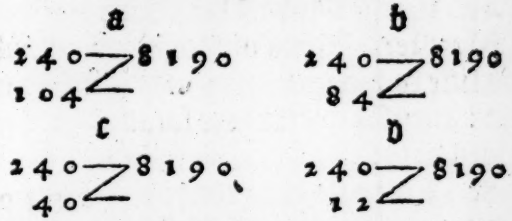
to your owne question, els it is not like to bee aunswered at this time.

Mayster. The towme to vnderstande the solution of this question, and all suche like, is this: Reduce all the Denominators into one numbre by Multiplication, except that anye of them be partes of some other of them, for all suche partes you may ouerpasse, and take for them all those nūbres, whose partes they be: as in this example ỹ shares be these $\frac{1}{3}$ $\frac{1}{10}$, $\frac{1}{4}$ $\frac{1}{10}$, $\frac{1}{6}$ $\frac{1}{20}$, if I multiply all the Denominatours together, beginning with 3, and so go on vnto 20, it will make 144000: but considering that 3 is à parte of 6, I shall omit that 3, and likewaies 10 whiche is à parte of 20 I may ouerpasse also, and then is there but 3 denominatours to multiply, that is 4, 6, and 20, whiche make 240, whiche summe I take for my woorke, bycause all the denominatours will be found in it. Then I take suche partes of it as the question importeth, that is for the firste man, $\frac{1}{3}$ $\frac{1}{10}$, the $\frac{1}{3}$ is 80: the $\frac{1}{10}$ is 24: whiche I put in one summe for the firste mannes share, and it maketh 104. Then for the second mans share, I take $\frac{1}{4}$, whiche is 60, and $\frac{1}{10}$ whiche is 24, and that maketh in the whole 84. Now for the thirde

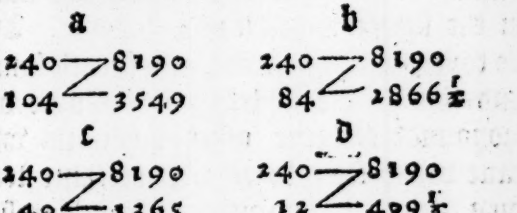
man whiche muste haue $\frac{1}{7}$ I take 40. And for the fourth man there remayneth but 12, whiche is $\frac{1}{20}$ of the whole summe: so that if the whole pray had been but 240 li, then weare the question answered: but bicause the summe was of greater value, by this meanes nowe shall I knowe the partition of it. I muste set my numbres by the ordre of the Golden rule, puttinge in the firste place the numbre that I founde by multiplyinge the Denominatours, and in the second place the summe of the booty. And loke what proportion is betweene that firste numbre and that seconde, the same proportion shall bee betweene the partes of the firste numbre and the partes of the seconde, comparing eche to his like. Therefore I muste put in the thirde place, one of the partes or shares, and then worke by the former rule of proportion or Golden rule. And bycause I haue 4 severall partes of the firste numbre, by whiche I would finde out 4 like partes of the second numbre, therfore muste I make 4 severall figures.

The reason of this rule.

Scholer. Nowe I truste I can aunswere to your question, as by youre fauoure I will proue.



And to trie it, I set the 4 figures thus, marked with a, b, c, d, to shewe their order. And then in eche of them I multiplye the seconde number by the thirde, and diuide their totall by the firste, & so amounteth the fourthe summe whiche I seeke for, for if I do multiply 8190 by 104, it maketh 851760, whiche beinge diuided by 240, maketh in the quotient 3549 for the firste mans portion. And so worckinge with the other three figures, I finde for the seconde man 2866½, and for the thirde man 1365: and then for the fourthe man 409½. And so is euery mans share set forth in the figure here annexed.



And thus I thinke I haue done well.

Mayster. If you misdoubt your worcking and liste to proue it, adde all the shares togither: and if they make the totall, then seemeth it well done.

The proofe by Addition.

Sc. I may set them thus: and then by Addition the iuste summe dothe amounte, that is 8190, and therefore (as you saye) it seemeth to be well wrought.

3549	
2866	1/2
1365	
409	1/2
8190	

But I beseche you, is there anie doubt in this triall, that you vse that worde, Seemeth?

Mayster. You may easily coniecture, that if you did assigne the firste mans share to the laste, & so chaunge all the rest, that one had an others share, yet woulde the Addition appeare all one, and therefore is not the proofe exact.

The iuste proofe.

But if you will make a iuste proofe, for the firste mannes parte take 1/9 1/10 of the whole summe, and if it agree with the number in the figure, then it is well done. And so do for the seconde, thirde, and fourth summes, and this proofe fayleth not. Nowe will I propounde certaine other questions whiche haue beene sette foorthe by certaine learned men, all be it not withoute some ouersighte, whiche

whiche questions I protest hartily I do not repeate to depraue those good men, whose labours and studies I muche praise and greately delyght in, but onely accordinge to my profession, to seeke out truthe in all things, and to remoue all occasions of errour, as muche as in me lyeth: and for that cause I will onelye name the questions without hurtinge the Authours name. The first question is this.

A questiō of buildinge.

Foure men did builde a house, whiche coste them 3000 Crounes, their shares were suche, that one man shoulde paye $\frac{1}{2}$ of the summe, and 6 crownes ouer: the seconde should pay $\frac{1}{3}$, and 12 crownes ouer: the third man must lay out $\frac{2}{7}$, abatinge 8 crounes, and the fourth manne shoulde pay $\frac{1}{8}$, and 20 crownes more, can you aunswere to this question?

Scholer. No in good faithe sir, and that you know beste of any man, for I knowe no more then you haue taught mee.

An impossible question.

Mayster. Then I dare saye you can not do it, nother yet ỹ beste learned man that euer did propose it, for the question is impossible: for declaration whereof I will be bolde to vse suche the representation of the numbers in their aptest fourme, (although I haue not yet taughte you that manner of worke) by-

cause it may appeare plainely that the question is not possible, for here I haue set the partes, and added them, and they make the hole summe and $\frac{3}{4}$, and 30 more. Nowe how is it possible to diuide truly eyther gaynes eyther charges so, that the particulars shall be more then the totall?

$$\left\{\begin{array}{rcr} \frac{1}{2} & + & 6 \\ \frac{1}{3} & + & 12 \\ \frac{2}{3} & - & 8 \\ \frac{1}{4} & + & 20 \\ \hline 1\frac{3}{4} & + & 30 \end{array}\right\}$$

Scholer. It is agaynste the forme of proofe by Addition of the partes.

Mayster. You say truthe. And bicause you shall perceaue it the better, I will trye it after the vulgare forme, as in this figure you see, where the $\frac{1}{2}$ with 6 ouer, is 1506: for the totall is as you hearde before 3000: the $\frac{1}{3}$ and the 12 more, is 1012: the $\frac{2}{3}$ woulde be 2000, but then abating 8, it is but 1992, and then laste of all, the $\frac{1}{4}$ is 750, and the 20 more maketh 770: whiche all being added in one summe, do make 5280, where ẏ totall sum should be but 3000, which sum if you diuide by $\frac{8}{3}$, so shall you haue $\frac{3}{4}$ of it, that is 2250, & thereto adde 30 more, then will

$$\begin{array}{r} 1506 \\ 1012 \\ 6992 \\ 770 \\ \hline 5280 \end{array}$$

will those 3 sūmes make 5280:
whereby you may see howe this
forme as well as the other, wthe
declare that the particulars in
that question woulde make
more then the whole summe by

3000	
2250	
30	
5280	

$\frac{1}{20}$, and 30 more: and therefore can that question not be accepted as à possible thinge, but yet doe certayne learned menne propounde suche questions, and aunswere to them. Therfore somewhat to saye to their excuse rather of their good meanynge then for theire doinge, I will anone declare what may be said for theire defence: but in the meane season I will propounde the question as it maye bee wroughte by good possibilitye. As if foure menne buylde à house togither, and it coste them 3000, and then for the particion they agree thus: that as often as the firste manne doothe paye 6 crownes, so often the seconde shall paye 4, the thirde man 8, and the fourthe man 3. Or els thus: that the firste man shall pay double so muche as the iiij. and the second man shall paye $\frac{2}{3}$ of the firste mans charge: the thirde man shall paye double so muche as the seconde: (And these two wayes are to one ende) but farther for their agreement it is apointed also, that the firste man shall giue 6

crownes ouerplus, and the seconde 12, and the iiij. shall geue 20, but the third man shal giue no ouerplus, but shall haue 8 crownes abated of his charge. Now is the question possible to be soyled, and this is the way to wo it. Marke the proportion of the seuerall charges, and set out small numbres in that rate, by which you may reduce the worke to the Golden rule, as here in the first forme, the numbres are already named, 6, 4, 8, 3: and in the seconde forme, although they bee not playnly named, yet they may be the same numbres: for 6 is double to 3, and 4 is $\frac{2}{3}$ of 6: and agayne 8 is double to 4. Now adde these togither, and they make 21, which 21 muste be set as the firste numbre in the Golden rule: for if it with the ouerplus of ech mannes charge woulde make the totall summe of the charges, then were those seuerall summes, the charges of ech man, beside hys ouerplus: but nowe it is not so.

The rule. But yet this is trewe, that loke what proportion ech of these seuerall summes dothe beare to 21, the same proportion wthe the iuste charges of euerye man (beside his ouerplus) beare to the totall of the charges, the ouerplus beyng deducted: wherfore this may you note, that before you w apply the totall of the charges

ges to the Golden rule, you must deducte the ouerplus whiche is 6, 12, and 20, that is in the whole 38: but then 8 must bee restored for the abatement of the thirde man, and than remayneth to be deducted 30. Take 30 therefore out of 3000, and there will reste 2970, whiche I must sette in the Golden rule for the seconde summe: and for the thirde summe I must put eche of the small numbres before mẽtioned, whiche although they bee not the seuerall charges, yet they represent them in proportion. And so makinge for euerye mannes charge à seuerall question, the figures will bee 4, whiche I marke with foure letters, a.b.c.d, thus.

a.

21 — 2970
6 — 848 4/7

b.

21 — 2970
4 — 565 5/7

c.

21 — 2970
8 — 1131 1/7

d.

21 — 2970
3 — 424 2/7

where I haue set for briefnes the summe of euery mãs charge in the fourth place, presuppsinge that you can tell howe to trye out that fourthe summe by so manye exaumples as yee haue had.

Scholer. As I truste that I doe vnderstande this forme, so I desire muche to knowe what may be saide for them that misstoke this question.

Mayster. You seeme so desirous to knowe this errour, that you haue forgotten to examine whether this worke be without errour.

Scholer. Mee seemeth this woorke to bee well done, bycause the Addition of the 4 seuerall numbres dothe make the totall summe of 2970, whiche was to be diuided into suche fourepartes.

Mayster. But then haue you forgotten that the firste man muste paye 6 Crownes more besides this share, and the seconde man 12 crownes more: the thirde man 8 Crownes lesse: and the fourth man 20 Crownes more, for without these, youre firste totall of 3000 crownes will not be made.

Scholer. Then muste I adde to the firste mannes summe 6 more, and it will be 854 $\frac{4}{7}$: and to the second sum I muste adde 12, and it will be 577 $\frac{5}{7}$: from the third summe I must abate 8, and then will the sum bee 1123 $\frac{3}{7}$. then adding vnto the iiij. summe 20, it will be 444 $\frac{2}{7}$: and these 4 sūmes will make 3000, whiche is the whole charge, as in this example

ample it may appeare, where firſte I gather the $\frac{14}{7}$, that maketh 2, and ſo proceede I in the Addition to the ende.

	854 $\frac{4}{7}$
	577 $\frac{5}{7}$
	1123 $\frac{3}{7}$
	444 $\frac{2}{7}$
	3000

Mayſter. Now haue you well done, & this worke in the ſame ſummes is brought of other learned men for the true ſolution of the queſtion as it was firſte propoſed, whiche (as I ſaide) was impoſſible: and nowe examine it by theſe ſeuerall ſummes, and ſee whether it do agree with the ſummes in the queſtion propoſed.

The firſte man muſte paye $\frac{1}{2}$ and 6 ouer of the totall ſumme: howe thinke you, is 854 $\frac{4}{7}$ the halfe and 6 more of 3000?

Scholer. No that is it not, for it woulde be 1506: and for the ſecond man 1012: and for the thirde man 1992: and for the fourthe man 770, whereof not one ſumme agreeth to this worke. But I maruaile that ſo wiſe men coulde be ſo much ouerſeene.

Mayſter. It is commonlye ſeene, that when men will receaue thinges from elder wryters, and will not examyne the thinge, they ſeeme rather willinge to erre with their auncientes for company, then to bee bolde to examine their workes or wrytinges, whiche

ſcrupuloſitty

ſcrupuloſitie hathe ingendred infinite errours in all kindes of knowledge, and in all ciuile adminiſtration, and in euerye kinde of arte: but theſe learned menne did not meane anye other thinge by this queſtion, then to fynde ſuche numbers as ſhoulde beare the ſame proportion togither, as thoſe numbers in the queſtion proponed dyd beare one to an other: whiche thinge you ſhall perceaue more plainelye by an other queſtion of theirs, that is this.

A queſtiō of a teſtament.

A man lyinge vpon his deathe bed, bequeatheth his goodes (whiche were worthe 3600 Crownes) in this ſorte. Bicauſe his wyfe was greate with childe, and hee yet vncertaine whether the childe were a male or a female, hee made his bequeſt conditionally, that if his wife bare a daughter, then ſhoulde the wyfe haue halfe his goodes, and the daughter $\frac{1}{3}$, but if ſhe were deliuered of a ſonne, thē that ſonne ſhoulde haue $\frac{1}{2}$ of the goodes, and his wife but $\frac{1}{3}$. Now it chaunced her to bringe forthe bothe a ſonne and a daughter, the queſtion is: How ſhall they parte the goodes agreable to the teſtatour his will.

Scholer. If ſomme cunninge Lawyers had this matter in ſcanninge, they woulde determine

termine this Testament to be quite voide, and so the man to dye vntestate, bicause the testament was made vnsufficient, sithe this condition was not expressed in it, and also it might haue chaunced that shee shoulde haue brought foorthe neyther sonne nor daughter, as often hath bene seene, so is the will vnsufficiente in that poynte also.

Mayster. Such scanners shoulde seeme to cunning, and yet not so cunning as cruell: for the mind of the Testatour is to be taken fauorably, for the ayde of the legatories, when there ryseth such doubtes. But let vs try this worke, not by force of lawe, but by proportion Geometricall, seeinge the Testatour did minde to prouide for eche sorte of theim.

Scholer. If the sonne shall haue $\frac{1}{2}$ by force of the Testament, so muste the mother haue $\frac{1}{3}$. Againe bicause she hath à daughter also, therfore ought shee to haue $\frac{1}{2}$, and the daughter $\frac{1}{3}$: that is bothe wayes $\frac{1}{2}$ $\frac{1}{3}$ and $\frac{1}{2}$ $\frac{1}{3}$, whiche commeth to the whole gowdes, and $\frac{2}{3}$ more. Wherfore it seemeth also impossible.

Mayster. In this matter the minde of the Testatour is so to be vnderstãd, that such proportion shoulde bee betweene the portion of the wyfe and the sonne, as is betweene $\frac{1}{2}$

and

and $\frac{2}{1}$, that is, the sonne muste haue $\frac{1}{2}$ for $\frac{2}{2}$ to his mother, so shall hee haue 3 to 2, that is as muche as his mother, and halfe as muche more: and the mother muste haue the lyke rate in comparison to her daughter. Then muste I finde out 3 numbres in suche proportion, that the firste maye bee as muche as the seconde, and halfe as muche more (that is) in proportion *sesquialtera*, and the seconde to the thirde in the same proportion, suche numbres be 9,6,4.

Scholer. I pray you sir, how shall I finde out those numbres?

To finde three nũbers in any proportion.

Maister. That will I gladlye tell you. Whatsoeuer the proportion bee for anye three numbers, multiply the Termes of that proportion togither, and the number that amounteth, shalbe the middle number of the 3: then multiply that middle number by the lesser Terme, and diuide that totall by the greater, and the least number of the 3 will amount. So yf you multiplye that middle number by the greater extreme, and diuide that totall by the lesser extreme, then will the greatest number of that progression amount.

To finde the proportion

Scholer. Then in this exaumple, to finde the proportion of $\frac{1}{2}$ to $\frac{1}{3}$, I muste diuide (as you

you taught me in Diuision) $\frac{1}{2}$ by $\frac{1}{3}$, and the quotient will bee $\frac{1}{2}$, that is 1 $\frac{1}{2}$, whereby I perceaue that the proportion in this question is, as 3 to 2. Therefore (as you taught mee euen nowe) I multiply 3 by 2, & the summe is 6, whiche must bee the middle number: then I multiply the middle number 6 by 2, whiche is the least Terme, and the summe is 12, that me I diuide by 3, beynge the greater Terme, and the quotient is 4, so is 4 the least number of the 3. Then I multiplye 6 by 3, whereof commeth 18, and that I diuide by 2, and so haue I 9, whiche is the greatest number of the 3. betweene two numbers.

Mayster. An other way yet may you finde the thirde number in anye progression, if you haue 2 of them: for if the middle number bee one of them whiche you haue, then multiplye it by it selfe (as in this exaumple 6 by 6 maketh 36) and that totall diuide by the other number whiche you haue, and the thirde number will be the quotient.

Scholer. Then if I diuide 36 (whiche cōmeth of 6 multiplyed by it selfe) by 4, the quotient will be 9: and if I diuide 36 by 9, the quotient will be 4. But what if I knowe the first number & the thirde, and woulde haue the middle

middle numbre?

Mayster. Multiply the ij, numbers togither, and in their totall you muſt ſeeke ẏ roote of that number, & it ſhall bee the middle numbre: but bycauſe as yet you haue not learned howe to extracte rootes, therefore vſe the firſte forme whiche I haue taught you, till I teache you to extract rootes. And now goe forewarde with the aunſwere to the ſame queſtion.

Scholer. I perceaue then that the ſonne muſte not haue $\frac{1}{2}$ of the goods, nother ẏ mother $\frac{1}{3}$, nor yet the daughter $\frac{1}{2}$, but yet muſte the goods be diuided in ſuche proportion, that the ſonne ſhall haue 9 crownes for 6 to his mother: and the mother ſhall haue 6 crownes for euery 4 to the daughter. Then I applye it to the Golden rule in 3 examples thus: where the firſte number is the Addition of thoſe three nũbers, 9, 6, 4, and the thirde is one of them ſeuerally: the ſeconde is the totall of the goodes in the teſtament: & then by the worke of the Golden rule I finde out ẏ fourth numbre in euery worke, that is for the ſonne

19 Z 3600
9

19 Z 3600
6

19 Z 3600
4

sonne 1705 $\frac{5}{19}$: for the mother 1136 $\frac{16}{19}$: and for the daughter 757, $\frac{17}{19}$, which three summes added togither we make the sum of the whole goodes, as may bee seene by this example.

1705	$\frac{5}{19}$
1136	$\frac{16}{19}$
757	$\frac{17}{19}$
3600	

And this (me thinketh) I we perceaue, that bicause in this case there is à necessary remedy diuided againste an vrgent inconuenience, therfore those learned men thought they might vse the like liberty in that other question.

Mayster. Your gesse is good, but they had so good reason for them in the one, as they haue in the other: as in an other example of theirs it may better appeare, that is this.

A man left vnto his iij. sonnes 7851 crounes, to be parted in this sorte, that the firste sonne should haue $\frac{1}{2}$, the second sonne $\frac{1}{3}$, and the thirde sonne $\frac{1}{4}$ which is not possible, for $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ dothe make $\frac{20}{24}$: or $\frac{13}{12}$, that is 1 $\frac{1}{12}$, so is it more then the whole: but reduce these fractions into one denomination, and they will be $\frac{6}{12}$, $\frac{4}{12}$, $\frac{3}{12}$, and so may you parte the goods in such proportion as these 3 Numeratours beare togither: that is, the firste to haue 6 for euery 4 to the seconde: and the seconde to haue 4 as often as the thirde hath 3: and so theyr

An other question of a Testament.

portions will be for the first, $3623\frac{7}{13}$: for the second $2415\frac{9}{13}$: and for the thirde $1811\frac{10}{13}$, and those iij. shares added togither will make the totall sum of the whole goodes, as you may easily see in this example. An other question is there proponed thus.

$$\begin{array}{r} 3623\frac{7}{13} \\ 2415\frac{9}{13} \\ 1811\frac{10}{13} \\ \hline 7851 \end{array}$$

An other like question.

There is 450 Crownes to be diuided betwene 3 men, so that the firste man must haue $\frac{1}{2}\frac{1}{3}$, the second man $\frac{1}{3}\frac{1}{4}$, the thirde man shall haue $\frac{1}{4}\frac{1}{5}$.

Scholer. I maruaile that any man should be so ouerseene to propounde that question as à thing possible, sithe $\frac{1}{2}\frac{1}{3}$, $\frac{1}{3}\frac{1}{4}$, $\frac{1}{4}\frac{1}{5}$, we make $1\frac{1}{3}\frac{1}{5}$, that is almost double the whole summe.

☞ But I perceaue it might be thus proponed, that as often as the firste man did receaue 50 Crownes, so often the seconde man should receaue 35, and the thirde man 27, for $\frac{1}{2}\frac{1}{3}$, is equall to $\frac{50}{60}$, and so is $\frac{1}{3}\frac{1}{4}$ equall to $\frac{15}{60}$, and $\frac{1}{4}\frac{1}{5}$ is $\frac{27}{60}$, and so workinge the question, the 3 figures will appeare in this forme: whereby the firste mannes portion is founde to be $200\frac{50}{56}$: the seconde mannes part

$$\begin{array}{rcl} 112 & \Sigma & 450 \\ 59 & & 200\frac{50}{56} \end{array}$$

is

is 140 $\frac{35}{56}$: the thirde mans share is 108 $\frac{24}{76}$: which in ye whole do the make 450 crounes, that was ye whole summe to be diuided betweene them.

112	450	
35	140 $\frac{35}{56}$	
112	450	
27	108 $\frac{27}{50}$	

Mayster. And thus you are (I thinke) sufficiently instructed in the rule of felowshippe.

THE RVLE OF ALLIGATION.

NOwe will I go in hande wish the rule of Alligation, whiche hathe his name, for that by it there are diuers parcels of sundry prices, and sundry quantities alligate, bounde or mixed together, wherby also it might be well called the rule of mixture, and it hathe great vse in composition of medicines, and also in mixtures of metales, and some vse it hathe in mixtures of wines, but I wishe it were lesse vsed therein than it is now a dayes. The ordre of the rule is this: When any summes are proponed to be mixed set them in ordre one ouer an other, and the common numbre whereunto you will reduce them,

The rule of Mixture.

The reason of this rule.

them, set on the lefte hande, then marke what summes be lesser then that common numbre, & whiche be greater, and with a draught of youre penne, euermore linke ij. numbres togither, so that one be lesser then the cõmon numbre, and the other greater then hee, for two greater or two smaller can not well bee linked togither, and the reasõ is this, that one greater and one smaller may bee so mixed, that they will make the meane or common numbre verye well: but ij. lesser can neuer make so many as the common numbre, beynge taken ordrelye: no more can two summes greater then the meane, neuer make the meane in one ordre, as it shal appeare better to you hereafter. And as it is of necessitye to linke euerye smaller (once at the leaste) with one greater, and euerye greater with one smaller, so it is at libertye to linke them oftener then ones, and so maye there bee to one question manye solutions. When you haue so linked them, then marke howe muche eche of the lesser numbres is smaller then the meane or common numbre, and that difference set againste the greater nũbres whiche bee linked with those smaller, eche with his matche still on the right hande, & likewayes the excesse of the greater numbres aboue the meane, you

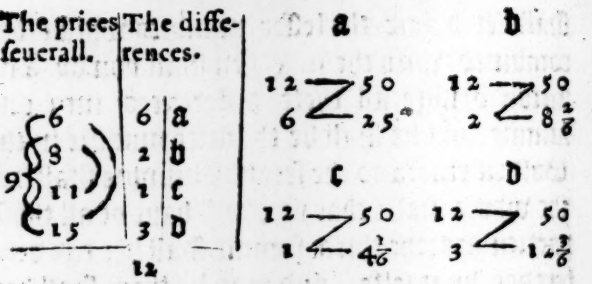
fy all

shall set before the lesser numbres whiche bee combined with them. Then shall you by Addition bringe all these differences into one summe, whiche shall be the firste numbre in the Golden rule: and the seconde numbre shall bee the whole masse that you will haue of all those particulars: the thirde summe shall bee eche difference by it selfe, and than by them shall bee founde the fourthe numbre, declaringe the iuste portion of euerye particular in that mixture. As nowe by these exaumples I will make it playne.

There is foure sortes of wyne of seuerall prices, one of 6 d, a galon, an other of 8 d, the thirde of 11 d, and the fourthe of 15 pence the galon, of all these wines would I haue a mixture made to the summe of fifty gallons, and so that the price of eche gallon may be ix. pence. Now demaunde I howe muche masse bee taken of euery sorte of wyne? A questiō of mixing of Wine.

Scholer. If it shall please you to worke the firste exaumple, that I maye marke the applyinge of it to the rule, then I truste I shall bee able not onely to we the like, but also to see reason in the order of the worke.

Mayster. Marke then this forme, and the placing of euery kinde of numbre in it.



Heere (you see) I haue set downe the seuerall prices whiche bee 6, 8, 11, 15, and haue linked togither 6 with 15, and 8 with 11. The common price 9, I haue set on the lefte side: And the difference betweene it, and euery particular price, I haue sette on the righte hande, not agaynst the summe, whose difference it is, but agaynst the summe that it is linked withall: so the difference of 15 aboue 9, is 6, whiche I haue set not agaynst 15, but agaynst 6, that is linked with 15, and the difference betweene 6 and 9 (that is 3) I haue set agaynste 15. So lykewayes the differenee betweene 8 and 9 is but 1, that haue I set agaynste 11, and the difference of 11 aboue 9 (whiche is 2) I haue set agaynste 8. Then adde I all those 4 differences, and they make 12, whiche I sette for the firste numbre in the Golden rule: the seconde numbre I make 50, whiche is the summe

of

of the gallonnes that I woulde haue, and the thirde summe is euerye particular difference. Nowe if you worke by the Golden rule, you shall finde the number of gallons that shall be taken of eche sorte of wyne: For the better distinction whereof, I haue sette these letters, a, b, c, d, bothe against the numbers for which the workes we serue, and ouer the workes also, which severallye serue for eche of them. And nowe if you liste to examine the truthe of these workes, adde those iiij. summes togither, and they will make 50, that is the totall whiche I woulde haue, as by this exaumple you may easilye perceaue. And for to proue how the prices we agree, do this. Multiplye this totall summe 50, by the common price 9, and it will make 450: than keepe that summe by it selfe, and afterwarde Multiplye euerye severall summe of Gallons, by the price belonginge to the same Gallons, and if that summe doe agree with this, which you haue kepte firste, then is your worke well wne. As here, 25 is the number of gallons of 6 d price, mul-

The proofe of this rule.

a	25
b	$8\frac{2}{6}$
c	$4\frac{1}{6}$
d	$12\frac{3}{6}$
	50

tiplye then 25 by 6, and it maketh 150, whiche you shall set downe: then multiply $8\frac{2}{6}$ by 8, whiche is the price for that numbre of galons, and it will make $66\frac{4}{6}$: so agayne $4\frac{1}{6}$ multiplyed by 11, wthe make $45\frac{5}{6}$. And laste of all $12\frac{3}{6}$ multiplyed by 15, maketh $187\frac{3}{6}$. And these added together dothe make 450, as in ẏ example annexed you may see: wherfore seing it wth agree with ẏ former summe of 50, multiplied by 9, I may iustly affirme this worke to be good and well wne.

$$\begin{array}{r} 150 \\ 66\frac{4}{6} \\ 45\frac{5}{6} \\ 187\frac{3}{6} \\ \hline 450 \end{array}$$

The variation of this question.

And now to prove how you can we ẏ like, I propounde the same question, onely willinge you to vse some other forme of combininge or linking the summes.

Scholer. That shall I prove with your fauoure, and therefore I combine 8 with 15, and 6 with 11, and then the forme will be thus.

whereby

whereby amounteth the same summe in totall of the differences, as did before: and yet now the differences be altered, as the combination is chaunged, whereof I vnderstande the reason by youre fourmer woorke. And therefore heere appeareth no straunge thinge, but that nowe I must haue $8\frac{2}{6}$ galons, of 6 pence, and 25 galons of 8 ō. and 12 galons and $\frac{3}{6}$ of 11 ō, and so consequently 4 galons and $\frac{1}{6}$ of 15 ō, so that multiplying $8\frac{2}{6}$ by 6, it maketh 50, and then 25 multiplyed by 8, maketh 200: likewaies $12\frac{3}{6}$ multiplied by 11, yeldeth $137\frac{3}{6}$, and $4\frac{1}{6}$ multiplyed by 15, maketh $62\frac{1}{6}$, whiche 4 summes added in one, will yelde in the totall 450, whiche agreeth with the Multiplication of 50 (being the totall summe of gallons) by 9 the common or meane price.

	50
	200
	$137\frac{3}{6}$
	$62\frac{1}{6}$
	450

Mayster. Seing you conceaue this woorke so well, I will propounde an other example vnto you of more varietie in the Alligations or combinings: Is thus.

A queſtiō of ſpices.

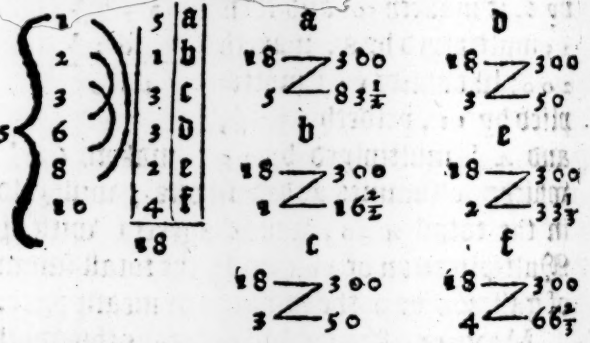
I Marchaunte beinge minded to make a bargaine for spices in a mixt masse, that is to saye, of Cloues, Nutmegges, saffron, pper, Ginger,

Ginger, and Almondes, the cloues beinge at 6 ℔. a pounde, the Nutmegges at 8 ℔. Saffron at 10 ℔. Peper at 3 ℔. Ginger at 2 ℔. and Almondes at 1 ℔.

Now would he haue of ech sorte some, to the valew of 300 li, in the whole, and eche pound one with an other to beare in price 5 ℔, how muche shall he haue of ech sorte?

Scholer. That will I trie thus.

First I set downe those sixe seuerall prices, and at the lefte hand



I set the common price 5 ℔. Then I lynke them thus, 1 with 10, 2 with 6, and 3 with 8.

Mayster. I had minded to haue combined them in more varietie, but I am content to see your owne worke firste, and then more varieties in combination may follow anone.

Scholer.

Scholer. Then to continewe as I began, I seeke the difference betweene 1 and 5 (whiche is 4) and that I set againste 10: then agaynst 1 I set 5, whiche is the excesse of 10 aboue 5: so I gather the difference betweene 2 and 5, whiche is 3, and that I set agaynste 6, bicause it is combined with 2: and likewayes the difference of 6 aboue 5 (whiche is 1) I set againste 2. Then take I the difference of 3 from 5, whiche is 2, and that I set againste 8, and before that 3 I set the difference of 8 aboue 5, whiche is 3. Then gather I all these differences by Addition, and they make 18, whiche I set for my firste numbre in the Golden rule, and so appeareth by those workes, that of Almondes I must take 83 li $\frac{1}{3}$, of Ginger 16 li $\frac{2}{3}$, of pepper 50 li, of Cloues 50 pounde, of Nutmegges 33 pounde $\frac{1}{3}$, of Saffron 66 pounde $\frac{2}{3}$.

Then for tryall hereof, I multiply euery parcelle by his seuerall price, as 83 $\frac{1}{3}$, which is the summe of Almondes, I multiplye by 1, whiche is theire price.

Also 16 $\frac{2}{3}$ the summe of Gynger, I multiplye by 2,

83	$\frac{1}{3}$
33	$\frac{1}{3}$
150	
300	
266	$\frac{2}{3}$
666	$\frac{2}{3}$
1500	

whiche

whiche is the price of it. And so eche other in his kinde, as this table annexed dothe represente: and then addinge them altogether, I finde the totall to be 1500, whiche also will amounte by the multiplication of the grosse masse of 300 by the common pryce 5, wherefore it appeareth well wroughte.

Mayster. Nowe will I make the Alligation to proue youre cunning somewhat better, but bycause you shall not thinke youre selfe pressed to muche, I will also note the differences, as in this example you may see.

1	1.3	4
2	3.5	8
3	5	5
6	4	4
8	4.3	7
10	3.2	5
		33

a

33 — 300
4 — $36\frac{4}{11}$

b

33 — 300
8 — $27\frac{8}{11}$

c

33 — 300
5 — $45\frac{5}{11}$

d

33 — 300
4 — $36\frac{4}{11}$

e

33 — 300
7 — $63\frac{7}{11}$

f

33 — 300
5 — $45\frac{5}{11}$

where I haue alligate 1 with 6 and 8: and therefore haue I set agaynst 1, bothe theire differences: that is 1 and 3. Likewayes bicause 2 is combined with 8 and 10, I set before him

theire

theire differences, 3 and 5. Againſt 3 I haue ſet onelye 5, whiche is the difference of 10, with whome 3 is combined onelye: lykewayes 6 is onelye Alligate to 1, and therefore is the difference of 1 onelye ſet againſte it: 8 is linked with 1 and 2, and therefore hathe hee againſte him their bothe differences 4 and 3: and 10 is ioyned with 2 and 3, therefore hathe he their differences 3 and 2. And becauſe of eaſe for you, in an other columne I haue ſet the differences reduced into one numbre, for euery ſeuerall ſorte, and haue alſo added them togither, whereby appeareth that they make 13, and ſo conſequently you ſee the workes of the Golden rule ſet forthe for the fyre ſeuerall drugges: I haue added letters, a, b, c, &c. as before. But I woulde not withe you to cleaue ſtill to theſe elementary aydes, but accuſtome memory to truſt to her ſelfe, ſo that occaſion of negligence bee beſte auoyded. And as for the proofe, trye it at a more leyſure, bycauſe ẏ time nowe is ſhorte, and you ſufficientlye inſtruſted in that proofe. And there reſteth diuers thinges behinde yet, of whiche I woulde gladly giue you ſome taſte before our departure.

Scholer. But if it maye pleaſe you to lette me ſee all ẏ variations of this queſtion, before

you

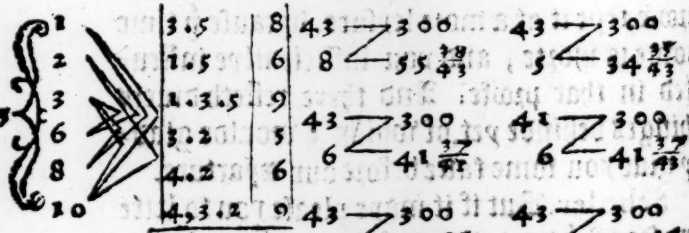
you go from it, for me thinketh I could varye it two or three wayes more yet.

Mayster. I am contente to see you make two or three variations, but I would be lothe to stay to see all the variations, for it may bee varied aboue 300 wayes althoughe manye of them would not well serue to this purpose.

Scholer. I thoughte it impossible to make so many variations.

Mayster. Marvayle not thereat, for some questions of this rule maye bee varyed aboue 1000 wayes, but I would haue you forget such fantasies, till a time of more leisure. And now goe forwarde with some variation of this question.

Scholer. For the firste variation, I linke the firste numbre 1 with 8 and 10, and 2 I combine with 6 and 10, then ioyne I 3 with 6, 8, and 10, as in this forme.

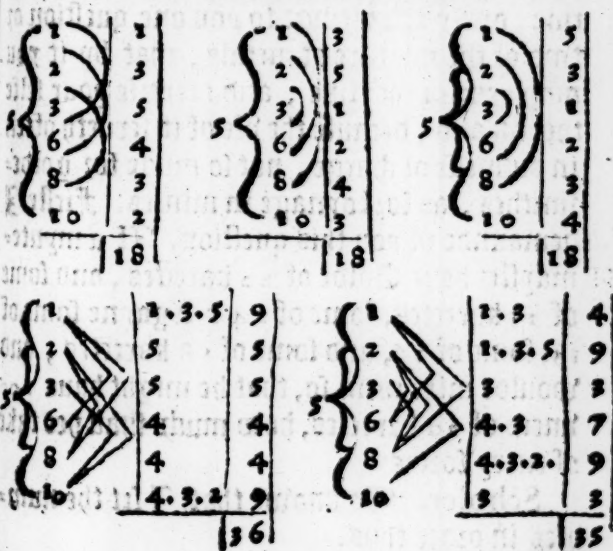


And

And so dothe there appeare the portion of weight for euery kinde of drugge in this mixture. Now for the triall.

Ma. May staye there, you shall not neede to make triall in one exãple so often, or if you list to we it by your selfe, I am content. But now set forth (for declaration that you conceaue the rule) two or three examples of seuerall combinations, and then will we passe to some other example, and so ende this rule.

Scholer. As it pleaseth you so will I doe. And these bee the varieties in whiche as the



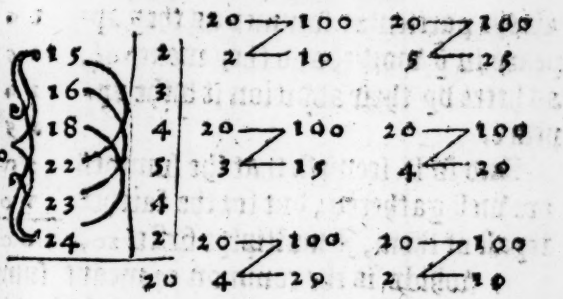
conti-

combinations are seuerall, so wthe it plainely appeare, that the differences by which the proportion of edr seuerall kinde is taken, are also seuerall. And yet I see in the iij. firste of these v. varieties, and in one other before the totall summe of the differences to be one, that is to say 18, whereby I perceaue that the varietie of theire mixture dothe depende of the varietie of their differences seuerall, and not of the varietie of their totall summe.

Mayster. So is it. And seeinge you conceaue it so well, I will make an ende of this rule, onely exhibitinge to you one question or two of the mixture of metals, that by it you may deuise other like, and exercise your selfe therein also, because the vse of it serueth often in business of charge, not so much for golde-smithes, as for coynage in mintes. Firste I demaunde of you this question. If a mynte-mayster haue Golde of 22 karectes, and some of 23 karectes, some of 24: Agayne some of 15, some of 16, and some of 18 karectes, and woulde mixe them so, that he might haue 100 vnces of 20 karectes, how muche shall hee take of euery sorte?

A questiō of mixing of Golde.

Scholer. To knowe that, I set the numbres in order thus.



Mayster. You haue wrought the question well, but how chaunced you made no doubt of that new name, Karecke?

Scholer. Bicause I thought it out of time to demaunde such questions now, seinge you make so much haste to ende: and againe in this case the proportion of the numbers is sufficiēt for my purpose in this worke, trusting that an other tyme you will instructe mee as well of this, as of sundry other thinges, which I haue heard you to talke of, so I haue a great desire to know them.

Mayster. Your aunswere is reasonable: and your request and trust with Gods helpe I intende to satisfie. And nowe to go forwarde with this matter, let me see your examination of this laste worke.

Sch. First for the one parte I adde together

all the particular summes as they appeare in ỳ worke, and they make 100, as heere by their addition it wthe appeare.

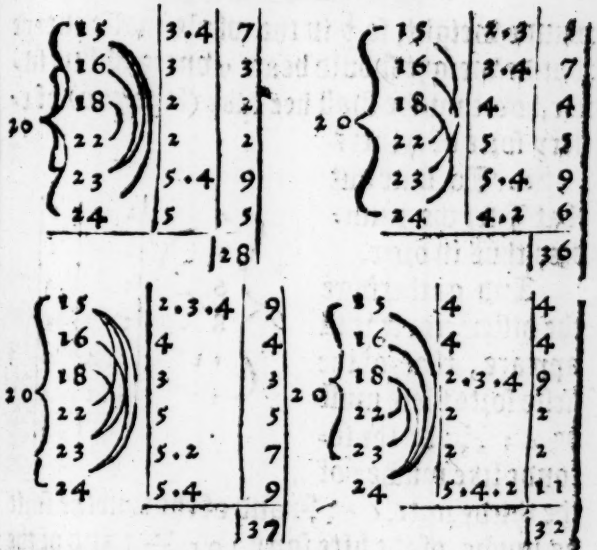
And so it seemeth that the summes are well gathered, but for the farther tryall of them, I multiplye firste 20, which is the common or meane summe of the karectes by 100, which is the summe of the whole masse which I would haue, and it maketh 2000. Thẽ I multiply euery particular sume by the karectes that it dothe contayne, as 10 by 15, and that maketh 150.

Likewise I multiplye 15 by 16, and it yeldeth 240: so 20 by 18 maketh 360. And 25 by 22 yeldeth 550: likewayes 20 by 23 bringeth forthe 460: and last of all 10 multiplyed by 24, yeldeth 240: whiche summes all ioyned togither make 2000. that dothe agree with the like summe before: wherefore I maye well saye that the worke is good. And now if it please you I wolde set forth some varieties of this question, to proue my witte.

10	150
15	240
20	360
25	550
20	460
10	240
100	2000

Mayster. Go to, let me see.

Scholer. Heere bee foure varieties.



And more yet I coulde make, but not lyke to the number that you spake of in the variation of the other question.

Mayster. That will I teache you at more leasure, seyng it is a thinge rather of pleasure, then of any necessitye.

But nowe for youre exercise in this rule, one other question I will propose. A minte mayster hathe 6 ingottes of siluer of sundrye fineneſſe, some of 4 vnces fine, and some of 5 vnces, some of 6, and other of 8, some of 11, and other of 12: and his desire is to mixe 500

pounde weight, so ℔ in the whole masse euerye pounde weight should beare 9 vnces of fine siluer, howe muche shall hee take (saye you) of euery sorte of syluer?

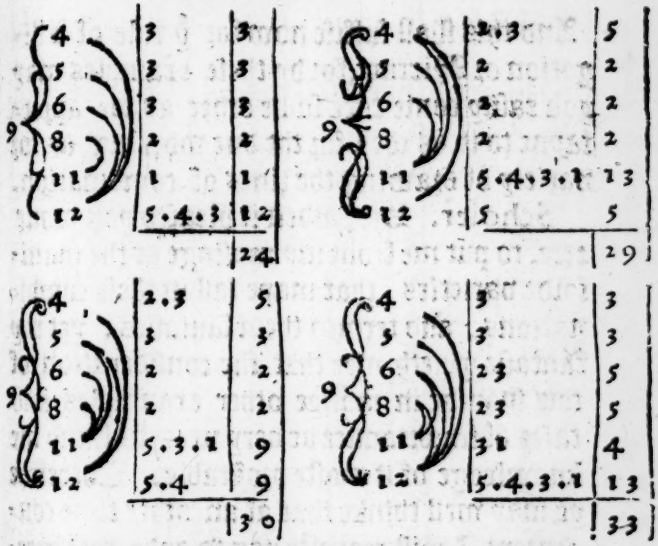
Sc. To finde out that I set the numbres thus in order.

4		2	2
5		2	2
6		3	3
8	9	3	3
11		5.4	9
12		3.1	4
			23

And gatheringe the differēces, it will appeare, that of the firste sorte there must be 43 $\frac{1}{23}$: of the seconde like muche: of the thirde sorte, 65 $\frac{5}{23}$: and of the fourthe sorte as muche, of the fifte sorte 195 $\frac{15}{23}$: and of the sixt sorte 86 $\frac{22}{23}$, whiche in ℔ whole will make 500 li waight: and in vnces 4500, that is of the first sorte 173 $\frac{21}{23}$: and of the seconde sorte 217 $\frac{9}{23}$: of the thirde sorte 391 $\frac{7}{23}$: of the fourthe sorte 521 $\frac{17}{23}$: of the fifte sorte 2152 $\frac{4}{23}$, and of the sixte sorte 1043 $\frac{21}{23}$, whiche all together we make 4500 vnces, agreable to the multiplication of 9 by 500.

Mayster. This is well wne of you, therfore nowe make three or foure varieties, and to an ende of this rule.

Sc. These 4 varieties I set for exaumple.



Mayster. And by these it appeareth that you can finde out more, wherein I will not now meddle, saue only for to shew you an easy helpe in drawinge the lines of Combination, I will set forthe two varieties heere.

And this shall suffise now for ỹ rule of Alligation or Mixture, for by these examples may you easily conjecture suche other as we apperteyne to it, as well for the due working, as for variety of drawing the lines of combination.

Scholer. Sir, albeit it pleased you while ere, to put me from my musinge at the manifolde varieties, that maye fall in these combinations, and termed them fantasies, yet my fantasie giueth mee that the consideration of this shoulde in manye other exaumples and cases of importaunce be very needefull, and the knowledge of it moste profitable. Therefore ye may well thinke that at an other time conuenient I will requeste you to ayde mee herein.

Mayster. Truthe it is, that this consideration may fall in practise as well politike, as philosophicall, and sundrye wayes in them bee applyed, therefore when time shall fall fitte for the discussing of this consideration, you shall not want my helpinge hand.

NOw will I briefly also teach you somwhat of ỹ rule of Falsehod, whiche beareth his name, not for that it teacheth any fraude or Falsehode, but for that by false numbers taken at all aduentures, it teacheth how to finde those true numbers that you seeke for. The occasion of the name.

Scholer. So might any other rule be called the rule of Falsehode, for they worke by wronge numbres, and by them finde out the right numbers, so dothe the rule of Alligation, the rule of Fellowship, and the Golden rule partely.

Mayster. In the Golden rule, the rule of Felowship, and ỹ rule of Alligation, although the numbers that you worke by, be not the true numbres that you seek for, yet are they numbers in iuste proportion, and are founde by orderly worke: where as in this rule, the numbres are not taken in any proportion, nor founde by orderly worke, but taken at all

aduentures.

And therefore I sometimes beinge merrye with my friendes, and talking of suche questions, haue caused them that proponed suche questions to call vnto them suche children or ydeotes; as happened to be in the place, and to take theyr aunswere, declaring ẏ I woulde make them to solue those questions that semed so doubtfull.

And in deede I did answere to the question and worke the triall thereof also, by those answeres whiche they happened at all aduentures to make: whiche numbers seing they be taken as manifest false, therefore is this rule called the rule of False positions, and for breifnesse, the rule of Falshode, whiche rule for readinesse of remembraunce, I haue comprised in these fewe Verses followinge, in fourme of an obscure Riddle.

Gesse at this worke as happe doth leade,
By chaunce to truthe you may proceede.
And firste woorke by the question,
Although no truthe therein be done.
Suche falsehode is so good a grounde,
That truthe by It will soone be founde.
From many bate to many mo,

From

From to fewe take to fewe alſo.
With to muche ioyne to few agayne,
To to fewe adde to many plaine.
In croſſewaies multiply contrary kinde,
All truthe by falſehode for to finde.

The ſence of theſe Verſes, and the ſumme of this rule, is this.

When any queſtion is propoſed apperteninge to this rule, firſte imagine any number that you liſte, whiche you ſhall name the fyrſte poſition, and put it in ſtede of the trewe number, and then woorke with it as the queſtion importeth: and if you haue miſſed, then is the laſte number of that woorke other to greate or to little: that ſhall you note as hereafter ſhall be taught you, and you ſhall call it the Firſte errour.

Then beginne agayne, and take an other numbre, whiche ſhall be called the Seconde poſition, and woorke by the queſtion: if you haue miſſed agayne, note the exceſſe or defaulte as it is, and call that the Second errour. Then multiplye croſſewayes the Fyrſte poſition by the Seconde errour, and agayne the Seconde poſition by the Firſte errour, and note their totalles ſeuerallye by the names of Totalles.

Then

Then marke whether the two errours were bothe like, that is to saye, bothe to muche or bothe to little: or whether they be vnlike, that is, the one to muche, and the other to little, for if they be like, then shall you subtracte the one totall from the other (I meane the lesser from the greater) and the Remainer shall bee your diuidend, so muste you abate the lesser errour out of the greater, & the residew shall be the diuisour. Nowe diuide the diuidende by that diuisour, and the quotient will shewe you the true numbre that you seeke for. But and if the errours bee vnlike, then muste you adde bothe those totals (whiche you noted) togither, and take that whole number for the diuidend, so shall you adde bothe the errours togither, and that whole number shall bee the diuisour, and the quotient of that Diuision shall giue you the true numbre that the question seeketh for: and this is the whole rule.

Scholer. This rule seemeth so vnlike any other, that without some example I shall not easily vnderstande it.

A question of Masonrye.

Mayster. Therefore take this example: I Mason was bounde to builde a wall in 40 dayes, and it was couenanted so with him, that euery day that he wrought he should haue

for

for his wages 2 ſ, 1 penny, and euery day that he wrought not, he ſhould bee amerced 2 ſhillinges, 6 d. ſo that when the wall was made, and the reckeninge taken of the dayes that hee wroughte, and of the other that hee wroughte not, the Maſon had clearely but 5 ſ, 5 d, for his worke. Now we I demaunde, howe manye dayes did hee worke of thoſe 40, and howe manye did he not worke?

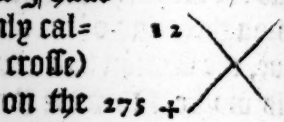
Scholer. I praye you expreſſe the order of the worke, that I maye partely by imitation, and partely by comparing it with the rule, bee able agayne to woe the like.

Mayſter. This order ſhall you keepe in the worke of this rule: firſte take ſome numbre (as you liſte) at aduenture, as for exaumple: I ſay he played 12 dayes, and wrought 28 dayes. Now caſt you the wages of euery day and ſee whether it will agree with the ſumme of 5 ſ, 5 d.

Scholer. The 28 dayes that hee wrought after 25 pence by the day, yeldeth 700 d. Then the 12 dayes that he wrought not, at 30 pence eche day, bythe amount to 360 pence, whiche if I abate out of 700 pence, there reſteth 340: but you ſay he had not ſo manye.

Mayſter. Hee had but 65 pence, and by this

this suppoſition hee ſhoulde haue had 340: therefore is this ſumme to muche by 275, whiche ſumme I muſte ſet wone after this ſorte as you ſee heere, where firſt I haue made a croſſe (commonly called Saint Andrewes croſſe) and at the ouer corner on the lefte hande I haue ſette the firſte poſition 12, and at the other corner vnder it, I haue ſet 275 whiche is the firſte errour, with this figure 4, whiche betokeneth to muche, as this lyne, ——— playne without a croſſe lyne, betokeneth to little. On the righte hande of the croſſe I haue lefte two lyke roomes for the ſeconde poſition and his errour. Therefore to proſecute the worke, I ſuppoſe he played 16 dayes, and wrought 24.



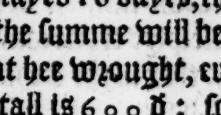
Scholer. I was a while in doubte why you named the dayes of his worekinge, ſeynge they bee not ſet in the figure: and I doubted how you knew them, or elſe whether that you did ſuppoſe them at all aduentures, as you did ỹ dayes that he played: but now I gather, that ſeynge 40 dayes is the whole time limited, then the dayes that hee played beyng ſuppoſed, the reſte of 40 muſte needes bee the dayes

that

that hee wrought, and therefore 28 followed 12 of necessitye, and 24 foloweth 16 also of necessitye: but yet I feare perceaue why you set not in the figure as well 28 as 12.

Mayster. It forceth not whiche of them I take, so that in the seconde position I take the numbres of the same nature that is heere bothe of working daies, or bothe of ydle daies: but now examine you this second position.

Scholer. If hee played 16 dayes, then abating 16 times 30 d. the summe will bee 480 d. And for 24 dayes that hee wrought, euery day yelding 25 d, the totall is 600 d: so that abatinge 480 out of 600, there resteth 120: and as you saye it should bee but 65, therefore it is to muche by 55, that muste bee set on the right hande of the figure at the nether part, and ouer it on the same side 16, whiche is the seconde position, thus.



And as I gather by youre wordes, it were all one if I did set 28 in steede of 12, and 24 in steede of 16.

Ma. So were it. But this shall you marke, that of what nature so euer the two positions bee, of the same nature is the Quotient. Therefore when the positions in this question

are

The profe of this rule.

are 12 and 16, whiche bothe beynge numbres of the playing dayes, the quotient shall declare the trew numbres of the playing dayes, where as if the positions had beene 28 and 24, whiche are supposed to bee the worckinge dayes, then woulde the quotient declare the trewe numbre of the worckinge dayes, and not of playinge dayes as it will be nowe. And therefore to continue the worke of this question, and to finde the trewe numbre of playinge dayes, I muste multiplye crossewayes the firste position 12 by 55, that is the seconde errour, and the totall will be 660: then I multiplye 275 by 16, and it yeldeth 4400. Nowe bycause the errours are like, that is to saye, bothe to muche, I must subtracte 660, out of 4400, and so remayneth 3740, whiche is the diuidend. Agayne, I must subtract the lesser errour 55 out of 275, that is the greater errour, and there will remayne 220, whiche shall be ỹ diuisour, then diuidinge 3740, by 220, the quotient will be 17. Wherefore I say nowe constantlye, that 17 is the trewe number of dayes that the Maion played: and then it followeth, that he wrought 23 dayes, and so is the question aunswered.

Nowe for the order of triall of this worke

there

there neveth none other proofe but onelye this, to worke with this number accordinge to the question, and if it agree, then appeareth the number to be it that you would haue. As here nowe seinge he wrought 23 dayes, and muste haue for euery daye 25 pence, the whole sūme cōmeth to 575. Then agayne seeinge he playd 17 dayes, and muste abate 30 pence for euery day, the whole summe of the abatemente will bee 510: therefore I subtracte 510 out of 575, and there will remayne 65, whiche maketh 5 ℔, 5 ß, the cleere wages of the Mason for his worke, accordinge to the question.

Scholer. Nowe I truste I vnderstande the worke and the rule so well (and the better by this proofe) that I can bee able to doe the like. And for a proofe I take the same question all saue the laste number, where I will suppose that he had 10 ℔, for his wages cleere. And now to gesse at the number of the dayes that hee wrought, I suppose firste that he wrought 20 dayes, then say I, if he wrought 20 dayes, his wages muste be 500 ß, then did he playe other 20 dayes, for whiche muste bee abated 600 ß, and then hee leeseth 100 ß. And so am I at a staye, for it is not like to youre former worke.

Mayster.

Mayster. You should haue required of me some question, and not haue taken à question of youre owne fantasyinge, vntill you were more expert in this arte: for so might you as well happen on an impossible question as on a possible: but nowe to go forwarde, consider that this number is to little by 220, seeing he should gayne by your supposition 1200, and in this position he leeseth 100, those bothe make 220, which you shall set downe for the firste errour with this signe —, betokening to litle, as here in this forme folowing bothe appeare. And now for the rest go forward your selfe once againe.

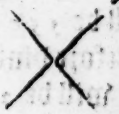
20

220—

Scholer. As my errour hath bttered my follye, so it hath procured me better vnderstanding. Now therefore considering this position not to solue the question, I take an other, supposinge that he wrought 30 dayes, then for his wages he must be allowed 750 ð, and for the 10 dayes which he wrought not, he muste abate 300 ð, and so remaineth clere 450 ð: but it should be onely 120 ð, therefore is it to muche by 330, whiche I set downe in the figure with the former position, and his errour and the figure appeareth thus.

Nowe

Nowe muste I multiply in crosse waies 220, by 30, & it will be 6600. Then agayne I multiplye 330 by 20, and it will be also 6600. Wherefore if I shall subtracte the one out of the other, there will remaine nothing to be the Dividend.



Mayster. In this you forget your selfe againe: for in as muche as the signes of the errours be vnlike, therefore muste you worke by Addition, addinge together those two totalls to make the dividende, and addinge also the two errours to make the divisour. And bicause you shall no more forget this part of the rule, take this briefe remembraunce:

Vnlike require Addition,
And like desire Subtraction.

Scholer. You meane, that if the errours haue like signes, then must the dividend and the divisoure bee made by Subtraction, as is taught before: And if those signes be vnlike (as in this laste example they be) then must I by Addition gather the Dividend and the Divisour. Therefore must I adde 6600 to 6600, and it will be 13200, whiche shalbe the divi-

dend. Then againe I adde 220 to 330, and it will be 550, which muste be the diuisour: wherefore diuiding 13200 by 550, the quotient will be 24, whereby I knowe that the Mason wrought 24 dayes: and then it followeth that he played 16 daies.

Mayster. Examine your worke whether it be agreable to the question or no.

Scholer. For 24 daies worke, the wages muste be 600 d. and for 16 daies which the Mason wrought not, there muste bee abated 480 d, & then remaineth cleare to the Mason 120 pence, as the question importeth, wherefore it is euident, that 24 is the true number of the daies that he wrought.

Mayster. Although you seeme now to vnderstande this worke, yet to acquainte your minde the better with ỳ new trade of this rule, I thinke it good to propone to you fiue or sixe exaumples more, before I make an ende of it.

Scholer. Sir I thanke you, that you weil consider my commodity and profit in knowledge, for vndoubtedly it is practise and exercise that maketh men prompte and experte in euery kinde of knowledge.

Mayster. You say well, so that they follow some certaine precepts to gouerne and rule

their

their practise by, elles maye practise procure custome of errour, and a repugnaunce to exactnesse of knowledge; namelye as longe as the errour is not playnely knowen to the vulgare sorte. But to returne to our worke. There is a seruaunt that hath bought of veluet and damaske for his mayster 40 yardes, the veluet at 20 ſ, a yarde, and the Damaske at 12 ſ, and when he commeth home, his mayster demaundeth of him, how much he hathe bought of ech sorte: I can not tell (sayth he) exactly, but this I know, that I payd for damaske 48 ſ. more then I payde for veluet, nowe muste you gesse how many yardes there is of ech sorte. A question of wares.

S. Although the gesse seemeth difficulte, yet I will proue what I can we: for I remēbre youre sayinge, that it forceth not howe fonde or false the gesse be, so it bee somewhat to the question, and not an aunswere of a contrary matter.

Therefore firste I imagine that he bought 20 yardes of Damaske, for whiche hee shoulde paye after the former price 240 shillinges: then muste hee needes haue of veluet other 20 yardes (to make vp the 40 yardes) and that would coste 400 ſ. So that the totall of the price of the damaske is lesse then the sum payde for veluet 160 ſ, and shoulde be more by 48.

therefore the firſte erroure is 208 to little. Then begin I agayne, and ſuppoſe he boughte of Damaſke 30 yardes that coſte 360 ℔, then had hee but 10 yardes of Veluet, whiche coſte 200 ℔: and nowe the prycе of Damaſke is greater then the price of the Veluet by 160 ſhillinges, and ſhould bee but 48, therefore is the ſeconde erroure 112 to muche, whiche I ſette in fourme of a figure as heere dothe appeare. Then we I multiply in croſſe wayes 208 by 30 & the ſumme will be 6240. Alſo I multiplye 112 by 20, and there will amount 2240. And in as muche as the ſignes of the erroures bee vnlike, I knowe I muſte worke by Addition, therefore adde I theſe two totalls togither, and they make 8480, whiche is the diuidende: then adde I alſo the two erroures togither 208 and 112, and they make 320, whiche is the diuiſor. Wherefore diuiding 8480 by 320, the quotient will be 26½, whiche is the trewe ſumme of yardes of Damaſke that hee boughte: and in Veluet 13 yardes ½, and that appeareth by examination, thus: 26 ½ yardes of Damaſke at 12 ℔.

20 — 30

280 — 112

the

the yearde, maketh 318 ℔, then in Ueluet hee had but 13 yeardes and ½, that coste 270 ℔. at 20 ℔. the yarde. Now Subtract 270 out of 318, and there will remayne 48, whiche is the numbre of shillinges that the Damaske did coste more then the Ueluet.

Mayster. Nowe shall you haue à question of an other kinde. A questiō of debte.

There are three men that owe monye to me, and I haue forgotten what ỳ totall sūme is, and what the particulars be.

Scholer. Whye? then it is impossible to knowe the debte.

Mayster. Peace, ye are to hastye: there is more helpe in it then you yet see. I haue three seuerall notes, whereby it appeareth that I did confer their debts togither, and found the debte of the first and the seconde to amounte to 47 li. the debte of the first man & the thirde did make 71 li, & the seconde man his debte to the third did rise to 88 li. Now can you tell what euery mā did owe, & what was ỳ whole total?

Scholer. Naye in good sayth: but as I perceaue that it must be foūd by coniecture, so wil I gesse at it, supposing ỳ the first man did owe 20 li. and the seconde man 30, and the thirde

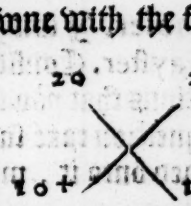
Mayster. Naye staye there, for you are to

farre gone all ready, you may not suppose a seuerall summe for euery man, for it is ynoughe to suppose one summe for the firste man, and let the other rise as the question importeth. Therefore seynge you sette the firste man his debte to bee 20 li, the second man can not owe 30 li, for the declaration is that theire debtes added togither, did make 47 li, so muste the seconde man his debt be but 27 li. Now this second debt with ỹ thirde muste make 88, therefore subtracte 27 out of 88, and there will remayne 61, as the thirde man his debte. Then sayeth the declaration, that the firste and thirde mennes debtes we make 71: but by this supposition they make 81, ỹ is 10 to muche: whiche I muste set for the firste errour. Nowe worke you the seconde position.

Sc. I suppose the firste mannes debte to be 24 pounde, then muste the seconde mans debte (by youre declaration) bee but 23 li. seynge bothe they we make but 47 li. Also ỹ seconde man his debte with the thirde, we make 88 li, and the seconde man oweth but 23, therfore the third man muste owe 65 li. Nowe the thirde mans debte with the firste, shoulde make by the declaration 71 li, and they we make 89 li: that is 18 li to muche, and that is the seconde

errour,

errour, which I set downe with the first, and their position in this forme: & then we I multiply in crosse waies 20 by 18, and it is 360. Also 10 by 24 maketh 240.



And bicause the signes of the errours be like, I must worke by subtraction: therefore I subtract 240 out of 360, and there resteth 120, which is the diuidende: then doe I subtracte 10 out of 18 by the same reason, and so is the diuisour 8, whiche is founde 15 times in 120, therefore I say, that the first man did owe 15 li, and then the second man must owe 22 li, for those 2 doe make 37 li, and the thirde man his debt is 56, for so much remaineth if I bate 15 out of 71, or if I take 32 out of 88.

Mayster. For the third example take this easie question for the varietie in worke. Two men hauing seuerall summes which I knowe not, doe thus talke togither: the first sayeth to the seconde, If you giue me 2 s of your money, than shall I haue 3 times so much money as you: the seconde answereth: If weare more reason, that ours summes weare made equall, & so will it be, if you giue me 3 s of your mony. Now gesse what eche of them had. The third question.

Note. Scholer. I imagine that the firſt had 9 ſ.

Mayſter. Conſider euermore in your imaginations that you take a likely ſumme, as in this queſtion take ſuche a ſumme that hauinge 2 added vnto it, may be diuided into 3 partes euen.

Scholer. Why? I remember you ſaide before, it ſorced not howe fondely ſo euer I gelled.

Mayſter. As for the poſſibilitie of the ſolution it is truthe, but for eaſineſſe in worke, the apteſt numbers are moſt conuenient.

Scholer. I thought no leſſe, and therefore I toke 9 as an apte number to be parted into 3: but I perceaue I ſhoulde haue conſidered the aptneſſe of that partition after the Addition of 2 vnto it, and then 7 had ben more meeter.

Mayſter. That is truthe, and then ſhoulde the ſeconde mans ſumme be 5: for although he haue now but the thirde parte of 9, that is 3, yet you muſt remember that he lent the firſte man 2, and ſo had he 5.

Scholer. Then to go forwarde: if the ſeconde man had 3 of the firſt man, then ſhoulde he haue 8, and the firſt man but 4, ſo hath he double to the firſt man: yet he ſaide in the queſtion, they ſhould haue equall: wherefore it appeareth

peareth that he hathe 4 to muche. Therefore I note that error with his supposition, and gesse againe that he hathe 10 s. whereunto I adde 2 shillinges borowed of the seconde man, and then hathe hee 12 shillinges, so the seconde man hath remayninge but fower, whereunto if I adde the 2 that he lente to the firste man, so had he but 6 s at the beginninge. Then take 3 shillinges from the firste man, and giue to ẏ seconde, then hath the firste man but 7, and the seconde hathe 9, whiche are not equall, but there are 2 to many, wherefore I set downe bothe the positions wt their errors as heere you see, and multiply à crosse, so commeth

7 10

44 24

there 40 and 14: and bycause the signes be lyke, I take 14 out of 40, and so resteth 26 to be the diuidende, then likewayes I take 2 out of 4, and there resteth 2, by whiche I diuide 26, and the quotient will bee 13, whiche is the summe that the firste man had. And so appeareth that two beynge added thereto, the summe will bee 15, so hath the seconde man now but 5, and before he had 7: then take three from the firste, and put to his seuen, so haue eche

of them 10, and that is equall, as the question woulde.

The fourthe example.

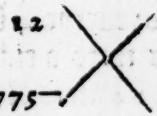
Mayster. For the fourth example take this question. One man saide to an other: I thinke you had this yeare two thousande lambes: so had I saide the other: but what with payinge the tythe of them, and then three seuerall losses, they are muche abated: for at one tyme I losse halfe as many as I haue now lefte: and at an other time the thirde parte of so manye: and the thirde time $\frac{1}{4}$ so many. Nowe geesse you how many are lefte.

Scholer. Bicause heere is mention made of certaine partes, I must take a number that may haue all those partes: that is to say, $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$, whiche will bee 24, howe bee it 12 hath the same partes. Therefore firste I take 12 to be the number that wth remaine, so hath hee losse 6, 4, and 3, that is 13: and in the whole 25, but it should bee 1000.

Mayst. Ye are deceiued yet still: you haue forgotten the 10 parte, whiche must bee defalked, that is 100, so there remaineth but 1800: and now go on againe.

Sc. Then to finde the errour, I take 25 out of 1800, and there remaineth 1775 to fewe, whiche I set for the firste erroure. Then for

for the seconde psition I take 24, whose halfe is 12, the thirde parte 8, & the quarter 6, where by riseth 50, whiche is to little by 1750, therfore I set wone bothe ỳ psitions with theire errours thus.



And multiply in crosse wayes 1775 by 24, wherof cometh 42600. Also I multiplye 1750 by 12, and there resteth 21000. And bicause the signes are like, I we subtracte the one from the other, & so remayneth ỳ diuidente 21600: then we I subtracte 1750 out of 1775, and there resteth 25: by whiche I diuide 21600, & the quotient is 864, whereof the halfe is 432, and the thirde parte is 288, ỳ quarter is 216, whiche all beyng added togither, will make 1800. And if you adde thereto the tenthe whiche was abated before, then will the whole sume be 2000. And nowe dothe there come a question to my memory whiche was remaunded of me, but I was not able to answere to it, and now me thinketh I could solue it.

864
432
288
216
1800

Mayster. Propone youre question.

Scholer. There is supposed a Lawe made, that A questiō

of Sheepe and Tillage.

that (for furtheringe of tillage) euerye man that wthe keepe sheepe, shall for euerye 10 sheepe eare and sowe one acre of grounde: and for his allowance in sheepe pasture, there is appynted for euerye 4 sheepe 1 acre of pasture: Nowe is there a riche sheepemayster whiche hathe 7000 acres of grounde, and woulde gladlye keepe as many sheepe as hee mighte by that Statute, I demaunde howe many sheepe shall hee keepe?

Mayst. Answere to the question your selfe.

Scholer. Firste I suppose hee maye keepe 500 sheepe, and for them hee shall haue in pasture after the rate of 4 sheepe to an acre, 125 acres, and in earable grounde 50 acres, that is 175 in all: but this errour is to little by 6825. Therefore I gesse agayne, that hee may keepe 1000 sheepe, that is in pasture 250 acres: and in tillage 100 acres, whiche maketh 350: that is to little by 6650.

500 — 1000

6825 — 6650

These bothe errours with their positions I sette wwne as you see, and multiplye in crosse 6825 by 1000, and it maketh 6825000. Then I multiply 6650 by 500, and it wthe amounte

amounte to 3325000, whiche summe I doe subtracte out of the firste, and there remayneth 1500000 as the diuidende. Also I we subtracte the lesser erroure out of the greater, and so remayneth 175, by whiche I diuide ỹ sayde dividend, and the quotient will be 20000, so that I see, that by this rate hee ỹ hathe 7000 acres of grounde, may keepe 20000 sheepe: and thereby I coniecture, that many men may kepe so many sheepe, for many men (as the common talke is) haue so many acres of grounde.

Ma. That talke is not likelye, for so muche grounde is in compasse aboue 48 ¾ miles, but leaue this talke, & returne to youre questions, leaste your poynting be scarse well taken.

Scholer. In dede we I remembre, that the Egiptians did grudge so muche agaynste shepherdes, till at lengthe they smarted for it, and yet they were but smale sheepemaisters to some men that be now, and the sheepe are wexen so fierce now and so mighty, that none cã withstande them but the Lyon.

M. I perceaue you talke as you heare some other: but to the worke of your question, bothe this last question and the nexte before mighte bee wrought without the seconde position, by the rule of proportion, as this. When in this question

An other way of working.

question ye found in the firste erroure, that for 500 sheepe there muste bee 175 acres, then might you reduce it to the Golden rule, thus. If 175 acres will admit in allowance 500 sheepe, than 7000 will haue 20000. And so by one position with the helpe of the Golden rule may you answere ẏ question. Likewayes for the question of lambes, when you hadde found that 12 came of 25, you might haue set the fygure thus as you see, and haue sayd: If 25 doe leaue but 12, what shall 1800 leaue? & it would appeare to be 864.

175	Z	500
7000		20000

25	Z	12
1800		864

Scholer. Sir, I thanke you for this ayde, for it dothe muche shorten the worke of this rule.

An other way yet.

Mayster. Yet agayne I will shewe you an other waye to aunswere to this laste question without this rule of Falsse position, and that by the rule of Felowshippe, for it appeareth in the proponinge of the question, that 10 sheepe muste haue in pasture 2 acres and $\frac{1}{2}$, and for them must there be eared but one acre: so it foloweth that for two acres eared, there must bee fiue set to pasture. And if you put them bothe into one sum, they will make 7. Therfore loke what

what proportion 7 being this totall, doth beare to 5 and to 2, suche proportion shall any totall in this question beare to the pasture grounde and the eared ground.

Scholer. This serueth wonderous aptlye. Therefore to proue it, I demaunde this by the former supposition: If à man haue 300 acres, how muche shall he leaue in pasture, and howe muche shall he turne to tillage? You say that as 7 is to 5, so shall 300 be to the acres of pasture: & as 7 is to 2, so is 300 to the acres of tillage, whereof for bothe I haue set examples here folowing, wherby appeareth ye of pasture there shall be $214\frac{2}{7}$ acres, and of tillage $85\frac{5}{7}$, whiche both summes added togither, doe make 300.

$$\begin{array}{ccc} 7 & & 5 \\ 300 & \text{Z} & 214\frac{2}{7} \end{array}$$

$$\begin{array}{ccc} 7 & & 2 \\ 300 & \text{Z} & 85\frac{5}{7} \end{array}$$

Mayster. Now take an other example: A man hath three siluer cups with one couer, the couer wayeth 18 vnces, the second cup wayeth euen halfe the waight of the first and the third. Now if the couer be put to the firste cup, they weye iuste as muche as all the three cuppes doe weye: and the couer be ioyned with the seconde cup, they weye as muche as the second twise [illegible]e third: and if the couer be put to ye third cup, An other question.

cup, they will make twise as muche as the first and the seconde cup. Now trye you what was the iuste waight of euery cup.

Scholer. I we set the waight of the firste cup to be 9 vnces: then in as muche as these two (that is to say, the couer and the first cup) we wey the waight of the three cups, I se that the three cuppes muste wey 27 vnces, for so muche is 18 and 9. Also bicause the firste and the third we wey double so muche as the second, therefore is it the thirde parte of that waighte, that is 9, and then woulde it follow, that the third cup also shoulde wey 9 vnces, but then the question sayeth, that the couer being ioyned to the second cup, they weye as muche as the seconde twise and the thirde once, that shoulde be 27, and so it dothe: then being ioyned with the third cup, they shoulde wey twise as muche as the firste and the seconde, that shoulde bee 36, and they weye but 27, so is that erroure 9 to little. The beginne I againe, and say, that the firste cup doth wey 12 vnces, whiche I ioyne with the couer, and they make 30 vnces: then seinge the seconde is $\frac{1}{3}$ of that waight, it muste needes weye 10 vnces, and the thirde muste weye 8 vnces, seeinge the firste and the third must wey 20 vnces. Now put I the couer to

the

the seconde cuppe, and they weye 28 vnces, whiche should be euen so: then ioyne I the couer with the thirde cuppe, and so should it weye twise the firste & the second, that is 44 vnces, and they doe weye but 26, that is 18, to little. those erroures with their positions I set wwne and multiply in crosse waies 9 by 12, whereof commeth 108. Also 9 by 18, and that yeldeth 162, and in as muche as the signes bee like, I abate the lesser out of the greater, and there wth remaine 54. Then doe I also abate the lesser erroure from the greater, and so remaineth 9, by whiche I diuide 54, and the quotient is 6: whiche I take for the true weight of the first cup: whiche beinge ioyned with the couer, muste weye as muche as the three cuppes, so we they wey but 24 vnces. Then seinge the seconde cuppe is the third part of that weight, for the other two cuppes (you say) must wey double his weight, the weight of the seconde cuppe is 8 vnces, and so the weight of the thirde muste bee 10 vnces. Now put the couer to the second cuppe, and it will make 26 vnces: that muste bee the weighte of the seconde twise, and the thirde

9 12

9— 18—

ones, that is twise 8, and once 10, and so is it. Againe, put the couer to the thirde cuppe of 10 vnces, and they muste weye twise as muche as the firste and the seconde, that is 28: and so is all agreable.

Mayster. Then aunswere to this questiō.

A questiō of water. There is à Cesterne with iiij. cockes, containing 72 barrels of water, and if the greatest cocke bee opened, the water will auoyde cleane in vj. houres: at the seconde cocke it will aske 8 houres: at the thirde cocke it will auoyde in no lesse then 9 houres: and at the smallest it will require 12 houres: Nowe I demaunde, in what space will it auoyde, all the cockes being set open?

Scholer. Firste I imagine that it will auoide in 2 houres.

Mayster. Then muste there auoide by the firste cocke $\frac{1}{2}$ of the water, that is 24 barrels, and by the seconde cocke $\frac{1}{4}$, that is 18, and by the third cocke $\frac{2}{3}$ that is 16 barrels, and by the smallest cocke $\frac{1}{6}$ that is 12 barrels, all whiche summes put togither, we make 70, as by their addition it dothe appeare, but it should be 72, therefore the errroure is 2 to fewe.

Scholer.

Scholer. Then I begin agayne by youre fauoure, bicause I thinke I vnderstande the worke, & put three houres for the due time: so shall there run out at the greatest cocke $\frac{1}{2}$, ỳ is 36 barrels, and at the second hole $\frac{1}{3}$, that is 27, and at the third cocke $\frac{1}{4}$ that is 24, and at ỳ smallest hole $\frac{1}{12}$, that is 18 barrels, whiche altogither we make 105, and should bee but 72, so is it to muche by 33, therefore we I set the errors in order of ỳ figure, with their positions, & worke by multiplication, in crosse, sayinge: 2 times 3, is 6, & 2 times 33 maketh 66: and bicause the signes are vnlike, I muste adde those two totalls togither, whiche make 72: also I adde the two erroures, and they make 35, by whiche I diuide 72, and the quotient riseth $2\frac{2}{35}$, whereby I see, that all the cocks beyng set open, the water will auoide in 2 houres and $\frac{2}{35}$ of an houre.

	24
	18
	16
	12
	70

2 3

2 — 33 +

Mayster. This exercise maketh you to growe expert in the rule. Therefore I will inure you somwhat more with a questiō or two.

There were two men that had bene parteners, and had in accompte betweene them 300

duckets: whereof the one should haue for his parte 180, and the other 120: but in the parting of them they fell at variaunce, so that ech of them catched as many as he could: yet after warde beyng reconciled, they agreed that he whiche had gotten most parte of them, shoulde laye downe $\frac{3}{4}$ of them agayne, and he that had gotten leaste, shoulde laye downe $\frac{1}{3}$ of those whiche he had taken, then parting them vnto two equall partes, ech man to haue halfe thereof, and so had they their iuste portions as they ought: now I demaunde of you what ech of them had gotten by the scamblinge?

Scholer. I suppose hee that had leaste, gatte 108 duckets, then the other had 192: wherefore in laying downe againe of the 192, there was put downe $\frac{3}{4}$, that is 144, and so had he left but 48. Also of the 108 there was layde downe 36, that is $\frac{1}{3}$, and so he had lefte 72. Then I put togither 144, and 36, and it maketh 180, whiche I parte into two partes euen, and so commeth 90 to bee geuen to ech of them: whiche summe put to 72, maketh 162 and ioyned to 48, it maketh 138: and nowe I wuot how I shall goe forwarde.

Note. Mayster. You neede not to take but one of them whiche you list, the greater or the smaller,

for

for all commeth to one purpose: and so maye you compare it that you take to anye of the other summes, remembringe that you make comparison to the same in the seconde worke: as for example of the firste parte. If you compare 138 with the lesser summe due, that is 120, so is it 18 to muche: and if you compare it with the greater summe, than is it 42 to little. Agayne, if you compare 162 to the greater summe, the erroure will bee 18, as it was in the other: but it will haue a contrarye signe, & if you compare it with the lesser sume, it will bee 42 to muche: so that the erroure bothe wayes is eyther 18 or 42: and as for the signes it little forceth, for in them is nothinge considered here, but likenesse and vnlikenesse, whiche in this case wthe neyther further nor hinder. But now go on with the worke.

Scholer. If it bee so, then am I out of my greatest doubt. Then I ioyne that 90 (whiche I found as the halfe of ỹ latter particion) vnto 48, whiche is lefte with one man, and so hathe he 138, whiche (I may say) is 18 to many, for the leaste should bee but 120, that erroure I note, & then make a new position, supposing the one man to haue 104, and the other to haue 96, wherfore of ỹ 104 there must bee layde

wone 153, and so remayneth with him 51. Also of the 96 there must be layde wone $\frac{1}{2}$, that is 32, and so resteth with that man 64. Now of the 153 and 32 I make one summe as 185, whiche I must diuide into 2 equall partes, and so eche man shall haue $92\frac{1}{2}$, whereunto if I adde theire former portions reserued, then the one shall haue $156\frac{1}{2}$, & the other hathe $145\frac{1}{2}$. Wherefore I take the lesser summe nowe agayne as I did before, that is $143\frac{1}{2}$, and finde yᵗ he hathe to many by $23\frac{1}{2}$, for he should haue but 120, so haue I for my two positions ij. errours, whiche I set wone as here may be sene, eche erroure vnder hys position, and than by ꝑthe rule I we multiply in cros wayes 108 by $23\frac{1}{2}$, and there ryseth 2538 whiche I note: then agayne, I multiplye 96 by 18, and thereof amounteth 1728. Nowe bicause the signes are bothe like', that is bothe to many, I muste worke by Subtraction, & so abating 1728 out of 2538, there will reste for the Diuidend 810: then for the diuisor I subtract 18 out of $23\frac{1}{2}$, and there remayneth $5\frac{1}{2}$, by whiche I diuide 810, and the quotient will

be 147 $\frac{3}{13}$, whiche is ỹ iuſt portion of him that had ỹ leaſt ſum. And if I we ſubtract it out of 300 being ỹ totall ſum, thẽ will there remaine 152 $\frac{8}{13}$ as the portion that the other did get.

Mayſter. For the proofe of this worke, you maye choſe whether you will examine thoſe numbres accordinge to the forme of the queſtion, or els worke by other two poſitions for to finde the ſeconde number: and if thoſe poſitions bringe the ſame numbers that did amount by the firſt two poſitions, then wthe eche worke confirme other.

Scholer. By your patience, I will proue bothe wayes, not onelye to ſee their agreement, but alſo to accuſtome my minde to thoſe workes: for I perceaue it is exerciſe that muſt be the chiefe engrauer of theſe rules in my memory.

Mayſter. You conſider it well: then go to.

Scholer. Firſte I will by twoo other poſitions trye to finde the portion of him whiche had moſte.

Mayſter. Although you may we it with any poſitions, yet to ſee the agreement of your worke the better, take the ſame poſitions that you did before, comparinge them nowe to the greater, as you did before vnto the leſſer.

Scholer. Then I suppose that he that had moste, had 192, so had the other 108. Nowe if I take $\frac{3}{4}$ of 192, that will be 144, & there will rest to that man but 48. And from the seconde whiche had 108, if I take $\frac{1}{3}$, that is 36, there will remaine to him 72: thē ioyning 144 with 36, it will make 180, the halfe whereof beinge 90. If I adde to eche of those two mens portions remaininge with them, the one shall haue 138, and the other 162, of whiche two I take the greater (that is 162) and see it to be 18 to fewe, for it shoulde bee 180, that error I note vnder his position. Then for the second position I take (as I did before) 204 for the one, & so resteth 96 for the other: then take I $\frac{3}{4}$ of 204, and it will be 153, and there resteth to him 51. Also of the 96 I take $\frac{1}{3}$ that is 32, and there remayneth to him 64. Nowe put I that 32 vnto 153, and it yeldeth 105: whiche beinge parted in equall valewes, maketh $92\frac{1}{2}$, to bee added to eche mannes remaynder, and so the one hathe $143\frac{1}{2}$, and the other $156\frac{1}{2}$: wherefore I take the greatest summe, and it is $23\frac{1}{2}$ to little, that doe I note also, and set bothe their errors vnder their positions, as in this exaumple folowing bothe appeare.

And

And then multiplyinge 192 by $23\frac{1}{2}$, there wth arise 4512. Agayne, I multiply 204 by 18, and

it maketh 3672, whiche I doe subtracte out of 4512, bicause the signes be like, and there resteth 840 for the diuidend: then subtractinge 18 out of $23\frac{1}{2}$, there will remayne $5\frac{1}{2}$, whiche I muste take for the diuisor. And so diuiding 840 by $5\frac{1}{2}$, the quotient will bee $152\frac{8}{11}$. whereby I haue founde an agreable summe to that whiche I founde by the former positions, for him that had moste, whiche if I doe subtracte out of 300, that is the totall, there will reste $147\frac{3}{11}$, whiche was the portion of him that had the leaste parte.

Mayster. So by diuers positions you see, that one dothe confirme the worke of the other. Now examine those two numbers by the forme of the question, and so shall you proue your worke good also.

Scholer. If that hee whiche gat moste, had $152\frac{8}{11}$, then muste hee laye downe $\frac{3}{4}$ of his summe, that is $114\frac{6}{11}$, and so shall remayne with him but onely $38\frac{2}{11}$. The other whiche had leaste, that is $147\frac{3}{11}$, muste put downe

of his summe $\frac{1}{2}$, that is $49\frac{1}{11}$, and so dothe there remayne with him yet $98\frac{2}{11}$. Then we ✠ adde together $114\frac{6}{11}$, and $49\frac{1}{11}$, ✠ it will make $163\frac{7}{11}$, whiche ✠ muste parte into ij. equall partes, and that will be $81\frac{9}{11}$, to bee giuen to ech of them: so puttinge $81\frac{9}{11}$, vnto $38\frac{2}{11}$, there dothe amounte 120 iuste, wiche is the trewe portion of him that shoulde haue the lesser sum: ✠ adding $81\frac{9}{11}$ to $98\frac{2}{11}$, the totall will bee 180, the trewe portion of the other. And so is the worke by this proofe also tryed to bee good. And this ✠ marke by the way, that in their scamblinge, he gat moste (as it chaunceth often) that ought to haue had leaste by iuste partition.

Mayster. Let youre studye bee to learne truthe and iust arte of proportion, and to distribute and parte accordinge thereunto, as often as occasion shalbe ministred. And heere woulde ✠ make an ende of this rule, saue that ✠ remember one pleasaunte question whiche ✠ can not ouerpasse, whiche ✠ will declare somewhat largely, bicause you shall as well vnderstande some reason in the pleasant inuention, as apte procedinge in the witty workinge thereof.

Anexam- Hiero Kinge of the Syracusans in Sycilia

tilia, hadde cauſed to bee made a Crowne of Golde of a wonderfull weyght, to bee offered for his good ſucceſſe in warres: in makinge wherof, the Goldeſmith fraudulently toke out à certayne portion of Golde, and put in ſyluer for it, ſo that there was nothinge abated of the full wayght, although there was muche of the valewe diminiſhed. Whiche thinge at lengthe beynge vttered, (as no euill can alwayes lye hidde) the Kinge was ſore moued, and beynge deſirous to trye the truthe without breakinge of the Crowne, proponed the wubte to Archimedes, vnto whoſe witte nothinge ſeemed vnpoſſible, whiche although preſently he coulde not aunſwere vnto, yet hee had good hope to deuiſe ſome pollicy for that inuention. And ſo muſing thereon, as he chanced to enter into a bayne full of water to waſhe him, he obſerued that as his bodye entred into the bayne, the water did runne ouer the tubbe: whereby his readye wit of ſuche ſmall effectes coniecturing greater workes, cõceaued by and by à reaſon of ſolution to the Kings queſtion, & therfore reioycinge exceedingly more then if he had gotten the Crowne it ſelfe, forgat ỳ he was naked, & ſo ran home, cryinge as hee ran, εὕρηκα, εὕρηκα, I haue found, I haue founde. ple of mixture of Golde and ſiluer

And

And there vpon caused ij. massye peces one of golde, and on other of siluer to bee prepared of the same weight that the sayd crowne was of: & considering that golde is heauier of nature then siluer, and therefore golde of like weighte with siluer, muste needes occuppe lesse rowme, by reason it is more compacte and sounde in substance, hee was assured that puttinge the masse of golde into a vessell brimme full of water, there woulde not so muche water runne ouer, as when hee shoulde put in the siluer masse of the like weight. Wherefore hee tryed bothe, and noted not onelye the quantities of the water at eche tyme, but also the difference or excesse of the one aboue the other, whereby hee learned what proportion in quantity is betweene golde and siluer of equall weight. And then puttinge the crowne it selfe into the vessell of water brimefull (as before) marked howe muche water did runne out then, and comparinge it with the water ỳ ran out when the golde was put in, noted howe muche it did exceede that, & likewayes comparing it to the water that ran out for the siluer, marked how muche it was lesse then that: and by those proportions found out the iuste quantity of golde, ỳ was taken out of the crowne, &

howe

how muche Syluer was put in steede of it. But seyng Vitruuius whiche writeth this historye, wolde not declare the particular woorke of this tryall, it shall bee no inconuenience to suppose an exaumple for declarations sake, wherein although the trewe and iuste proportions be not expressed, yet the fourme of tryall shall be trewly set forthe. And for an exaumple, I suppose the weight of the Crowne to bee 8 li, and so of ech of ỹ other two masses. And when the masse of Golde was putte into the water, I imagine that there ran out 2 pounde of water: and when the masse of Syluer was put in, I suppose there ranne out 3 pounde $\frac{1}{2}$. Agayne when the Crowne was put in, there ranne out 2 pounde $\frac{1}{2}$. Nowe to knowe what quantity of Syluer was in ỹ Crowne, woorke by the rule of false position, and imagine that there was 2 pounde of Syluer: then must there be 6 pounde of Golde. Then saye thus by the rule of Proportion: If 8 pounde of Golde do expel 2 li. of water, what shall 6 li. expell? and it will be 1 pounde $\frac{1}{2}$. Agayne for the Syluer: If 8 li. of Syluer expell 3 li, $\frac{1}{2}$ of water, what shall 2 li of Syluer put out? it will be $\frac{7}{8}$. Now adde those ij. weightes of water together, and they will make 2 li $\frac{3}{8}$ and it shoulde be by the

suppositiõ

ſuppoſition 2 li. $\frac{1}{4}$, ſo is it to muche by $\frac{1}{8}$.

Scholer. Now we I vnderſtande yͤ worke as I thinke, therfore I pray you let me worke the reſt of the queſtion. And bycauſe this firſte ſuppoſition did erre, I note that poſition and his erroure, and take a newe poſition, eſteeminge the Syluer to be but one pound, ſo muſt there be in Golde 7 pounde, Then ſaye I: If 8 li of Golde yelde 2 li of water, what ſhall 7 li yelde? and it will be 1 li. $\frac{3}{4}$. Agayne if 8 li, of Siluer expel 3 li. $\frac{1}{2}$ of water, what ſhall 1 li, expell? and it will be $\frac{7}{16}$. Nowe muſte I adde thoſe two ſũmes togither, & they make 2 li, $\frac{1}{16}$, & they ſhould make 2 li $\frac{1}{4}$, ſo is it to little by $\frac{3}{16}$, Therefore I ſette the poſitions with theire erroures in ordre, as heere foloweth. And thẽ I multiply in croſſewayes 2 by $\frac{1}{16}$, and it maketh $\frac{1}{8}$: likewayes 1 multiplyed by $\frac{1}{8}$: maketh $\frac{1}{8}$. And bicauſe the ſignes bee vnlike, I muſt adde thoſe ij. ſummes, whiche make $\frac{1}{4}$, and that is the diuidende. Agayne I muſt adde $\frac{1}{8}$ to $\frac{1}{16}$, and it will be $\frac{3}{16}$, that is the diuiſour. Nowe I ſhall diuide $\frac{1}{4}$ by $\frac{3}{16}$, and the quotient will bee $\frac{16}{12}$, that is, 1 $\frac{1}{3}$, whereby I knowe that there was put 1 li and $\frac{1}{3}$ of ſiluer into the Crowne,

Crowne, and so much golde taken out for it.

Mayster. Proue it nowe by examination accordinge to the question.

Scholer. If theee weare 1 pounde $\frac{1}{3}$ of Syluer, then was there of Golde 6 pounde $\frac{2}{3}$. Now saye I by the rule of proportion: if eyght pounde of Golde expell two pounde of water, what shall 6 pounde $\frac{2}{3}$ expell?

$$8 \sim 2 \qquad 6\tfrac{2}{3} \sim 1\tfrac{2}{3}$$

It will be one pounde $\frac{2}{3}$. Agayne if 8 li of Syluer expell 3 li $\frac{1}{2}$ of water, what shall 1 $\frac{1}{3}$ expell? It will bee $\frac{7}{12}$. Nowe must I adde togither 1 li $\frac{2}{3}$ and $\frac{7}{12}$, & they will make 2 li, $\frac{3}{12}$, that is 2 li $\frac{1}{4}$, accordinge to the supposition of the question, wherby I perceaue the worke to be well done. And as I can not but much reioyce of this excellent inuention, so my desire is kindeled vehemently to be perfectly instructed in euery parte thereof, and namely in this poynte, whether ỹ proportion betweene water & Golde be such, ỹ for 8 li of Golde put into a vessell full of water, there shal run out 2 li of water: & for as much sluer, whether 3 li. $\frac{1}{2}$ of water would auoyde?

$$8 \sim 3\tfrac{1}{2} \qquad 1\tfrac{1}{3} \sim \tfrac{7}{12}$$

Mayster. I perceaue your meaninge, and coniecture youre imagination to bee thus: that

that if you knewe the exacte proportion betweene Golde and Syluer and Water bothe in their waight and in their quantities, then coulde you eallye finde out the mixtures of them, whiche thinge I haue reserued for an other woorke that intreateth suche matters specially. And at this time you must consider, that you learne Arithmetike, whiche intreateth of the manner to solue doubtefull questions touchinge number, without regard what matter is signified by that number, els weare it necessarye in Arithmetike to teache all artes, seinge in it may bee moued questions of all artes. But seinge you are so desirous to know this thinge, I will tell it you in suche à sorte, that you shall practise youre arte in findinge it, & propounde it in forme of à question. Golde beareth greater proportiou to water then Syluer bothe, and their two proportions be in proportion together as $\frac{48}{25}$. But to helpe you somwhat in this riddle, you shall note that ỹ proportion of quicke Siluer vnto water, is the iuste middle number proportionall in Progression Geometricall, betweene the proportions of Golde and Siluer vnto water. And his proportion is, as $\frac{200}{121}$. Nowe if you will know the iuste numbers of these 3 proportions, then

A question of the proportiō of Golde, Syluer, & quicke syluer vnto water.

must

must you finde out 3 numbers in Progression Geometricall, whereof the middemost must be $\frac{200}{21}$, and the firste muste bee vnto the laste, as 25 to 48. And thus I will leaue you to finde those numbers when you be at leasure.

Scholer. Yet sir I thanke you hartely for this mudye, for nowe I see the possibilitye to finde them out. How be it bicause this question seemeth straunge, if it might please you to instruct me somewhat in the order of worcking for it, I shoulde the more easily finde the trewe worckinge.

Mayster. You desire to mudye ease if you will studye for nothing: therefore to occasion you to study the better, I will leaue this wubt wholy to your owne leardye. But as touching the generalty of the rule, Archimedes needed not to take two masses of golde and siluer equall in waight with the crowne, for the proportion might as well bee founde in any other waight, yea although the masse of golde were of one waight, and ỳ masse of siluer of another As for example. If the crowne were of 8 vñce waight, as I did suppose, and I haue not so mudye other fine golde, but onely 1 li. and trying that by water, and finding that it with expell but $\frac{1}{4}$ of an vnce of water, yet then by

it may I inferre, that 8 pounde of golde would expell 6 vnces of water. And likewayes of the siluer: whereof if I had but 2 pounde, and finde that it doth expell iii. vnces of water, then might I affirme ẏ 8 pounde would expell 12 vnces, that is 1 li. weight. And so is it as good as if the 3 masses were all of one weight. And thus for this tyme I will make an end of this other part of Arithmetike.

Scholer. Although I can not sufficiently thanke you for this, yet your promise made me to looke for the arte of Extraction of rootes, whereof hitherto I haue learned nothing.

Mayster. I will not breake my promise, but intend (God willing) to performe it with in these three or foure monethes, if I perceaue this my paines to be well taken in the meane season. And you shall not repent the taryinge for it, for it shall be encreased by the tarying. And in the meane time, you shall take this Addition not for the second parte of Arithmetike whiche I promised, but for an augmentation of the first parte, vnto whiche I woulde haue annexed the extraction of Rootes, square and cubike, namely for Examples of the Statute of Assise of woode, but that in the seconde parte I muste write of diuers other Rootes,

and

and therefore thought yt beste to reserue those Rootes also with their examples to the same Second parte.

Scholer. Sir, although I cannot recompence your goodnesse, yet I shall alwaies doe mine indeuour to occasion you not to repente your benefite on me thus imployed.

Mayster. That recompence is sufficient for your parte.

FINIS.

The Valuation of Englishe, Flemishe and Frenche mony, and how eche of them may easily be brought to other valew.

How briefly to reduce ℔, ß, and ₰ Flem. into ℔, ß, and ₰ Englishe, or Sterlinge.

IT is to be noted, that 7 ₰ Flemish maketh but 6 ₰ sterlinge, 7 ß flem. maketh 6 ß sterl. and 7 ₰ fl. 6 ₰ ster. So that 7 yeldeth but 6. Wherein is euident that there is losse $\frac{1}{7}$, (if it may be so called) when it is reduced into Englishe mony. Wherefore to knowe howe

much 233 l'i, 13 ſ, 4 d, fl. maketh engliſhe. you muſte ſubtract from it $\frac{1}{7}$, beginninge with the pounds, &c. and that which reſteth after this ſubtraction, is the ſumme required: ſo that 233 lb, 13 ſ, 4 d fl. maketh 200 lb, 5 ſ, 8 $\frac{4}{7}$ d. ſterlinge.

Example.

lb.	ſ.	d.
233	13	4
$\frac{1}{7}$ 33	7	7 $\frac{3}{7}$
200	5	8 $\frac{4}{7}$ ſter.

An other example

lb.	ſ.	d.
311	0	0
$\frac{1}{7}$ 144	8	6 $\frac{6}{7}$
266	11	5 $\frac{1}{7}$

To reduce lb. ſ. and d ſter. into lb. ſ. d. flem.

Note that a l'i, ſterling maketh 1 l'i, 3 ſ, 4 d, flem. that is 1 lb $\frac{1}{6}$: 1 ſ ſter. maketh 1 ſ $\frac{1}{6}$ flem. and 1 d. ſter. maketh 1 d $\frac{1}{6}$ fl. So that there is gained (if it may ſo be called) $\frac{1}{6}$ of the ſumme being thus reduced to fl. For of $\frac{6}{6}$ is made $\frac{7}{6}$, which is 1 whole and $\frac{1}{6}$. Then to know how much 237 lb, 7 ſ, 6 d ſter. maketh flem. ſubtract from your ſter. the $\frac{1}{6}$ of the whole ſumme, and adde it to the ſame ſumme, and it maketh 276 lb, 18 ſ, 4 d ſter. which is the ſumme required.

Example.

lb.	ſ.	d.	
237	7	6	
$\frac{1}{6}$ 39	11	3	
276	18	9	ſterl.

An other example.

lb.	ſ.	d.
337		
$\frac{1}{6}$ 56	3	4
393	3	4

Ye

Ye shall note, that the equality of Flemishe and Frenche mony is this, that is to say: the l'i, flem. maketh 7 l'i $\frac{1}{5}$ frendx or tournois: 1 ß, fle. maketh 7 ß $\frac{1}{5}$ frendx, and a grote flem. maketh 7 d $\frac{1}{5}$ frenche.

Wherefore to knowe howe muche 143 l'i, 4 ß, 9 d flem. maketh fr. yee must multiplye the whole number twise by 6, beginning at d. and so forwarde: and the produce of youre seconde multiplication, diuide by 5, so that worke is finished. Or multiply the sayde summe by 7, and take out of it $\frac{1}{5}$ adding it to ỹ produce of your multiplication by 7, and that is youre numbre required. So that as well by the one as by the other, 143 l'i, 4 ß, 9 d, flemishe maketh 1031 l'i, 6 ß, 2 d $\frac{2}{5}$ frendx, or tournois.

Exaumple.				An other exaumple.		
lb.	ß.	d.		lb.	ß.	d.
143	4	9	flem.	143	4	9
		6				7
859	8	6		1002	13	3
		6		$\frac{1}{5}$ 28	12	11 $\frac{2}{5}$
$\frac{1}{5}$ 5156	11	0		1031	6	2 $\frac{2}{5}$
1031	6	2 $\frac{2}{5}$	fre.			

An other exaumple.	or thus.
143 ℔. flem.	143.
6	7
858.	1001.
6	$\frac{1}{5}$— 28. .12.
$\frac{1}{5}$— 5148	1029 ℔. 12 fr.
$\frac{1}{5}$— 1029 ℔ $\frac{1}{5}$, oz 12 ß fre.	

A breife Reduction of ℔. ß. and ȝ. frenche into ℔. ß. and ȝ. flemiſhe.

Multiplply 233 l'i, 8 ſ, 4 ȝ, fr. by 5, and diuide the product twiſe by 6, that is the ſayde number by 6, and the product agayne by 6: and the quotient of this ſeconde diuiſion is the thinge required. So that 233 l'i, 8 ſ, 4 ȝ, fren. maketh 32 l'i, 8 ſ, 4 ȝ, $\frac{5}{6}$ flemiſhe.

Exaumple.				An other.			
℔.	ß.	ȝ.		℔.	ß.	ȝ.	
$\frac{1}{6}$— 233	8	4	fren.	753			fren.
		5		5			
$\frac{1}{6}$— 1167	1	8		3765.			
$\frac{1}{6}$— 194	10	3	$\frac{1}{3}$	$\frac{1}{6}$— 627.	10.		
$\frac{1}{6}$— 32	8	4	$\frac{10}{18}$, oz $\frac{5}{6}$	$\frac{1}{6}$— 104.	11.	8.	

To

To reduce ℔. ß. and ð. Sterlinge, into ℔. ß. & ð. Frenche or Tournois.

The ℔ ster. maketh 8 ℔, 8 ß frenche. that is to say, 8 ℔ $\frac{2}{5}$: the ß maketh 8 ß $\frac{2}{5}$, and the peny 8 ð $\frac{2}{5}$ frenche. Wherfore to knowe what 231 ℔, 13 ß, 4 ð ster. maketh fren. ye muste multiply your whole summe by 42, that is by 7, and ye product of it by 6, and diuide this second product by 5, and that is the summe required. Otherwise multiply the summe ster. by 8, and adde twise to the product $\frac{2}{5}$, and it shall produce the summe required. So that bothe wayes, 231 ℔, 13 ß, 4 ð ster. maketh 1946 ℔ frenche. As here vnder foloweth.

the same otherwise.

℔.	ß.	ð.		℔.	ß.	ð.	
231	13	4	sterl.	231	13	4	ster.
		6				8	
1390	0	0		1853	6	8	
		7		46	6	8	
$\frac{2}{5}$ - 9730	0	0		46	6	8	
$\frac{1}{5}$ - 1946	0	0	fren.	1946	0	0	fr.

An other example.				The same.			
753			ster.	753			
6				8			
4518				6024			
7				1/3	150	12	
31626				1/3	150	12	
1/3	6325	4	fr.		6325	4	fren.

To reduce ℔, ß, and ɗ fr. into ℔, ß, and ɗ ster.

To know how much 1256 ℔, 12 ß, 6 ɗ fr. maketh in sterlinge mony, multiply the sume by 5, and diuide the product by 7 ℣ 6 at twise, and the laste quotient shalbe the thinge required, that is to say, 1256 ℔, 12 ß, 6 ɗ, maketh 149 ℔, 11 ß, 11 ɗ 4/7 sterlinge.

Example.				An other example.			
℔.	ß.	ɗ.		℔.	ß.	ɗ.	
1256	12	6	ster.	2531			fren.
		5		5			
6283	2	6		12655			
1/6 1047	3	9		1/6 2109	3	4	
1/7 149	11	11 4/7		1/7 301	6	2 2/7	

Note that when any mony is giuen by exchange at London for Roane, at 71 ɗ 1/2: or rather 71 3/7, for the crowne of 50 ß frenche. There is

is neyther gayne nor losse, for it is one mony for an other, accomptinge 8 ℔, 8 ß fren. for 1 ℔ sterling. So the gyuer loseth the ryme of payment, whiche is aboute 15 dayes, & he that taketh it, hath the gaine of the same.

They of Roan, that put forthe or take mony by exchange for London, ought to haue like consideration.

Item, when any man giueth at Lõdon 64 õ $\frac{1}{4}$, or rather 64 õ $\frac{2}{7}$, to haue at one of ẏ fayres of Lyons a crowne de marc. he that so gyueth his mony, loseth the time, and he that taketh it gayneth the same: for 62 õ $\frac{2}{7}$ is equall in value to 45 ß frenche. He that putteth or taketh mony at Lyons for London, ought to consider the same. A crowne de marke, is 45 ß. fr. or torn.

Item, when any deliuer in Andwerpe 75 õ, to receaue at Lyons à crowne of marcke, hee that putteth it forth loseth the time, and he that taketh it, gayneth the same. For 75 grotes fl. is equall in valew to 45 ß frenche.

Thus I make an ende of the practise for exchange, and instruction thereto belonginge, mindinge to shewe the gentle Reader sondry other necessary practises for weight and measures of sundry Contries and townes.

The equalitye, or valuation, weightes and measures of sundry Countries and Townes, as foloweth.

ANDVVARPE.

An hundreth elles Andwarpe measure, maketh 60 elles of London measure cõmõly vsed in measuring of linnẽ clothe, & such like, & for the measure of all other things commonly sente, 75 Uerges or yardes of Seuille, or other places of Spayne 81 Uarres, in Lisbone 60, in the Isle of Madera 62, in Lions 60 elles, in Paris 57, at Roan 52, at Mylan 138 braces, at Genes 288½ palmes, at Uenis for wollen and linnen cloth 108 braces, at Florens 122½ braces, at Luke 120, at Nuremberg 104 elles, at Franckforte, Leypzig and Prellawe 125 elles, at Danlicke 83, at Uienna in Austria 87.

It is to be noted, that all silkes be bought at Andwerpe by the Bruges elle, whiche is greater then the common measure, by whiche they retayle, by 2 in the C.

Item 100 li. at Andwerpe, maketh at London 104, at Franckforte 92 li, at Collen and Augsburg 95, at Nuremberg 93, at Roan 91, at Lyons betwixte 110 and 111, at Paris 95, at Dieppe 93, some holde 90, at Amsterdã 95, at Geneua 84, at Toulouse 114, at Rochell 116,

at

at Merſeille 115, at Seuille and other places of Spayne 102, at Uenis, ala ſotilla, or ſmall wayght 155, by groſſe wayghte 98, at Iquila in Abruzzo 146, at Uienna 83, at Prellau 125, at Leypzig 94, at Danſicke 120, at Lubec, 98½, at Barcellone 133⅓, at Liſbone, ſmalle wayght 92, great wayght 86, at Gennes 146.

At Andwarpe, Golde and Siluer is weyghed by the Mark, the marke is 8 vnces, the vnce 20 eſtrelins, and the eſtrelin 32, as our graynes, ℈ goldeſmithes diuiſe that into ſmaller weyght, but not the Marchantes. The pꝛoofe of Golde is made by Karackes, whereof 24 maketh a marke of fine golde, the karacke is 24 graynes, the pꝛoofe of the money is made by deniers, whereof 12 ₰, is 1 ℔ fine, that is a marke of fine ſiluer, the ₰ is alſo diuided into 24 graynes, and the grayne into 4 quarters.

Item 100 markes in Andwarpe Troy weyght, maketh at Lyons 103 markes, 2 vnces, and 20 graynes 23 p̈. at Nuꝛemberge 103 markes 2½ vnces, 2 quints, 3 ₰, at Francfoꝛt 105 marks, at Augſburg 104 marks, 3 vnc. 1 quint, at Uenis 103 markes, 1 vnce, 7 ℥, 18 graynes, at Bourgos in Spayne 116⅔, at London 66lb.

The Marke of ſiluer oꝛ golde at Andwerpe Troy weight, whiche is 8 vnces, maketh 7½ vnces common weight, with whiche all other marchaundiſe

marchaundise is weyghed, so that the Troy weyght is greater then the common by 6 ½ in the C. By this weyght of Troy, they weyghe Muske, Amber and Pearle. &c.

LONDON.

OF all kinde of marchandise solde by weight at London, they sell or bye alwayes 112 ℔. for the Hundred, for half an hund. 56 ℔. and so after the rate: but in bying or selling by the ℔, they giue but 1 ℔. for all to witte 16 vnces.

Golde and Siluer is soulde by the ℔, contayning 12 vnces, whiche weyghe more then ỳ common vnces: for 100 markes, at 8 vnces to the marke, maketh at Lions or Andwerpe 101, at Noremberge 103 markes.

A hundred ℔. common weyght at London, maketh at Andwerpe 96: at Lyons 106: at Roan 90: at Millan 142. And of silke weyght 154, at Vincense silke weight 130, at Marseille 116 ½, at Francfort and Noremberge 89, at Rochell and Marseille 112 or 114, at Paris 91, at Venise ala Sotilla 151, at Castill and other places in Spayne 102: moreouer 96 ℔. Saffron weyghte, maketh at Francfort and Nuremberge 100 ℔. At London is vsed two sortes of measures, to wytte the elle and the yarde, by

the

the elle is vsuallye measured linnen clothe, as Canuas, &c. and by the yarde, silkes, woollen clothes. &c. Item 3 elles Englishe, maketh at Andwarpe, and almoste ouer all Flaunders, Hollande and at Maklyne 5 elles, or 60 elles maketh 100 elles. Item 100 yardes of London, maketh at Andwerpe 133 $\frac{1}{3}$, at Nuremberge 142, at Francford 166 $\frac{2}{3}$ elles, at Uenise 133 $\frac{1}{3}$ braces, at Millan for clothe 133, at Uincens for cloth as muche: at Florens 162 braces, at Lukes 160 braces, at Gennes 384 pal. $\frac{2}{3}$, at Roan 83 elles, at Paris 76, at Lions 80, at Rome 44 $\frac{1}{2}$, Cannes in Castille, at Siuille, and sundry townes in Spayne 108 Uarres.

LIONS.

AT Lions is vsed 3 sortes of weyghtes, whereof the firste is the common Towne weighte, with which they weghe all kinde of Spycery, and diuers other marchaundise. The second is called Geneua weyghte, which is 8 in the 100 greater then the cõmon weight wt which they weyghe silkes, &c. The thirde is Frenche weyght, called commonly the mark weyght, and 100 li. thereof maketh 106 $\frac{1}{4}$ li. Geneua, and 114 $\frac{3}{4}$ common weyght, with whiche Frenche weyght is wayed all thinges that payeth Custum or tolle.

Item

Item 100 li. common, maketh at Marseille 106 li. at Andwerpe 90, at Roan 82 ½, at Luke weyght de casse 126 li at Paris 86 lb.

Item 108 weyghte of Geneua, maketh at Uenis ala sotilla 148, at Gennes smale weighte 133 ⅓, In Calabria silke weyght 126 ½, 100 li. common of Lions weyghte, maketh at Uenis 141, at Brugis 89, at London 95, at Franckfort, Nuremberge and Basill 85 lb.

At Lions is also 2 sortes of elles or aulnes, the one wherewith they commonly measure grosse clothes, as canuas and suche lyke, the other is called the frenche elle or aulne, wt whiche they measure all other kinde of marchaundise, whereof 7 common towne elles, maketh 11 ordinary Frenche elles.

Item 100 ordinary elles of Lyons, maketh at Andwarpe 164, at London 123 yardes, at Roan as well for clothe as silke 104, at Nuremberge 84 ⅔, at Franckforte 101 ⅔, at Augsburge as muche, at Gennes 472 ½ palmes, at Florence 120 braces, at Luke as muche, or one or two more.

NVREMBERGE.

AN hundred elles of Nuremberge, make at Andwarpe 95 ½, at London 71 ½, at Roan 50 alnes, at Paris 54 ½, at Lyons,

Franckforte

Francfoꝛte and Augſburge 108 ½, at Pꝛeſleau and at Pꝛaga in Bohemia 116 ⅔, at Gennes 266 ⅔ palmes, at Florence 115 ½ braces, at Luke 117, at Rome 32 cannes.

Item 100 ℔. of Nuremberge maketh at Andwarpe 108, at London 112, at Roan 98, at Paris 102, at Lyons 120, at Gennes 158, at Aquill in Abruzo 142, at Uenis of ỹ Sotille 168, at Millane 155, at Pꝛaga 115, at Leyplig 111, at Francfoꝛte and Baſell 100, at Geneua greate weyght 90, at Augſburge 104, at Toulouſe 123, at Rochell and Marſeille 125, at Uienna in Auſtria 91, at Liſbone 100 ℔.

VENIS.

AT Uenis be 2 ſoꝛtes of weightes, the one is called La groſſe, the other la Sotille,

With theſe greate is weyed all kinde of greate wares, and with the ſmall all kinde of ſpicery, with ſuch like. 100 ℔ of ſmall weight maketh at Frankfoꝛt 59 ℔, at Nurembꝛge 59, oꝛ 60, at Lyons 70, at Marſeille 72, at Paris 60, at Rochell 74, at Roan 58, the 100 ℔ of ſpicery without tare oꝛ allowance is at London 64, and with tare 56, and 94 à la groſſe maketh 100 at London.

Item, 100 ℔ of ſpicerie à la Sotille tared, maketh at Andweꝛpe 54, without tare 63 ℔.

Florence

OF Florens measure 100 braces maketh at Andwarpe, Bruges measure 81 elles $\frac{2}{3}$, at London 61 yardes, at Nuremberge 85 elles $\frac{5}{7}$. 19 braces of Florens maketh at Andwerpe 15 $\frac{1}{2}$ elles, and 7 braces 6 elles at Nuremberge.

Item 113 $\frac{1}{3}$ ℔ Florens, make in Spayne, to witte at Valentia, Calis, and other places, 100 ℔.

Item 100 ℔ silke weight of Genua, maketh at Florence tared 127 ℔. The quintall or C. weight of Canary, to witte 4 Rowes maketh at Florence 133 ℔ $\frac{1}{3}$, and 1 Rowe 33 $\frac{1}{3}$ ℔. 100 ℔ of Towloule maketh at Florence 125 ℔, and ỹ charge of Towloule 375 ℔ of Florẽce.

Item 116 of Florence maketh at Marseille 100 ℔, and 145 of Florence maketh at Roan 100 ℔. Item 100 of Florence is at Aquilla for saffron 110 ℔. The weight of Florence and that of Luke, is all one.

GENNES, OR GENVA.

ITem 100 palmes of Gennes make of Andwerpe 34 $\frac{2}{3}$ elles, at London 26 yardes, at Franckforde 33 $\frac{1}{3}$ elles, at Ulmes 40 $\frac{2}{3}$, at Nuremberge 36, at Lyons 21, at Paris 19 $\frac{3}{4}$, at Roan 18 $\frac{1}{4}$ elles.

Luke.

The Luke Sattins are commonly soulde at Lyons by waight, and 133 ½ ℔ Lukes maketh at Lyons 100 ℔, so that 1 ℔ ⅓ maketh at Lyons 1 ℔. The 100 Lukes braces maketh in Andwarpe 83 ⅓ elles, at London 62 ½ yardes, at Nuremberge 86 ⅔, at Franckfort 104 elles.

AQVILA.

An hundreth ℔ of Saffron of Aquila in Abruzo maketh at Frãckfort 66 ½, at Andwerpe 72, at Nuremberge 66 ½. Item 136 ½ ℔ saffron of Aquila, maketh at Geneua 100 ℔, and 11 ℔ of Geneua, is 15 ℔ of Aquilla.

LISBONE.

At Lisbone their silke, cloth of golde, and wollen, is measured w̄ à measure whiche they call à Cubet, containinge aboute ¾ of a Uarre of Castill. The linnine clothe as well fine as course, soulde by measure called a Uarra, whiche maketh 5 palmes, and containeth neere 1 ¼ of a barre of Castille. ꝑ C. weight of Lisbone make 4 Rowes, euery Roue 32 li. so that the C. weight is 128 li. And the pũde cõtaineth 14 vnces of the same weight. Item 100 barres of Lisbone maketh at Andwerpe

166 ½ alnes. The elle of London commonly vsed for canuas, and such like, is all one with the barre of Lisbone. All kinde of marchandise brought from Flaunders, Roan, or Brytaigne, payeth at Lisbone as à dewty or custome to the kinge 20 in the hundreth, that is to say 10 in the hundreth, which they call the tenthe in marchandise, and the other tenthe in mony. Note that all kinde of marchandise comming to Lisbone by lande, payeth lesse in custome then that which commeth by water. The hundreth small weight of Lisbone, maketh at Andwerpe 108 li, at Lyons betwext 118 li, and 120, at Rochell betwext 112, and 114, at London 112 ½ ℔.

SEVILLE.

THe Robe of Seuille is 30 ℔. 4 Robes maketh an C. waight, and 100 ℔ maketh at Andwerpe 98 ℔. The 100 barres of Seuille, maketh at Andwarpe 119 ½ elles. At Castill the C. waight cõtaineth 4 Robes, the Robe 25 ℔. So the C. weight containeth 100 ℔, & maketh at Andwarpe 98, at Lyons 108, at London 102, at Roan 89. All kindes of marchandise is measured by the barre, whereof 100 barres maketh at Lyons 73 elles, at Andwarpe 120, at Florence 159 braces, at Rome 40 Cannes.

Valencia.

AT Valencia bee 2 sortes of waightes, to say the greate and small. The C. waight of the greate contayneth 4 Robes, the Robe 36 ℔, so ỹ 100 great waight is 144 ℔. The C. waight small is also 4 Roues, but ỹ Roue containeth but 30 ℔, so that the hũdreth waight small is 120 ℔. By this small is soulde the scarlet grayne, suger, with all kindes of spicerie. By the greate waight is soulde wolle, with such like grosse wares. The 1 ½ ℔ of silke of Valencia, maketh at Lyons 1 ℔ Geneua waight. The charge of greate marchandise at Valencia, is to be vnderstande 432 ℔, and in small wares 360 ℔. The waight of Valencia, and at Barcellone is all one.

ROAN.

AT Roan 6½ muyds of Salte beyng measure of that place, maketh an hundred at Irmuiden in Zelande, and ỹ C. of Brouage measure of that place, maketh at Roan 11 muides, 100 elles of Roan, maketh at Andwarpe 193 elles, 118 elles of Canuas of Roan, maketh at London by the corde 100 ℔, the corde is 12 elles, and 12 cordes is an hundred, 30 mynes maketh à laste of corne, and 16 a laste of otes.

At Fecane the myne of corne containeth 4 buſſhels, whiche is iuſte as muche as the barrell of Hearinge holdeth: at Diepe the myne containeth 4 ¾ buſſhels: whereof 4 filleth an hering barrell.

Item 100 ℔. Roan weight, maketh at Andwarpe 108 ℔, at London 113, at Nuremberge 120, at Geneua 92, at Lyons 120, at Baſell 120, at Vennis ala Sottila 170, at Franckfoyte 102.

DANSIC.

AT Danſik, a ſchippont maketh 320 ℔, and 20 Lyſpond 1 ſchippont, and 16 marke 1 Lyſponde Danſick weyght, whiche ſchippont maketh in Andwarpe 266 ⅔ ℔, and a 100 ℔. of Danſick, make at Andwarpe 83 ⅓, at London 86. And 22 ½ grotes maketh 20 ſtuuers Flemiſhe; 6 ß. maketh 1 ₣, 4 ₣. a grote, 15 grots a ſmale marke, and 20 grotes a great marke.

Item 1 Florin maketh at Danſick 30 grots of Holelande, and 33 of theſe grotes maketh a Daler of 30 ſtuuers Flemiſhe.

TOVLOVSE.

AT Toulouſe 6 cabas of woode maketh a charge. 2 Ceſtres of corne meaſure, maketh a charge

charge. The Cestre wayeth 160 ℔. waighte of that place, and 100 ℔, maketh at Andwarpe 68, at London 90.

FINIS.

Imprinted at London in Paules churche yarde, at the signe of the Brasen Serpent, by Reginalde Wolfe.

ANNO DOMINI M. D. LXXIII.

I. D. to the earneſt Arithmeticien.

My louing frende, to Science bent,
Something thou haſte by this boke wonne:
But if thou wilt be excellent,
An other race thou muſt yet runne.

Supplyes thereto (but fewe) do neede:
And none but ſuche, as in our phraſe,
(By Recordes pen) thou mayſt well reade:
Procede therefore: Be no ſtunt dwaſe.

The grounde moſte ſure, whereon this race
With ſpedefull courage muſte be paſte,
This yeare hath turned his Grekiſhe face,
By Engliſhe tylth, whiche aye will laſte.

The famous Greeke of Platos lore,
EVCLIDE I meane Geometer:
So true, ſo plaine, ſo fraught with ſtore,
(As in our ſpeache) is yet no where.

A treaſure ſtrange, that boke will proue,
With Numbers ſkill, matcht in due ſorte,
This I thee warne of ſincere loue,
And to procede doe the exhorte.

Plus oultre.

Τῶ θεῶ δόξα

The Grounde of [illegible]

teachinge the worke and practise of
Arithmetike, bothe in whole numbers
and Fractions, after a more ea-
sier and exacter sorte then
lyke hathe hitherto been
forth: Made by
M. ROBERT RECORDE
Doctor in Phisick, and nowe
diligently ouerseene and augmen-
ted with newe and neces-
sary Additions.

L. D.

That which my frende hath well begonne,
For very loue to common Weale:
Neede not all whole to be newe donne,
But new encrease I do reuocale.

Something herin, I once redreste:
And now againe for thy behoofe,
Of zeale I do, and at requeste,
Bothe mend and adde, fyll for all proofe.

Of Numbers vse, the endles might,
No witte nor language can expresse,
Apply and Try, bothe day and nyght:
And than this truthe thou wilt confesse.

LONDINI,
Anno Domini. 1573.

62-1234

The Bookes Verdicte.

To pleaſe or diſpleaſe ſure I am,
But not of one ſorte to euery man.
To pleaſe the beſt ſorte woulde I fayne,
The frowarde diſpleaſe ſhall I certaine.
Yet wiſhe I will, though not with hope,
All eares and mouthes to pleaſe or ſtoppe.

To the moſt mighty PRINCE EDWARDE the ſixt, by the grace of God Kinge of Englande, Fraunce, and Irelande. &c.

The excellencye of mans nature, beynge ſuche, as it is by gods diuine fauoure (moſt mighty prince) not onely created in highneſſe of degree far aboue al other corporall thinges, but by perfection of reaſon and ſcarche of witte muche approching toward the image of God, as not onely the holy ſcriptures do teſtify, but alſo thoſe naturall Philoſophers, whiche greatly did conſider the nature of man, namelye the farre reache and infinite compaſſe of the workes of the minde, were inforced to confeſſe, that man ſcarſely was able to know himſelfe. And if he woulde duelye ponder the nature of himſelfe, he ſhoulde finde it ſo ſtrange,

that it might seme vnto hym a verye miracle. And thereof sprang that saying: Magnum miraculum est homo, maximum miraculum sapiens homo. For vndoubtedly as man is one of the greatest miracles that euer God wrought, so a wyse man is plainely the greatest.

And therefore was it, that some dyd accounte ỹ head of a man to be ỹ greatest miracle in the world, bicause not onely of ỹ strange workemanshyp that is in it, but muche more of the efficacy of reason, wit, memory, imagination, and suche other powers and woorkes of the minde, whiche can more easily conceaue any thing in a maner, than vnderstand it selfe. And amongst all the creatures of God it fyndeth none more difficulte to be perceyued then these same powers of it selfe, whereby it wthe conceaue and iudge, as it may be well coniectured by the diuersitye of opinions that the wisest Philosophers dyd vtter, touching the spirit of man and the substaunce of it: whereof at this present I intend to make no rehearsall, but who so listeth to reade thereof, maye finde it largely set forth not onely in Aristotle hys bookes De anima, but also in Galen hys booke called Historia Philosophica, and agayne in Plutarche his woorke, De Philosophorum placitis,

citis, whose wordes are also repeated of Eusebius in the 15 booke, τῆς εὐαγγελικῆς προπαρασκευῆς, vnto whom I remit them that haue desire to vnderstande the intricate difficultie of knowing our owne selues, as touching our beste parte, and that parte whereby we deserue to beare the name of menne.

This matter seemed so obscure and difficulte, in knowledge, that Galen, who for his excellent wisdome and iudgement in naturall workes, is called of manye men a Miracle in nature, yet in searching the nature and substaunce of the spirite of man, hee not onelye confesseth himselfe ignoraunte, but counteth plain temerity to attempt to finde it: so far aboue ỹ hope of mans knowledge is that parte wherby mã wth know & iudge of things. And although ỹ ignorant sort (which hate al things ỹ they know not) do litle esteme ỹ profoundnes of mãs spirit, & of reason, the chiefe power & facultye of it, yet as there is a kinde of feare & obedience of all vnreasonable beastes vnto man by the worlinge power of God, so is there in those small reasoned persons a certaine kinde of reuerence towarde wisedome and reason, whiche they doe shewe oftentimes, & by power of perswasion are inforced to obeye reason,

wyll they nill they.

And hereby came it to paſſe, that the rudeneſſe of the firſte age of man was brought vnto ſomme more ciuille trade, as it ys well declared by Cicero in the beginninge of his firſt boke, de inuentione Rhetorica, where hee ſaythe thus.

Nam fuit quoddam tempus quum in agris homines paſſim beſtiarum more vagabantur, & ſibi victu ferino vitam propagabant, nec ratione animi quicquam, ſed plæráque viribus corporis adminiſtrabãt. Non dum diuinæ religionis, non humani officij ratio colebatur. Nemo legitimas viderat nuptias, non certos quiſquam inſpexerat liberos, non ius æquabile quid vtilitatis haberet, acceperat: ita propter errorem atque inſcitiam cœca ac temeraria dominatrix animi cupiditas, ad ſe explẽdam viribus corporis abutebatur, perniciosiſſimis ſatellitibus. Quo tempore quidam, magnus videlicet vir & ſapiens, cognouit quæ materia eſſet, & quanta ad maximas res oportunitas in animis ineſſet hominum, ſi quis eam poſſet elicere, & præcipiendo meliorem reddere. Qui diſperſos homines in agris, & in tectis ſyluestribus abditos, ratione quadam compulit in vnum locum, & congregauit: & eos in vnamquanque rem inducens vtilem atque honeſtam primo propter inſolentiam reclamantes. Deinde propter rationem atque orationem ſtudioſius audientes, ex feris & immanibus mites reddidit, & manſuetos.

This long repeticion of Tullies wordes wil ſeme tedious to them whiche loueth but little, and

and careth muche lesse for the knowledge of reason, but vnto your Maiesty (I dare saye) it is a delectable remembraunce, and vnto me it semed so pleasaunt, that I could scarse stay my pen from wrytinge all that mine eyes did so greedely read.

This sentence of Cicero am I lothe to translate into English, partely for that vnto youre Maiestye it needeth no translation, but especially knowing how farr ỹ grace of Tullies eloquence doth excell any Englishe mans tongue, and muche more exceedeth the basenesse of my barbarous style, yet for the fruite of the sentence, I hadde rather vnto my mere English country men vtter the rudenes of my translation, then to defraude them the benefite of so good a lesson, trusting, they will so learne to loue reason, that they will also gladlye and greedelye imbrace all good sciences that maye helpe to the iuste furniture of the same, when they consider that infourmed reason was the onelye instrumente, or at leaste the chiefe meane to bringe men vnto ciuile regiment, from barbarous manners and beastlye conditions.

For the time was (sayth Tullie) that men wandred abroade in the fieldes vp and downe

like beastes, & vsed no better order in feedinge then they, so that by reasons rule they wrought nothing, but most of their doing dyd they atchieue by force of strengthe. At this time there was no iuste regard of religion toward God, nor of duety toward man. No man had sene right vse of mariage, nother did anye manne knowe their owne children from other, nor no man had felte the commodity of iust Lawes: so that through errour and ignoraunce, wilfull lust, lyke a blynde and heady ruler, abused bodyly strengthe as a moste mortall minister for the satisfyinge of his desire. At that time was there one, whiche not onely in power, but also in wysedome was greate, And he considered how that in the myndes of men was bothe apt instrumentes, and great occasion to the due accomplishement of moste weightye affayres, if a man coulde applye them to vse, and by teaching of rules, frame them to better trade. This manne with persuasion of reason, gathered into one place the people that weare wanderinge about the feildes, and laye lurkinge in wylde cotages and woodes: And brynginge them into one common societye, dyd trade them to all suche thinges, as ether weare profitable or honest, although not without

out repining at the firſt, by reſon that they had not been ſo accuſtomed before. Yet at length through reaſon & perſwaſion of wordes they obeied him more diligently, and ſo of a wilde and cruell people, hee made them curteous and gentle.

Thus hathe Tullye ſet foorthe the efficacie of reaſon and perſwaſion, howe it was able to conuerte wylde people to a mylnes, and to chaunge their furious cruelnes into gentle curteſye: were it not now a great reproche in thys oure time (when knowledge raygneth ſo large) that men ſhoulde ſhewe them ſelfe leſſe obſequious to reaſon? Unleſſe it maye bee thoughte, that nowe euerye manne hauing ſufficient knowledge of him ſelfe, needeth not to hearken to the perſwaſion of others.

In dede he that thinketh himſelfe wiſe, will not eſteme the reaſon of any other, be he neuer ſo wiſe, ſo that of ſuch one it may wel be ſaid: Hee that thinketh himſelfe wyſer then hee is, may iuſtly be counted a double fole: wherfore ſuch men are not to be permitted in open audience to talke, but muſt be put to ſilence, and made to giue eare to reaſon, which reaſon conſiſteth not in a multitude of woordes, heaped

rashelye togither, and applyed for one purpose, but reason is therpressinge of a iuste matter w̄ witty persuasions, furnished w̄ learned knowledge: suche knowledge had Moyses, beynge experte in all learninge of the Egyptians, as the Scriptures declare, and therefore was able to perswade the stubborne people of the Iewes, although not without greate payne. Suche knowledge and suche reasos did Druys shewe, whiche was the firste lawemaker of all the west parte of Europe. Lyke reason & wisedome did Xamolxis vse amongest the Gothes: Lycurgus vnto the Lacedemonians, Zaleucus to the Locrians, Solon to the Athenienses, and Dumwallo Molmitius 2000 yeres paste, amongeste the olde Britaines of this Realme. And thereby came it to passe, that theire lawes continued longe, till more perfect reason altered manye of them, and willfull power oppressed moste of them.

Druys was sone to Kinge Sarron & succeded him in his kingdome.

At the beginninge when these wise men perceiued how harde it was to bringe the rude people to vnderstande reason, they iudged the beste meanes to attayne this honeste purpose to vpende of learninge in euery kynde, for by learning, as Ouide sayeth: *Pectora mollescunt, asperitásque fugit*: Stoute stomackes we waxe mylde,

mylde, and sharpe fiersenes is exiled. Therfore as Berosus wythe testify, Sarron, that was ỳ thirde Kinge ouer all this weste parte of Europe, for to bringe the people from beastly rage to manly reason, did erecte Scholes of liberal artes, whiche toke so good successe, ỳ his name continued in that sorte famous aboue 2000 yeares after: for Diodorus Siculus, whiche was in the time of Julius Cesar, maketh mẽtion of the learned men of the Celtes, and nameth them Sarronides, that is to say, Sarron his Scholers and folowers.

Sarron died before Christes byrthe, 1913 yeares, after he had raigned 48 yeares.

Amonge these artes ỳ thẽ were taught, some did informe the tongue, and make men able bothe to vtter aptely their minde, and also to perswade, as Grammar, Logyke, and Rhetorike, although not so curiously as in this time: some other did appertaine to the iuste order of partition of Landes, the trew vsinge of weyghtes, measures, and reckeninges in all sortes of bargaynes, and for order of buyldinge and sundry other vses, those were Arithmetike and Geometry. Agayne to incourage men to the honoure of God, they taught Astronomye, whereby the wonderfull workes of God were so manifestly set forthe, that no mans tongue nor penne can in lyke sorte expresse his infinite

power,

power, his vnspeakable wysedome, & his exceedinge goodnesse towarde man, wereby he doth bountifully prouide for mã all necessaries, not onely to lyue, but also to lyue pleasantly. And so was theire confidence in Gods prouidence strongly stayed, knowinge his goodnes to bee such, that hee woulde helpe man as hee coulde, and his power to bee so greate, that hee coulde do what he would: and thirdely his wisedome to be so pure, that he woulde doe nothinge, but that that was best. Besides these scyences they taught also Musike, which moste commonlye they didde applye partelye to Religious Seruices, to drawe men to delighte therein, and partely to Songes made of the manners of men, in prayse of Uertue, and discommendation of Vice, whereby it came to passe, that no man woulde displease them, nor doe anye thinge euill that might come to theyr hearing: for theyr songes did make euill men more abhorred in that tyme, then anye excommunication dothe in this tyme. The posterity of these Musitians continewe yet bothe in Wales and Irelande, called Bardes vnto this daye, by the aunciente name of Bardus, theyr fyrste founder.

This Bardus Druid.

And as these sciences did increase, so dyd vertue

vertue increase thereby. Againe as these sciences did decay, so vertue loste her estimation, and consequently was little in vse: wherof to make a full declaration were a thing mete for a Prince to heare, but it would require a peculiare treatise. Wherefore at thys present I count it sufficient lightly to haue touched this matter in generall woordes, and to saye no more of the particularities thereof, but onelye touching one of those sciences, that is Arithmetike, by whiche not onely iuste partition of landes was made, but also touching buyinge & selling, all Assises, weightes, and measures were diuised, and all reckninges and accountes driuen, yea by proportion of it were the true orders of Justice limitted, as Aristotle in his Ethikes dothe declare, and the degrees of estates in the common wealthe established. Althoughe that proportion bee called Geometricall, and not Arithmeticall, yet dooth that proportion appertayne to the arte of Arithmeticke: and in Arithmetike is taughte the Progression of suche proportions, and all thinges thereto belonging. Wherefore I may well saye, that seing Arithmetike is so manye waies needefull vnto the firste plantyng of a common wealth, it muste needes be as muche

dius the fifth king of the Celtes reigned 69 yeares and dyed 1832 yeares before Christ.

required

required to the preseruation of it also, for by the same meanes is any common wealthe continued, by whiche it was erected and established. And if I shall in smale matters in appearance, but in verie very weighty, put one exaumple or two, what shall wee saye for the Statutes of this Realme, whiche bee the onely staye of good order in manner nowe? As touching the measuring of grounde by lengthe and bredthe, ther is a good and an auncient Statute made by arte of Arithmetike, and nowe it shalbe to litle vse, if by the same art it be not practised and tryed: for the assise of breade and drynke, the two most common and most necessary thinges for sustentation of man, there was a goodly ordinaunce in the lawe made, which by ignoraunce hath so growen out of knowledge and vse, that fewe men do vnderstande it, and therfore the Statute bokes wonderfully corrupted, and the commons cruelly oppressed: notwithstandinge some men haue written, that it is to doubtfull a matter to execute those Assises by those Statutes, by reason they depende of the standerde of the coyne, whiche is much chaunged from the state of that tyme whan those Statutes were made. Thus shall euerye man rede (that liketh) in the Abridgement of Statutes,

tutes, in the tytle of Weightes and measures, in the seuenth numbre of the Englyshe booke, where he shoulde haue translated a good ordinaunce, whiche is set foorthe in the Frenche booke: but no meruayle if the Abridgemente dothe omitte it, seyng the greate boke of Statutes doth omitte the same Statute, as it hath done diuers other very good Lawes. And this is the fruict of ignoraunce, to reiecte and condemne all that it vnderstandeth not, although they vse some clokes for it: but suche clokes as beynge allowed, might serue to repell all good Lawes, whiche God forbyd.

Againe there is an auncient ordre for Assise of fyrewood, and coales, whiche was renued not many yeares paste, and nowe how Auarice and Ignorance doth canuas that statute, it is to pitifull to talke of, and more myserable to feele.

Furthermore for the statute of coynage, and the standerde thereof, yf the people vnderstode rightly the statute, they should not nor would not (as they often do) gather an excuse for theire folly thereby: but as I saide, these statutes by wisdome and good knowledge of Arithmeticke were made, and by the same must they be continued. And let ignoraũce no more

meddle

meddle with the vse of them, then it did with the makinge of them. Oh in howe miserable case is that Realme, where the ministers and interpreters of the Lawes are destitute of all good sciences, which be ỹ keyes of the Lawes. Howe can they eyther make good Lawes or mayntayne them, that lacke that trew knowledge whereby to iudge them. And happy may that Realme be accompted, where the Prince himselfe is studious of learninge, and desireth to vnderstande equitie in all Lawes. Therfore moste happy are wee the louinge subiectes of youre maiestie, whiche may see in youre highnes not only suche towardnesse, but also suche knowledge of diuers artes, as seldome hathe ben seene in any Prince of suche yeares, whereby wee are inforced to conceaue this hope: Certaynly, that hee whiche in those yeares seketh knowledge when knowledge is leaste esteemed, and at suche an age can discerne theim to bee enemyes bothe to his royall person and to his Realmes, whiche labourte to withdrawe him from knowledge to excessiue pastime, and from reasonable study to idle or noysom pleasures, he must nedes when he commeth to more mature yeares bee a moste prudente Prince, a moste iuste Gouernoure, and a righte Iudge,

not

not onely of his subiectes commonly, but also of the ministers of hys lawes, yea and of the lawes them selfe. And to be able to conceaue the true equity and exact vnderstanding of all his lawes and statutes, to the comforte of his good subiectes, and the confusion and reproche of them whiche labour to obscure or peruerte the equitye of the same lawes and statutes. How some of those statutes maye be applyed to vse as well in this our time, as in anye other time, I haue peculiarely declared in this booke, and some other I haue omitted for iust considerations till I may offer them firste vnto your Maiestie, to waye them, as to your highnes shall seeme good: for manye thinges in them are not to be published without your highnes knowledge and approbation, namely bicause in them is declared all the rates of aloies for all standerds from one vnce vpward, with other misteries of mynte matters, and also most part of ỹ varieties of coynes ỹ haue ben currant in this your Maiesties realme, by the space almoste of vi. hundred yeares laste paste, and many of them that were currant in the time that the Romaines ruled here.

All whiche with the aunciente discription of England and Ireland, and my simple cen-

sure of the same, I haue almoste completed to be exhibited to your highnes. In the meane season moste humblye beseeching youre Maiestye, to accept this simple treatise, not worthie to be presented to so highe a Prince, but that my lowely requeste to your Maiestye is, that this amongest other of my bookes maye passe vnder the protection of youre highnesse, whom I beseche God most earnestlye, and daily, accordinge to my duety, to aduaunce in all honour and Princely regality, and to increase in all knowledge, iustice, & godly pollicy.

AMEN.

Your Maiesties moste obedient subiecte and seruaunt, Robert Recorde.

TO THE LOVINGE READERS.

The Preface of M. Robert Recorde.

ORE OFTEN-times haue I lamē-ted wyth my self the infortunate condition of England, seing so manye great Clerkes to aryse in sundrye other partes of the world, and so fewe to appeare in this our nation: where as for pregnancy of naturall wit (I thinke) few nations do excell English men. But I cannot impute the cause to any other thing, than to the contempt or misregard of learning. For as Englishe men are inferioure to no men in mother witte, so they passe all men in vaine pleasures to whiche they maye attaine wythoute greate paine or laboure: and are as slacke to anye neuer so greate commoditye, if there hange of it anye painefull study or trauelsome laboure.

Howbeit, yet all men are not of that sorte, though

though the moste parte be, the more pitye it is: but of them that are so glad not onely wyth paineſull ſtudy and ſtudious payne to attaine learninge, but alſo with as great ſtudye and paine to communicate their learning to other, and make all England (if it mought be) partakers of the ſame, the moſte parte are ſuche, that unneth they can ſuppoꝛte their owne neceſſarye charges, ſo that they are not able to beare any charges in doing of that good, that els they deſire to do.

But a greater cauſe of lamentation is this: that when learned menne haue taken paynes to doe thinges foꝛ the ayde of the unlearned, ſcarſe they ſhall be allowed foꝛ their well doing, but deryded and ſcoꝛned, and ſo utterly diſcouraged to take in hand anye like enterpꝛice againe. So that if any be founde (as there are ſome) that doth fauour learning and learned wittes, and can bee contente to further knowledge, yea onely with their woꝛde, fewe perſons, though they be rare, yet than they encourage learned menne to enterpꝛiſe ſome thinges, at the leaſte, that England may reioyce of. And I haue good hope that England will (after ſhee hathe taken ſome fuell taſte of learning) not onely bꝛinge foꝛthe moꝛe fauou-